

Identification algorithm for a kind of fractional order system

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Abstract: The state-space representation of linear time-invariant (LTI) fractional order systems is introduced, and a proof of their stability theory is also given. Then an efficient identification algorithm is proposed for those fractional order systems. The basic idea of the algorithm is to compute fractional derivatives and the filter simultaneously, i.e., the filtered fractional derivatives can be obtained by computing them in one step, and then system identification can be fulfilled by the least square method. The instrumental variable method is also used in the identification of fractional order systems. In this way, even if there is colored noise in the systems, the unbiased estimation of the parameters can still be obtained. Finally an example of identifying a viscoelastic system is given to show the effectiveness of the aforementioned method.

Key words: fractional order systems; state-space representation; system identification; fractional order Poisson filter; least square method; instrumental variable method

Recently the focus of interest on fractional calculus and fractional order systems has become increasingly heated. One important reason is that many real phenomena, such as electrochemistry^[1], heat conduction^[2], diffusion^[3], and viscoelasticity^[4] are well suited to be represented by fractional order systems, i.e., they conform to the real situations better if the orders of these systems are non-integers rather than integers. For the identification of integer order systems, there are many mature algorithms: autoregressive with external input (ARX), autoregressive moving average with external input (ARMAX), least square method, prediction-error method, instrumental variable method^[5,6], etc., but currently there are no efficient methods for the identification of fractional order systems.

An efficient identification algorithm for fractional order systems is proposed in this paper. The basic idea of the algorithm is that the input and output data are first filtered by a fractional order Poisson filter, and then those filtered data are used for system identification. The least square method is used for system identification here. The originality of this paper lies in the combined computation of the filter and the fractional derivatives, i.e., the filtered fractional derivatives are obtained in just one step. Instrumental variable method is also used in the

identification of fractional order systems. In this way, even if there is colored noise in the systems, the unbiased estimation of the parameters can still be obtained. It is verified both in practice and theory that a fractional derivative is much more useful than an integer one in modeling viscoelastic systems^[4,7,8]. An example of identifying a viscoelastic system using the above method shows that even if the noise to signal ratio (NSR) reaches 70%, the given method can still guarantee a high identification precision.

1 State-Space Representation of Fractional Order Systems

1.1 Definition

The systems studied in this paper are commensurate order, linear time-invariant (LTI), fractional order systems. Commensurate means all the differential orders of the systems are the multiples of a rational number (integer or non-integer). The representation is defined by two equations: the state equation and the output equation, as in classical state-space representation, but the former is the generalization of the latter. The state vector of a fractional order state equation is different from that of an integer order state equation in that each component of the former is a fractional derivative while each component of the latter is an integer one; so we can denote the state vector of the former as the fractional order state vector. The state equation of a fractional order system can be written as

$$\left. \begin{aligned} D^\alpha \mathbf{x}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \right\} \quad 0 < \alpha < 1 \quad (1)$$

where scales $u(t)$ and $y(t)$ denote system input and

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output, respectively; $\mathbf{x}(t)$ is the non-integer state vector; \mathbf{A} , \mathbf{B} and \mathbf{C} are suitable dimension matrices. The α order derivative of the non-integer state vector $\mathbf{x}(t)$ is defined by

$$D^\alpha \mathbf{x}(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \left(\int_0^t \frac{\mathbf{x}(\tau)}{(t-\tau)^\alpha} d\tau \right) \quad (2)$$

where $t > 0$. Its Laplace transform with null initial conditions is

$$L\{D^\alpha \mathbf{x}(t)\} = s^\alpha \mathbf{X}(s), \quad \mathbf{X}(s) = L\{\mathbf{x}(t)\} \quad (3)$$

1.2 Stability conditions

Definition The fractional order LTI system

$$D^\alpha \mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) \quad 0 < \alpha < 1, \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (4)$$

is said to be

1) Stable iff $\forall \mathbf{x}_0, \exists M > 0, \forall t > 0,$

$$\|\mathbf{x}(t)\| \leq M \quad (5)$$

2) Asymptotically stable iff:

$$\lim_{t \rightarrow \infty} \|\mathbf{x}(t)\| = 0 \quad (6)$$

Lemma If $0 < \alpha < 2$, β is an arbitrary complex number and μ is an arbitrary real number such that $\pi\alpha/2 < \mu < \min\{\pi, \pi\alpha\}$, then for an arbitrary integer $\forall p \geq 1$, the following expansion holds:

$$E_{\alpha, \beta}(z) = - \sum_{k=1}^p \frac{z^{-k}}{\Gamma(\beta - \alpha k)} + O(|z|^{-1-p})$$

$$|z| \rightarrow \infty, \mu \leq |\arg(z)| \leq \pi \quad (7)$$

where $E_{\alpha, \beta}(z)$ is the two-variable Mittag-Leffler function^[9]. The proof can be found in Ref. [9].

Theorem System (4) is asymptotically stable iff all the eigenvalues λ_i of matrix \mathbf{A} , $\det(\lambda_i \mathbf{I} - \mathbf{A}) = 0$, satisfy

$$|\arg(\lambda_i)| > \frac{\alpha\pi}{2} \quad \forall i \quad (8)$$

Proof Suppose matrix \mathbf{A} has n different eigenvalues, $\lambda_i, i = 1, 2, \dots, n$, and then there exists a nonsingular matrix \mathbf{T} such that Eq. (4) can be changed into

$$D^\alpha \bar{\mathbf{x}}(t) = \mathbf{A} \bar{\mathbf{x}}(t), \quad \bar{\mathbf{x}}(0) = \bar{\mathbf{x}}_0 \quad (9)$$

where

$$\bar{\mathbf{x}}(t) = \mathbf{T}\mathbf{x}(t) \quad (10)$$

$$\mathbf{A} = \mathbf{T}\mathbf{A}\mathbf{T}^{-1} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n) \quad (11)$$

Suppose \mathbf{T} has the maximum and the minimum singular value σ_{\max} and σ_{\min} , then the following formula holds:

$$\sigma_{\min} \|\mathbf{x}(t)\| \leq \|\bar{\mathbf{x}}(t)\| \leq \sigma_{\max} \|\mathbf{x}(t)\| \quad (12)$$

Formula (12) holds, iff the following formula holds:

$$\lim_{t \rightarrow \infty} \|\bar{\mathbf{x}}(t)\| = 0 \quad (13)$$

Using function $E_{\alpha, \beta}(z)$, the analytical solution of Eq. (4) can be written as

$$\bar{\mathbf{x}}(t) = E_{\alpha, 1}(\mathbf{A}t^\alpha) \bar{\mathbf{x}}_0 \quad (14)$$

Thus $\|\bar{\mathbf{x}}(t)\| \rightarrow 0$ holds, iff the following formula

holds:

$$E_{\alpha, 1}(\lambda_i t^\alpha) \rightarrow 0 \quad \forall i \quad (15)$$

From the lemma we know that the above formula holds, iff formula (8) holds. In case of \mathbf{A} having the same eigenvalue, the proof is similar.

1.3 Numerical approximations of fractional derivatives

The approximate formula of the fractional derivative using the definition of Grünwald-Letnikov fractional derivative is written as

$$D^\alpha f(t) = \frac{1}{T_s^\alpha} \sum_{k=0}^n w_k^\alpha f((n-k)T_s) \quad \alpha > 0 \quad (16)$$

$$w_k^\alpha = \frac{(-1)^k \Gamma(\alpha+1)}{k! \Gamma(\alpha-k+1)} \quad (17)$$

Eq.(16) is only $O(T_s)$ if the weight coefficients w_k^α take Eq.(17). Lubich also gives the 2 to 6 orders of the weight coefficients for the approximation of the fractional derivatives^[10]. For example, Eq. (16) is $O(T_s^4)$ if its weight coefficients w_k^α are the coefficients of the Taylor expansion at zero of the following formula:

$$\omega_4^\alpha(z) = \left(\frac{25}{12} - 4z + 3z^2 - \frac{4}{3}z^3 + \frac{1}{4}z^4 \right)^\alpha \quad (18)$$

It is easy to obtain the approximate iterative formula of the fractional order system equation by using Eqs. (16) and (17).

2 Identification of the Fractional Order System Model

2.1 Identification model

The aim of any system identification is to establish a mathematical model capable of reproducing the system's physical behavior as faithfully as possible from a series of observations, so is the fractional order system identification. The identification model in this paper is a linear, constant coefficient, fractional order equation as follows:

$$D^{\alpha_0} y(t) + a_1 D^{\alpha_1} y(t) + \dots + a_n D^{\alpha_n} y(t) = b_0 D^{\beta_0} u(t) + b_1 D^{\beta_1} u(t) + \dots + b_m D^{\beta_m} u(t) \quad (19)$$

where the differential orders, $(\alpha_0, \alpha_1, \dots, \alpha_n, \beta_0, \beta_1, \dots, \beta_m)$, are real (integer or non integer) and supposed known by the user (as the orders of fractional order viscoelastic systems and many thermal systems^[4-6] can be determined by experiments). The case of identifying both the differentiation orders and the coefficients of fractional order systems will be given in other papers. The identification coefficients of Eq.(19) are denoted as parameter vector θ :

$$\theta = \{a_1, a_2, \dots, a_n, b_0, b_1, \dots, b_m\}^T \quad (20)$$

Applying a linear transformation (low pass fractional order Poisson filter $F(s)$ in our case) to Eq.(19) and isolating $D^{\alpha_0} y_f(t)$, we obtain a linear continuous regression, considering null initial conditions, of the form:

$$D^{\alpha_0} y_f(t) = \varphi_f(t) \theta \quad (21)$$

where

$$\varphi_f(t) = \{ -D^{\alpha_1} y_f(t), \dots, -D^{\alpha_n} y_f(t), \\ D^{\beta_0} u_f(t), \dots, D^{\beta_m} u_f(t) \} \quad (22)$$

where $u_f(t)$ and $y_f(t)$ are the filtered input and output of the system, respectively. Eqs.(19) and (21) are equivalent under general conditions. However, Eq.(21) permits the various derivatives in vector $\varphi_f(t)$ to be computed from observed data and filter $F(s)$, which is not permitted by Eq.(19).

2.2 Fractional order Poisson filter

Parameter estimation requires the computation of the fractional derivatives of the filtered input and output. As for classical methods, which use state variable filters $F(s)$, $F(s)$ is simulated using a state-space representation, fractional order state variable filter is used in this paper for fractional order systems. Fractional order Poisson filter is a special case of the fractional order variable filter, and its transfer function can be written as

$$F(s) = \frac{1}{\left(\left(\frac{s}{\omega_f} \right)^\alpha + 1 \right)^{n_f}} = \frac{\omega_f^{\alpha n_f}}{s^{\alpha n_f} + C_{n_f}^1 \omega_f^\alpha s^{\alpha(n_f-1)} + \dots + C_{n_f}^{n_f-1} \omega_f^{\alpha(n_f-1)} s^\alpha + \omega_f^{\alpha n_f}} \quad (23)$$

where frequency ω_f is fixed by the user according to the frequency characteristics of the system to be identified (close to the highest corner frequency of the system), αn_f is the order of the filter, and n_f is the dimension of the state-space representation of the fractional order Poisson filter. It is easy to find out from Eq.(23) that the fractional order Poisson filter is an extension of the classic integer order Poisson filter, i.e., they are equivalent when $\alpha = 1$. However, the fractional order Poisson filter must satisfy the following conditions:

1) As the fractional order state vector is composed of the fractional derivatives of the filtered input or output, α must be chosen such that

$$\alpha_i = \alpha k_{\alpha_i}, \beta_j = \alpha k_{\beta_j} \quad i=0, 1, \dots, n; \\ j=0, 1, \dots, m; k_{\alpha_i}, k_{\beta_j} \in \mathbf{N}; \\ n_f \geq \max(k_{\alpha_0}, k_{\beta_0})$$

2) The coefficients in Eq.(23) must satisfy the above stability theorem.

The state vector in Eq.(23), composed of the fractional derivatives of the filtered input or output, is defined by

$$\mathbf{x} = \{z_f(t), D^\alpha z_f(t), \dots, D^{\alpha(n_f-2)} z_f(t), \\ D^{\alpha(n_f-1)} z_f(t)\}^T \quad (24)$$

where $z_f(t)$ denotes $u_f(t)$ or $y_f(t)$. The state-space representation of Eq.(23) is

$$D^\alpha \mathbf{x}(t) = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -\omega_f^{\alpha n_f} & -C_{n_f}^{n_f-1} \omega_f^{\alpha(n_f-1)} & \dots & -C_{n_f}^2 \omega_f^{2\alpha} & -C_{n_f}^1 \omega_f^\alpha \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \omega_f^{\alpha n_f} \end{bmatrix} z(t) \quad (25)$$

Using numerical approximation described in section 1.3, the fractional order Poisson filter can be simulated and the fractional derivatives of the filtered input (or output) directly obtained simultaneously. **This is the originality of this paper.**

2.3 Parameter estimation

Suppose $u(t)$ and $y^*(t)$ are input and output respectively, and

$$y^*(t) = y(t) + p(t) \quad (26)$$

where $p(t)$ is a perturbation signal (measurement noise, for example). Parameter estimation is obtained using the classical least square method minimizing the quadratic norm of function $e(t)$ defined by

$$e(t) = D^{\alpha_0} y_f^*(t) + \hat{a}_1 D^{\alpha_1} y_f^*(t) + \dots + \\ \hat{a}_n D^{\alpha_n} y_f^*(t) - \hat{b}_0 D^{\beta_0} u_f^*(t) - \dots - \hat{b}_m D^{\beta_m} u_f^*(t) \quad (27)$$

where $(\hat{a}_1, \dots, \hat{a}_m, \hat{b}_0, \dots, \hat{b}_m)$ are the parameters to be identified (see Fig.1).

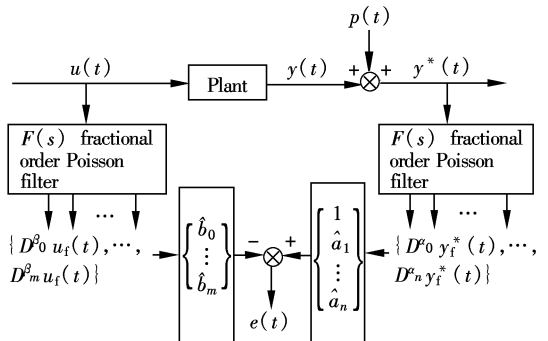


Fig.1 Parameter identification diagram of the fractional order system

Considering K observations on the system, parameter estimation is obtained by minimizing criterion J defined by

$$J(\hat{\theta}) = \mathbf{E}^T \mathbf{E} \quad (28)$$

where $\mathbf{E} = \{e(k_0 T_s), e((k_0 + 1)T_s), \dots, e((k_0 + K - 1)T_s)\}^T$, T_s is the sampling period, and k_0 is an integer number chosen such that $k_0 T_s \geq T_f$ where T_f denotes the settling time of filter $F(s)$. Using the linear continuous regression form (Eq. (21)), the optimal value of $\hat{\theta}_{\text{opt}}$ minimizing $J(\hat{\theta})$ is given by

$$\hat{\theta}_{\text{opt}} = (\Phi_f^{*T} \Phi_f^*)^{-1} \Phi_f^{*T} D^{\alpha_0} \mathbf{Y}_f^* \quad (29)$$

where

$$\begin{aligned} \Phi_f^* &= \{\varphi_f^{*T}(k_0 T_s), \varphi_f^{*T}((k_0 + 1)T_s), \dots, \\ &\quad \varphi_f^{*T}((k_0 + K - 1)T_s)\}^T \\ \mathbf{Y}_f^* &= \{y_f^*(k_0 T_s), y_f^*((k_0 + 1)T_s), \dots, \\ &\quad y_f^*((k_0 + K - 1)T_s)\}^T \end{aligned}$$

2.4 Estimation bias analysis

Suppose the true value θ of the estimated parameter vector $\hat{\theta}$ in Eq.(27) is determined by

$$D^{\alpha_0} y_f^*(t) = \varphi_f^*(t) \theta + e(t) \quad (30)$$

Substituting $D^{\alpha_0} \mathbf{Y}_f^*$ in Eq.(29) with the above formula, the classical estimation bias formula can be obtained:

$$\begin{aligned} \Delta \theta &= \hat{\theta}_{\text{opt}} - \theta = (\Phi_f^{*T} \Phi_f^*)^{-1} \Phi_f^{*T} \cdot \\ &\quad \begin{Bmatrix} e(k_0 T_s) \\ e((k_0 + 1)T_s) \\ \vdots \\ e((k_0 + K - 1)T_s) \end{Bmatrix} \end{aligned} \quad (31)$$

This classical relation reveals that the estimation is biased if $e(t)$ is not an output white noise. In this case, an instrumental variable (IV) method can be used. Parameter estimation is then obtained iteratively by

$$\hat{\theta}_{k+1}^{\text{IV}} = (\Phi_f^{\text{IVT}}(\theta_k^{\text{IV}}) \Phi_f^*)^{-1} \Phi_f^{\text{IVT}}(\theta_k^{\text{IV}}) D^{\alpha_0} \mathbf{Y}_f^* \quad (32)$$

where

$$\Phi_f^{\text{IV}}(\theta_k^{\text{IV}}) = \{\varphi_f^{\text{IVT}}(\theta_k^{\text{IV}}, k_0 T_s), \varphi_f^{\text{IVT}}(\theta_k^{\text{IV}}, (k_0 + 1)T_s), \dots, \varphi_f^{\text{IVT}}(\theta_k^{\text{IV}}, (k_0 + K - 1)T_s)\}^T \quad (33)$$

$$\begin{aligned} \varphi_f^{\text{IV}}(\theta_k^{\text{IV}}, t) &= \{-D^{\alpha_1} y_f^{\text{IV}}(\theta_k^{\text{IV}}, t), \dots, \\ &\quad -D^{\alpha_n} y_f^{\text{IV}}(\theta_k^{\text{IV}}, t), D^{\beta_0} u_f(t), \dots, D^{\beta_m} u_f(t)\} \end{aligned} \quad (34)$$

where $y_f^{\text{IV}}(\theta_k^{\text{IV}}, t)$ results from the simulation of Eq. (19) using parameter vector θ_k^{IV} .

3 Simulation Example

It is quite normal to build a mathematical model of a viscoelastic system using classical integer order differential equations. However, both theoretical and experimental studies show that the modeling precision is much higher by using fractional order differential

equations than that of integer order^[4-6] differential equations. Fig. 2 shows the schematic and working principle diagrams of a fractional order viscoelastic damper. G_0 and G_1 denote the elastic coefficient and the viscosity coefficient, respectively, and $\tau(t)$ denotes the exerted force. The mathematical models of both the integer and fractional order differential equations are written as

$$\tau(t) = G_0 \gamma(t) + G_1 \frac{d}{dt}(\gamma(t)) \quad (35)$$

$$\tau(t) = G_0 \gamma(t) + G_1 D^\alpha(\gamma(t)) \quad 0 < \alpha < 1 \quad (36)$$

When $\alpha = 1$ in Eq. (36), the above two formulae are equivalent. The value of α can be obtained by experiments, and its value is usually near 0.5. Fractional order viscoelastic damper can be used in big building shockproof^[8], car body shock absorption, etc.

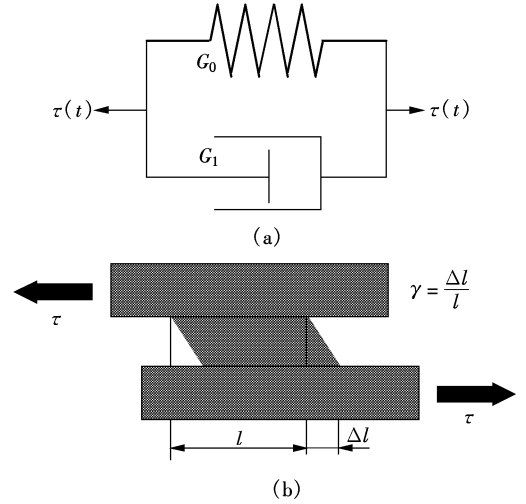


Fig.2 Fractional order viscoelastic damper. (a) Schematic diagram; (b) Working principle diagram

Suppose a mass-viscoelastic system can be described as

$$\begin{aligned} D^2 y(t) + 0.5 D^{0.5} y(t) + y(t) &= D^{0.5} u(t) + u(t) \\ y(0) = 0, \dot{y}(0) &= 0 \end{aligned} \quad (37)$$

Its transfer function is

$$G(s) = \frac{s^{0.5} + 1}{s^2 + 0.5s^{0.5} + 1} \quad (38)$$

Fig.3 shows its Bode plots. $\theta = \{0.5, 1, 1, 1\}^T$ is the parameter vector in Eq. (37). The system is simulated by using 600-point input signal, the first 300 points generated by unit step signal and the other 300 points by pseudo random binary signal (PRBS). The sampling period is fixed at $T_s = 0.05$ s. System output is noised by a Gauss white noise signal using various noise to signal ratios (NSRs), between 0 and 70%, defined by

$$\text{NSR} = \frac{\sqrt{\text{var}(p(t))}}{\sqrt{\text{var}(y(t))}} \times 100\% \quad (39)$$

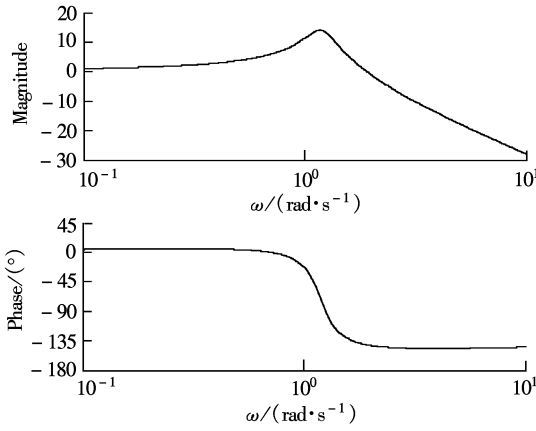


Fig.3 Bode plots of the system

The fractional order Poisson filter in Eq.(23) has the corner frequency $\omega_f = 0.1$ Hz, and $\alpha = 0.5$, $n_f = 5$. It is easy to know that the filter is stable according to the theorem. In order to evaluate the identification performances of each different NSR, the mean normalized errors (MNE) using 2-norm are also computed for each NSR:

$$\text{MNE} = \frac{\|\theta - \hat{\theta}\|}{\|\theta\|} \times 100\% \quad (40)$$

Weight coefficients w_k^α in Eq.(16) are the coefficients of the Taylor expansion of Eq.(18). Instrumental variable method is used in the process of parameter estimation. Tab.1 gives the identification results. From Tab.1 we can see that even if there is strong Gauss white noise mixed in the output, high precision of the identified parameters can still be guaranteed. Fig. 4 shows the simulation diagrams of both the real system and the identified system when $\text{NSR} = 70\%$, from which we can make a conclusion that the identification method proposed in this paper can restrain very strong noise, and the identified system can represent the real system well.

Tab.1 The identification results with weight coefficients taking the fourth order approximation

NSR/%	$\hat{\theta}^T$	MNE/%
0	{0.500, 1.000, 1.000, 1.000}	0.000
10	{0.504, 1.005, 1.003, 1.005}	0.480
30	{0.495, 1.017, 0.991, 1.019}	1.525
50	{0.483, 1.030, 0.971, 1.034}	3.131
70	{0.479, 1.040, 0.996, 1.046}	4.043

4 Conclusion

The identification algorithm for fractional order systems given in this paper combines the computation of fractional order Poisson filter and fractional derivatives, i.e., the filtered fractional derivatives can be directly obtained in just one step. It simplifies calculation and also improves the identification

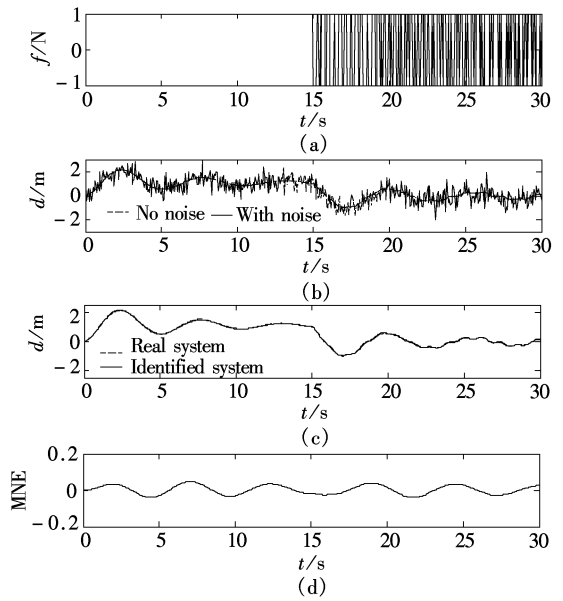


Fig.4 Identification outcomes of the system with NSR = 70% of the added Gauss white noise. (a) System input; (b) System output; (c) Identification outcome; (d) Mean normalized errors

precision. Especially the instrumental variable method is used in the identification of fractional order systems so that unbiased parameter estimation can be obtained. The simulation example also shows the effectiveness of this algorithm. In the future, it will be applied to the identification of distributed systems and other real systems like electrochemical systems.

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一类分数阶系统的辨识算法

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摘要: 介绍了分数阶线性定常系统的状态方程描述, 并给出了其稳定性定理的一个证明. 然后给出了线性定常分数阶系统的一个有效辨识算法. 其基本思想是利用分数阶泊松滤波器把分数阶导数和滤波计算合并起来, 只需计算 1 步就可以得到滤波后的分数导数, 再利用最小二乘法进行系统辨识. 还把辅助变量方法运用到分数阶系统的辨识上, 这样即使系统中存在有色噪声, 仍可以获得参数的无偏估计. 最后给出了一个粘弹性系统的辨识实例, 说明了上述方法的有效性.

关键词: 分数阶系统; 状态空间描述; 系统辨识; 分数阶泊松滤波器; 最小二乘法; 辅助变量法

中图分类号: TP13