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Finite element analysis of the free-damped beam-stiffened plate

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Abstract: A finite element model is presented for free-damped beam-stiffened plates. The nodes of the plate elements are treated as master-nodes, and the corresponding nodes of the beam elements are considered as slave-nodes. The stiffness and mass matrices of the elements are developed. Based on the analysis of the dynamic properties of the structures, modal loss factors are predicted by the modal strain energy method. Finally, an example is given to compare the results obtained from the proposed method with the results of the ANSYS software. The results show that the method in this paper is computationally efficient, simple and feasible with high precision and engineering practicability.

Key words: beam-stiffened plate; damping; finite element

The structures of beam-stiffened plates have the characteristics of high stiffness and light weight (saving materials), so they are widely used in the mechanical structures of machine tools, ships, planes, etc. The problem of controlling the noise and vibration of mechanical structures is very important. An effective way to solve the problem is to use the technique of surface damping treatment[1]. This technique can effectively restrain the resonance of structures by increasing the structure damping and has been broadly used in engineering. Free-damping treatment is one kind of surface damping treatment and can be used conveniently and economically. The finite element method is one of the important theoretical analysis methods used to analyze the surface-damping-treated structures. This paper proposes a method to establish a finite element model for the free-damped beam-stiffened plates using an integral-dividing element method[2-5]. Numerical examples show that the method can provide accurate estimates of the dynamic properties of the structures.

1 Modeling of the Finite Element

Ref. [6] indicated that, when we set up a finite element model for the beam-stiffened plates, there should be both plate and beam elements with strict attention paid to guaranteeing the compatibility of the displacements at the nodes of the beam and plate elements. Here the nodes of the plates are treated as master-nodes, and the nodes of the beams are considered as slave-nodes whose serial-numbers are dependent on the corresponding master nodes. The calculation scale is dependent on the quantities of the plate elements. The following is the modeling procedure. First, the free-damped plate element model is developed without regard to the beams, and the stiffness matrix is discussed. Secondly, the stiffness matrix of the beam element is properly transformed to combine with that of the plate element. Thirdly, the mass matrix is set up.

1.1 Basic assumption

It is assumed that each layer has the same transverse displacement as has been proved both experimentally and numerically[3], and that the deformation of each layer obeys thin plate theory.

1.2 Shape function and variables of the free-damped plate element

As shown in Fig.1, the free-damped plate element is rectangular with four nodes at each corner point. At each node, seven displacements are introduced. These are the transverse displacement w, the two rotations $\theta_x = \frac{\partial w}{\partial y}$ and $\theta_y = -\frac{\partial w}{\partial x}$, the in-plane extensional displacements of the base-plate-layer u_1, v_1, and the in-plane extensional displacements of the damping-layer u_2, v_2.

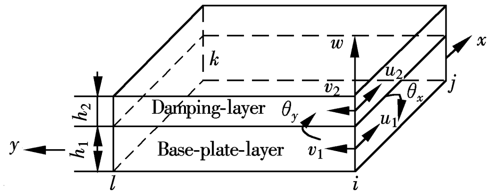

Fig.1 Definition of variables of free-damped plate element

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The displacements u_1, v_1 and u_2, v_2 are on the neutral surface of the base-plate and the damping-layer, respectively, and u_2, v_2 have relations with u_1 and v_1, so they are not included in the node degrees-of-freedom. Therefore the displacement vector of each node can be written as $\boldsymbol{\delta}_n^{\mathrm{T}} = \{u_{1n}, v_{1n}, w_n, \theta_{xn}, \theta_{yn}\}$, $n = i, j, k, l$, and the complete set of 20 nodal displacements of the element is $\boldsymbol{\delta}^{\mathrm{e}} = \{\boldsymbol{\delta}_i^{\mathrm{T}}, \boldsymbol{\delta}_j^{\mathrm{T}}, \boldsymbol{\delta}_k^{\mathrm{T}}, \boldsymbol{\delta}_l^{\mathrm{T}}\}^{\mathrm{T}}$.

We introduce $o\xi\eta$ coordinate system shown in Fig.2 to simplify the process. The length of the two edges of the rectangular are $2a$ and $2b$, and the two edges are parallel to x and y coordinate axes, respectively. The coordinates (x, y) which refer to the location of a certain point in the rectangular may be expressed as

$$x = x_0 + a\xi,\ y = y_0 + b\eta$$

where (x_0, y_0) are the coordinates of the point o.

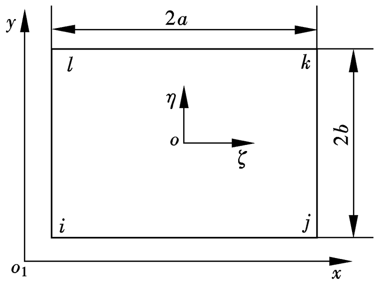

Fig.2 $o\xi\eta$ coordinate system

Suppose the displacements interpolation and we obtain

$$\left.\begin{aligned} u_1 &= \boldsymbol{U}_1\boldsymbol{\delta}^{\mathrm{e}} = \sum_{n=i,j,k,l} N_n u_{1n} \\ v_1 &= \boldsymbol{V}_1\boldsymbol{\delta}^{\mathrm{e}} = \sum_{n=i,j,k,l} N_n v_{1n} \\ w &= \boldsymbol{W}\boldsymbol{\delta}^{\mathrm{e}} = \sum_{n=i,j,k,l} \boldsymbol{W}_n\boldsymbol{\delta}_n = \\ &\sum_{n=i,j,k,l} W_n w_n + W_{xn}\theta_{xn} + W_{yn}\theta_{yn} \end{aligned}\right\} \tag{1}$$

where

$$N_n = \frac{1}{4(1+\xi\xi_n)(1+\eta\eta_n)}$$

$$\boldsymbol{W}_n = \{0,\ 0,\ W_n,\ W_{xn},\ W_{yn}\}$$

$$W_n = \frac{(1+\xi\xi_n)(1+\eta\eta_n)(2+\xi\xi_n+\eta\eta_n-\xi^2-\eta^2)}{8}$$

$$W_{xn} = -\frac{b\eta_n(1+\xi\xi_n)(1+\eta\eta_n)(1-\eta^2)}{8}$$

$$W_{yn} = \frac{a\xi_n(1+\xi\xi_n)(1+\eta\eta_n)(1-\xi^2)}{8}$$

1.3 Formulation of the stiffness matrix of the free-damped plate element $\boldsymbol{K}^{\mathrm{e}}$

$$\boldsymbol{K}^{\mathrm{e}} = \boldsymbol{K}_{\mathrm{flex}} + \boldsymbol{K}_{\mathrm{ext}}$$

The bending stiffness matrices of each layer can be obtained from the bending strain energies of the element.

$$\begin{aligned} \boldsymbol{K}_{\mathrm{flex}i} = \frac{E_i h_i^3\, ab}{12(1-\mu_i^2)}\int_A \Big[& \frac{1}{a^4}\frac{\partial^2 \boldsymbol{W}^{\mathrm{T}}}{\partial\xi^2}\frac{\partial^2 \boldsymbol{W}}{\partial\xi^2} + \\ & \frac{1}{b^4}\frac{\partial^2 \boldsymbol{W}^{\mathrm{T}}}{\partial\eta^2}\frac{\partial^2 \boldsymbol{W}}{\partial\eta^2} + \frac{\mu_i^2}{a^2b^2}\left(\frac{\partial^2 \boldsymbol{W}^{\mathrm{T}}}{\partial\xi^2}\frac{\partial^2 \boldsymbol{W}}{\partial\eta^2} + \frac{\partial^2 \boldsymbol{W}^{\mathrm{T}}}{\partial\eta^2}\frac{\partial^2 \boldsymbol{W}}{\partial\xi^2}\right) + \\ & \frac{2(1-\mu_i)}{a^2b^2}\frac{\partial^2 \boldsymbol{W}^{\mathrm{T}}}{\partial\xi\partial\eta}\frac{\partial^2 \boldsymbol{W}}{\partial\xi\partial\eta}\Big]\mathrm{d}\xi\mathrm{d}\eta \end{aligned} \tag{2}$$

where E_i is the elasticity modulus and μ_i is the Poisson's ratio ($i = 1, 2$ denotes the base-plate-layer and the damping-layer, respectively). Then $\boldsymbol{K}_{\mathrm{flex}} = \boldsymbol{K}_{\mathrm{flex1}} + \boldsymbol{K}_{\mathrm{flex2}}$.

The extensional stiffness matrices of each layer can be obtained from the extensional strain energies of the element.

$$\begin{aligned} \boldsymbol{K}_{\mathrm{ext}i} = \frac{abE_ih_i}{1-\mu_i^2}\int_A \Big[& \frac{1}{a^2}\frac{\partial \boldsymbol{U}_i^{\mathrm{T}}}{\partial\xi}\frac{\partial \boldsymbol{U}_i}{\partial\xi} + \frac{1}{b^2}\frac{\partial \boldsymbol{V}_i^{\mathrm{T}}}{\partial\eta}\frac{\partial \boldsymbol{V}_i}{\partial\eta} + \\ & \frac{\mu_i}{ab}\left(\frac{\partial \boldsymbol{U}_i^{\mathrm{T}}}{\partial\xi}\frac{\partial \boldsymbol{V}_i}{\partial\eta} + \frac{\partial \boldsymbol{V}_i^{\mathrm{T}}}{\partial\eta}\frac{\partial \boldsymbol{U}_i}{\partial\xi}\right) + \\ & \frac{1}{2}(1-\mu_i)\left(\frac{1}{b}\frac{\partial \boldsymbol{U}_i}{\partial\eta} + \frac{1}{a}\frac{\partial \boldsymbol{V}_i}{\partial\xi}\right)^{\mathrm{T}} \cdot \\ & \left(\frac{1}{b}\frac{\partial \boldsymbol{U}_i}{\partial\eta} + \frac{1}{a}\frac{\partial \boldsymbol{V}_i}{\partial\xi}\right)\Big]\mathrm{d}\xi\mathrm{d}\eta \end{aligned} \tag{3}$$

where $\boldsymbol{U}_1$ and $\boldsymbol{V}_1$ have already been obtained in (1), and $\boldsymbol{U}_2$ and $\boldsymbol{V}_2$ can be obtained below.

The relation between u_2 and u_1 shown in Fig.3 can be expressed as

$$u_2 = u_1 - \frac{h_1+h_2}{2}\frac{\partial w}{\partial x} \tag{4}$$

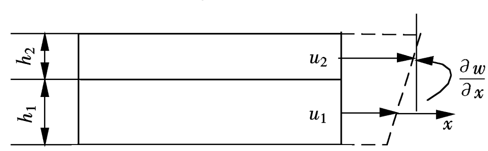

Fig.3 Relation between u_2 and u_1

A similar relation exists for v_2 and v_1:

$$v_2 = v_1 - \frac{h_1+h_2}{2}\frac{\partial w}{\partial y} \tag{5}$$

Consequently we can obtain

$$\left.\begin{aligned} u_2 &= \boldsymbol{U}_2\boldsymbol{\delta}^{\mathrm{e}} = \{\boldsymbol{U}_{2i}, \boldsymbol{U}_{2j}, \boldsymbol{U}_{2k}, \boldsymbol{U}_{2l}\}\boldsymbol{\delta}^{\mathrm{e}} \\ v_2 &= \boldsymbol{V}_2\boldsymbol{\delta}^{\mathrm{e}} = \{\boldsymbol{V}_{2i}, \boldsymbol{V}_{2j}, \boldsymbol{V}_{2k}, \boldsymbol{V}_{2l}\}\boldsymbol{\delta}^{\mathrm{e}} \end{aligned}\right\} \tag{6}$$

where

$$\boldsymbol{U}_{2n} = \left\{N_n,\ 0,\ -\frac{e_1}{a}\frac{\partial W_n}{\partial\xi},\ -\frac{e_1}{a}\frac{\partial W_{xn}}{\partial\xi},\ -\frac{e_1}{a}\frac{\partial W_{yn}}{\partial\xi}\right\}$$

$$\boldsymbol{V}_{2n} = \left\{0,\ N_n,\ -\frac{e_1}{b}\frac{\partial W_n}{\partial\eta},\ -\frac{e_1}{b}\frac{\partial W_{xn}}{\partial\eta},\ -\frac{e_1}{b}\frac{\partial W_{yn}}{\partial\eta}\right\}$$

where $n = i, j, k, l$; $e_1 = (h_1+h_2)/2$. Thus $\boldsymbol{K}_{\mathrm{ext}} = \boldsymbol{K}_{\mathrm{ext1}} + \boldsymbol{K}_{\mathrm{ext2}}$.

1.4 Stiffness matrix of the beam element

The coordinate systems $x_3y_3z_3$ and $x'y'z'$ are the local coordinate systems of the beam and the plate, respectively, which are shown in Fig. 4 (The viscoelastic damping-layer is not shown in Fig.4 for explicitness. The damping-layer and the beams should be on the opposite side of the plate). The two nodes of the beam elements i_3 and j_3 are slave nodes that have four displacements. They are u_3, w_3, θ_{x3}, θ_{y3}. The corresponding master nodes are i and j from the plate element. The displacement vector of the slave node of the beam is expressed as

$$\boldsymbol{\delta}_3^{\mathrm{e}} = \{\boldsymbol{\delta}_{3i}^{\mathrm{T}}, \boldsymbol{\delta}_{3j}^{\mathrm{T}}\}^{\mathrm{T}} \tag{7}$$

where $\boldsymbol{\delta}_{3i}^{\mathrm{T}} = \{u_{3i}, w_{3i}, \theta_{x3i}, \theta_{y3i}\}$, $\boldsymbol{\delta}_{3j}^{\mathrm{T}} = \{u_{3j}, w_{3j}, \theta_{x3j}, \theta_{y3j}\}$.

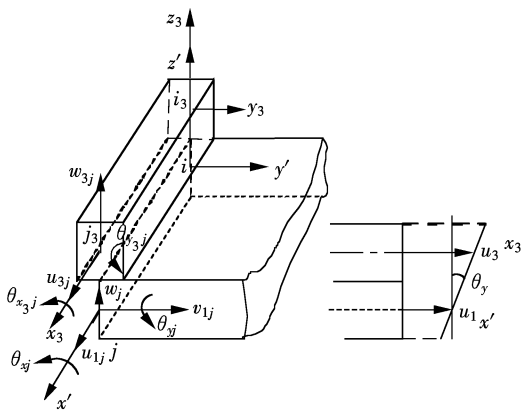

Fig.4 Master nodes and slave nodes

As can be seen in Fig.4, $w_3 = w$, $\theta_{x3} = \theta_x$, $\theta_{y3} = \theta_y$, $u_3 = u_1 + h_0\theta_y$, where h_0 is the distance from the center of the cross-section of the beam to the neutral surface of the plate. The relation between the displacement vector of a slave node of the beam $\boldsymbol{\delta}_{3n}$ and that of the corresponding master node of the plate $\boldsymbol{\delta}_n$ can be written as

$$\boldsymbol{\delta}_{3n} = \boldsymbol{\lambda}\boldsymbol{\delta}_n \qquad n = i, j$$

where

$$\boldsymbol{\lambda} = \begin{vmatrix} 1 & 0 & 0 & 0 & h_0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \tag{8}$$

Thus we can obtain

$$\boldsymbol{\delta}_3^{\mathrm{e}} = \begin{vmatrix} \boldsymbol{\lambda} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} \\ \mathbf{0}_{4\times5} & \boldsymbol{\lambda} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} \end{vmatrix} \boldsymbol{\delta}^{\mathrm{e}} \tag{9}$$

According to the coordinate transforming formula[6] of the stiffness matrix of the element, the relation between $\boldsymbol{K}_3^{\mathrm{e}}$ and $\boldsymbol{K'}_3^{\mathrm{e}}$ is as follows:

$$\boldsymbol{K}_3'^{\mathrm{e}} = \begin{vmatrix} \boldsymbol{\lambda}^{\mathrm{T}} & \mathbf{0}_{5\times4} \\ \mathbf{0}_{5\times4} & \boldsymbol{\lambda}^{\mathrm{T}} \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times4} \\ \mathbf{0}_{5\times4} & \mathbf{0}_{5\times4} \end{vmatrix} \cdot \boldsymbol{K}_3^{\mathrm{e}} \begin{vmatrix} \boldsymbol{\lambda} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} \\ \mathbf{0}_{4\times5} & \boldsymbol{\lambda} & \mathbf{0}_{4\times5} & \mathbf{0}_{4\times5} \end{vmatrix} =$$

$$\begin{vmatrix} \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{K}_{3ii}\boldsymbol{\lambda} & \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{K}_{3ij}\boldsymbol{\lambda} & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} \\ \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{K}_{3ji}\boldsymbol{\lambda} & \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{K}_{3jj}\boldsymbol{\lambda} & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} \\ \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} \\ \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} & \mathbf{0}_{5\times5} \end{vmatrix} \tag{10}$$

where $\boldsymbol{K}_3^{\mathrm{e}}$ is the stiffness matrix of the beam element in its own coordinate system $x_3y_3z_3$, and $\boldsymbol{K'}^{\mathrm{e}}{}_3$ is the stiffness matrix of the beam element transformed to the plate coordinate system $x'y'z'$. What needed to be done, when the whole stiffness matrix was assembled, is to impose $\boldsymbol{K}_3'^{\mathrm{e}}$ to the stiffness matrix of the corresponding plate element.

1.5 Mass matrix $\boldsymbol{M}^{\mathrm{e}}$

The mass matrix $\boldsymbol{M}^{\mathrm{e}}$ can be obtained from the kinetic energy of the element. It should be noted that rotary inertia has been omitted from the kinetic energy because earlier study[2,3] has shown that it has little influence on the results and the mass of the beams are also ignored here because they occupy a very small percentage of the whole mass. The mass matrix of the plate element is

$$\boldsymbol{M}^{\mathrm{e}} = \int_A [m_1\boldsymbol{U}_1{}^{\mathrm{T}}\boldsymbol{U}_1 + m_1\boldsymbol{V}_1{}^{\mathrm{T}}\boldsymbol{V}_1 + m_2\boldsymbol{U}_2{}^{\mathrm{T}}\boldsymbol{U}_2 + m_2\boldsymbol{V}_2{}^{\mathrm{T}}\boldsymbol{V}_2 + (m_1 + m_2)\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W}]ab\,\mathrm{d}\xi\mathrm{d}\eta \tag{11}$$

where $m_i (i = 1, 2)$ is the mass per unit area of each layer.

2 Calculation of Modal Loss Factors[4]

The modal loss factor is a very important parameter to indicate the ability of the structures to consume the vibration energy. Here, based on the analysis results of the dynamic properties of the structures, modal loss factors are predicted by the modal strain energy method. The expression of the modal loss factor η_r is

$$\eta_r = \frac{\beta \sum_{p=1}^{P} (\boldsymbol{U}_r^p)^{\mathrm{T}} \boldsymbol{K}_{\mathrm{d}}^p \boldsymbol{U}_r^p}{\boldsymbol{U}_r^{\mathrm{T}} \boldsymbol{K} \boldsymbol{U}_r} \tag{12}$$

where β is the loss factor of the viscoelastic damping material, $\boldsymbol{U}_r^p$ is the r-th mode shape vector of the p-th element, $\boldsymbol{U}_r$ is the r-th mode shape vector, $\boldsymbol{K}$ is the total stiffness matrix of the whole structures, $\boldsymbol{K}_{\mathrm{d}}^p$ is the damping part of the stiffness matrix of the p-th element, here $\boldsymbol{K}_{\mathrm{d}}^p = \boldsymbol{K}_{\mathrm{flex2}} + \boldsymbol{K}_{\mathrm{ext2}}$, η_r is the modal loss factor of the r-th mode, and P is the total number of the plate element.

3 Numerical Example

A rectangular beam-stiffened plate shown in Fig.5 is analyzed. The boundary condition is taken as clamped at one edge. The base-plate and the beams are of the same material. The geometrical and physical parameters are both shown in Fig.5 and as follows: $E_1 = 210$ GPa, $\rho_1 = 7.8 \times 10^3$ kg/m^3, $\mu_1 = 0.3$, $h_1 = 0.01$ m (the thickness of the base-plate), $b = 0.01$ m (the width of beams), $h = 0.01$ m (the thickness of the beams). The parameters of the viscoelastic damping material layer are: $E_2 = 499.2$ MPa, $h_2 = 0.01$ m, $\rho_2 = 1.05 \times 10^3$ kg/m^3, $\mu_2 = 0.3$, $\beta = 0.5$. The computed first-five-order frequencies of the structure with the finite element model in this paper are compared with the results obtained by using ANSYS. The results are listed in Tab.1. The modal loss factors are predicted by the modal strain energy method.

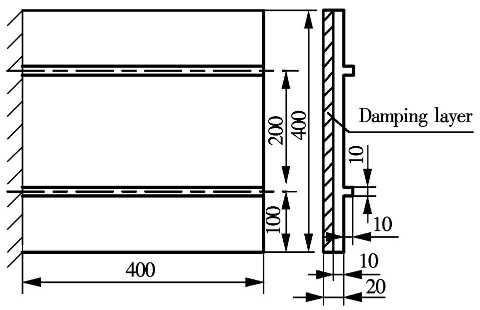

Fig.5 Free-damped beam-stiffened cantilever-plate(unit:mm)

Tab.1 Results comparison

No. of mode	Frequency f/Hz		Modal loss factor
	ANSYS	Proposed method	
1	7.044	7.052	0.031 90
2	17.540	17.68	0.034 12
3	44.053	44.82	0.034 65
4	56.133	57.29	0.034 91
5	63.704	65.10	0.035 00

It can be seen from Tab.1 that the results obtained with the two methods coincide fairly well. This illustrates that the modeling method here is suitable for analyzing the vibration characteristics of free-damped beam-stiffened plates.

4 Conclusion

The modeling method here is suitable for both free-damped and sandwich beam-stiffened plates, and it is much more convenient to predict the modal loss factors based on the analysis results of the dynamic properties of the structures using modal strain energy method. Although the free-damped plate element here is rectangular, it still can be applied to the plates with irregular boundaries by using the rectangular plate elements for the inner area and some free-damped triangular elements for the region around the outer edges. The modeling of the triangular elements for free-damped plates will be discussed later.

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壳梁组合结构自由阻尼处理有限元分析

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摘要: 构造了壳梁组合结构自由阻尼处理薄板结构的有限元模型, 将板单元的节点作为主节点, 梁单元的节点作为从属节点, 推导出相应的刚度矩阵和质量矩阵. 在用有限元法进行结构动态特性分析的基础上, 用模态变形能法估算了结构的模态损耗因子. 最后以一计算实例将本文方法所得结果与用 ANSYS 软件计算所得结果进行了比较, 结果表明本文方法计算效率高、简单可行, 且具有较高的精度和工程实用性.

关键词: 壳梁组合结构; 阻尼; 有限元

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