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Adaptive Volterra series model for nonlinear sensor compensation

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Abstract: A novel performance enhancement method of nonlinear sensor based on the Volterra series model is proposed. The Volterra series model, which is considered a nonlinear filter that can reduce sensor noise, presents an effective way for modeling and compensating a nonlinear sensor. In the experiment, the low accuracy pressure sensor MPX10 was used as the actual object, and higher accuracy sensor MPX2010 was used as the reference to provide the necessary teaching data for training the Volterra model. The simulation shows that the accuracy of MPX10 changes from $0.354-0.42$ to $0.041-0.053$ after the Volterra filter has been applied. Obviously this scheme can effectively improve the sensor performance. Moreover, the scheme provides greater accuracy and environmental suitability for a nonlinear sensor.

Key words: sensor; Volterra model; non-linearity

In modern applications, greater demands are made on sensor performances. Unfortunately, sensors inherently suffer from imperfections, such as non-linearity, cross-sensitivity, crosstalk, noise, etc. Improving sensor performance via better materials, new technologies and advanced design is generally expensive and limited. Another solution is the so-called "soft or intelligent" sensor design. Monitored correction (sensor-within-sensor) and tailored correction (using multi-dimensional look-up-tables for each sensor) are the main conventional solutions. However, there are several major drawbacks to these methods[1]. Another widely used approach is to use compensating algorithms. In this approach, the sensor output is modeled using polynomial fitting techniques. The memory requirements are dramatically reduced because only the polynomial coefficients need to be stored. It has been found that such an approach is incapable of producing the required performance accuracy even when using high order polynomials. In an attempt to find a compromise between the memory requirements of the look-up table approach and poor accuracy for the polynomial fitting method[1,2]. This paper presents a novel approach for intelligent sensor design based on the Volterra series model.

1 Basics of the Volterra Series Model

The Volterra series model is an exact mathematical approach for description of causal time-invariant systems, where dynamic and nonlinear phenomena are presented simultaneously. According to this model, the output signal of the nonlinear system or sensor can be expressed as a series of Volterra functions, i.e., by means of a series of multi-dimensional convolution integrals[2,3].

$$\hat{y}(t) = h_0 + \sum_{i=1}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_i(\tau_1, \tau_2, \cdots, \tau_i)\times \prod_{j=1}^{i} x(t-\tau_j)\prod_{j=1}^{i}\mathrm{d}\tau_j\right] \tag{1}$$

where $x(t)$ is the input signal, $\hat{y}(t)$ is the output signal, and $h_i(\tau_1, \tau_2, \cdots, \tau_i)$ is the Volterra kernel of the i-th order.

The discrete equivalent of the general formula is[3]

$$\hat{y}(n) = h_0 + \sum_{i=1}^{\infty}\sum_{m_1}^{\infty}\sum_{m_2}^{\infty}\cdots\sum_{m_i}^{\infty} h_{i_{m_1, m_2, \cdots, m_i}} \times \prod_{j=1}^{i} x(n-m_j) \tag{2}$$

where $x(n)$ is the discrete input signal, $\hat{y}(n)$ is the output sequence, and $h_{i\, m_1, m_2, \cdots, m_i}$ is the element of the i-th order Volterra kernel. Eq.(2) is the mathematical model of a Volterra series of infinite orders. In particular, the truncated model with a finite order M, and a finite memory of samples $N+1$ is considered.

$$\hat{y}(n) = h_0 + \sum_{i=1}^{M}\sum_{m_1}^{N}\sum_{m_2}^{N}\cdots\sum_{m_i}^{N} h_{i_{m_1, m_2, \cdots, m_i}} \times \prod_{j=1}^{i} x(n-m_j) \tag{3}$$

It is sufficient for the purpose here to consider the second-order Volterra model with finite memory N in Eq.(3).

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$$\hat{y}(n) = h_0 + \sum_{i=0}^{N} h_1(i)x(n-i) + \sum_{j=0}^{N}\sum_{k=0}^{N} h_2(j, k)x(n-j)x(n-k) \quad (4)$$

The optimization problem is to minimize the following cost function:

$$J = \sum_{i=1}^{n} \lambda^{n-i} |e(i)|^2 \quad (5)$$

where $e(n)$ is defined as

$$e(n) = \hat{y}(n) - y(n) \quad (6)$$

And, the forgetting factor λ chosen here has the following constraint:

$$0 < \lambda \leqslant 1 \quad (7)$$

The forgetting factor λ ensures that the data in the distant past are forgotten, in order to allow the possibility of tracking statistical variations in the observable data, especially when the signal to be modeled in nonstationary.

Using the steepest decent rule, the Volterra kernel neurons are updated by Eqs.(8) to (14)[4-6].

$$h_0(n+1) = h_0(n) + \mu \frac{\mathrm{d}h_0(n)}{\mathrm{d}t} \quad (8)$$

where μ is the step size parameter. Simplifying, the gradient is updated using

$$\frac{\mathrm{d}h_0}{\mathrm{d}t}(n) = \lambda(n)\frac{\mathrm{d}h_0}{\mathrm{d}t}(n-1) + 2e(n) \quad (9)$$

And,

$$h_1(n+1) = h_1(n) + \mu \frac{\mathrm{d}h_1(n)}{\mathrm{d}t} \quad (10)$$

The gradient can be updated using

$$\frac{\mathrm{d}h_1}{\mathrm{d}t}(n) = \lambda(n)\frac{\mathrm{d}h_1}{\mathrm{d}t}(n-1) + 2x(n)e(n) \quad (11)$$

And,

$$h_2(n+1) = h_2(n) + \mu \frac{\mathrm{d}h_2(n)}{\mathrm{d}t} \quad (12)$$

Using a recursion rule, the gradient at the $(n+1)$-th instant can be updated using

$$\frac{\mathrm{d}h_2}{\mathrm{d}t}(n) = \lambda(n)\frac{\mathrm{d}h_2}{\mathrm{d}t}(n-1) + 2\boldsymbol{x}(n)\boldsymbol{x}(n)^{\mathrm{T}}e(n) \quad (13)$$

The forgetting factor λ is updated according to

$$\lambda(n+1) = \lambda(n) + \mu \frac{\mathrm{d}\lambda(n)}{\mathrm{d}t} \quad (14)$$

2 Nonlinear Sensor Compensation Based on Volterra Series Model

We consider the output of sensor as the input signal corrupted by the sensor's noises, and a way to improve the accuracy is to employ an adaptive filter to reduce the sensor's noises. Based on this idea, our sensor performance or accuracy enhancement scheme is illustrated in Fig. 1. The Volterra series model connects to the sensor in the series. The Volterra series model is employed to filter the noise from the original sensor. Referring to Fig. 1, $x(n)$ and $y(n)$ represent the applied normalized sensor input and measured output, respectively. $y(n)$ is used as the input to the Volterra series model which generates an output $\hat{y}(n)$. If the Volterra series model is properly trained, $\hat{y}(n)$ represents an accurate estimate of the applied sensor input. After sufficient training, the estimated outputs should become more and more accurate.

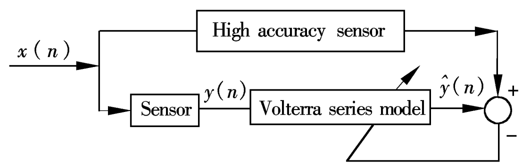

Fig.1 Learning procedure of Volterra series model

As this learning proceeds, the error E (mean-square-error, MSE) progressively decreases and finally attains a minimum value. At this stage the Volterra series model becomes an ideal adaptive filter that can reduce the applied sensor's noise and enhance the performance or the accuracy of the sensor. This allows the nonlinear sensor error to be corrected according to the model, as shown in Fig.2.

Fig.2 Principle of error to be corrected by Volterra series model

3 Simulation and Conclusion

In the simulation below, a second-order Volterra series model is chosen in sensor accuracy enhancement. It is observed that an MSE level of about −90 dB is attained at 2000 iterations in 300 to 350 ms with a learning rate constant of 0. 21. Achievement of such a low value of MSE ensures that the output $\hat{y}(t)$ is an accurate estimate of the high accuracy sensor's output. In the experiment, the low accuracy pressure sensor MPX10, which is fabricated by Motorola Co., is considered as the actual object. Another higher accuracy sensor MPX2010 is used as the reference to provide the necessary teaching data. The precision of the sensor can be defined by the ratio of the maximum error to full scale value as follows:

$$\theta = \frac{\max|y_i - \bar{y}|}{Y_{\mathrm{FS}}} \quad (15)$$

where y_i, $\bar{y}$ and Y_{FS} are the measure value of the sensor, arithmetic mean value of the measure value, and the full scale value, respectively. Through our experiments, we know that the original precisions θ of MPX10 and MPX2010 are about 0.354 − 0.42 and 0.032 − 0.043, respectively. The Volterra series model will gradually learn to reduce the sensor noise and this step is performed offline. Finally, according to the scheme of Fig.2, the precision of MPX10 is enhanced to 0.041 − 0.053 after Volterra series model filtering. The experimental results are shown in Fig.3, where the curve " + " denotes the precise behavior change of MPX10. Accordingly, we can conclude that the effective accuracy of the sensor is increased.

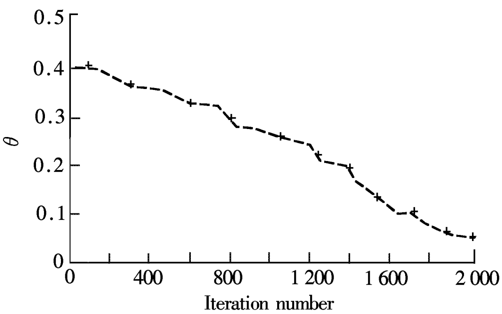

Fig.3 Dynamic behavior of sensor precision

This paper presents a new approach to sensor accuracy enhancement. It is believed that this scheme can be of great value, when applied to the sensors in practical control and measurement systems, because no extra hardware cost is needed. However, the computation burden will increase drastically. This technique will be useful for other types of sensors possessing similar nonlinear response characteristics. It has a potential future in the field of instrumentation and measurement.

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基于 Volterra 模型的非线性传感器补偿研究

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摘要: 提出了一种基于 Volterra 系数模型, 用于提高非线性传感器性能的新方法. Volterra 模型可作为一个非线性滤波器用于降低传感器的噪声, 并可对传感器进行非线性补偿. 在实验中, 采用精度较低的压力传感器 MPX10 作为实验传感器, 采用具有较高精度的传感器 MPX2010 产生构建 Volterra 模型的训练学习数据. 仿真实验表明, 利用 Volterra 模型进行滤波, 传感器 MPX10 的精度由原来的 0.354 ~0.42 变为 0.041 ~0.053. 由此可见该方法可有效地提高传感器的性能与精度, 并具有较高的环境适应能力.

关键词: 传感器; Volterra 模型; 非线性

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