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Wall pressure analysis in squat silos

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Abstract: Rankine theory and Coulomb theory are not suitable for the calculation of wall pressure by bulk materials, so this paper studies the actual distribution and calculation methods for the wall pressure in squat silos. Based on the limits equilibrium theory, the force on unit width of wall exerted by bulk materials is firstly obtained, and then the distribution of wall pressure is obtained by accurate mathematical deduction. It is proved that the results are in good agreement with those of the full-sized silo experiment, whether the top of the stored bulk materials is a horizontal plane or a conical pile.

Key words: curvy wall; squat silos; bulk materials; wall pressure; plane of rupture

There are several well-known approaches to calculating the wall pressure in squat silos[1]. The most popular is the Rankine earth pressure theory. However, the retaining wall is two-dimensional and the projection of the silo wall is circular on the plane. So the theory may not be applied to the silo wall. If applied reluctantly, it is only an approximate calculation. Furthermore, the wall pressures from the Rankine theory vary linearly with depth. Nonetheless, the wall pressure of a bulk solid may not distribute linearly with depth because a bulk solid behaves very differently from a fluid one. When the height of the bulk materials pile is lower, the error caused by the above reason may be smaller. But with the increase of filling height, the error may become greater and greater.

In China's code[2], the Rankine active earth pressure formula is adopted in the calculation of wall pressure in squat silos. But when the top of the stored bulk materials is a conical pile, the Rankine formula does not work, because the Rankine theory must be based on the assumptions that the interface between the retaining wall and the soil is erect and smooth, and that the top plane of the bulk materials behind the retaining wall is horizontal[3]. Some modifications for these methods are induced. The depth between the horizontal filling plane and the point which is calculated in the Rankine earth pressure formula is replaced by the distance below the centroid of the cone in the code(Eq.(3.2.6-1)). Consequently, it leads to a finite value of pressure at the intersection of the top surface with the silo wall, and the free surface boundary condition cannot be properly satisfied. Therefore, the above modification in the code[2] is not appropriate.

The USA's standard (ACI313—91) does not distinguish between deep silos and squat silos any more, and it uses the formula of deep silos uniformly, i.e. the Janssen formula, but given different modified coefficients for different ratios of height to diameter. And yet the error, which is caused by artificial factors about modified coefficients, is unavoidable.

It can be seen that the above theories are not suitable for squat silos. Two analytical methods for the wall pressure in squat silos have been proposed in Refs. [4 − 8]. The two methods both assume that the wall pressure of the stored bulk materials distributes linearly with depth, which is appropriate when the height of the bulk materials is lower, but with the increase of the height, the actual distribution law for the wall pressure of the stored bulk materials should be discussed in depth.

1 Analysis of the Bulk Solid Active Pressures on the Curvy Retaining Wall

For the curvy retaining wall that is concave to the bulk materials, the bulk solid inside will exert a pressure on the retaining wall. According to the limits equilibrium theory, the retaining wall's failure occurs and the wall offsets outwards, i.e. deviating from the bulk materials. On this circumstance, the sliding wedge is divided into countless small units by radial planes, one of which is shown in Fig.1.

When the curvy wall is erect and the top of the bulk materials is a conical pile (the angle of the top pile surface relative to the horizontal is β), the volume of the sliding wedge with unit width wall of bulk materials as shown in Fig.1(a) is

$$V = \left(\frac{1}{2}\tan\theta - \frac{1}{3}\delta\tan^2\theta\right)h^2 + \left[\frac{1}{2} - \frac{2\delta}{3}\tan\theta -\right.$$

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$$\frac{\delta}{3}\frac{\sin\theta\sin\beta}{\cos(\theta+\beta)}\tan\theta\Big]\frac{h^2\sin\theta\sin\beta}{\cos(\theta+\beta)}\tan\theta \tag{1}$$

where $\delta = h/D$, h is the height of the bulk materials, $D = 2R$, R is the curvature radius of the curvy wall unit, the inclination angle θ is shown in Fig.1.

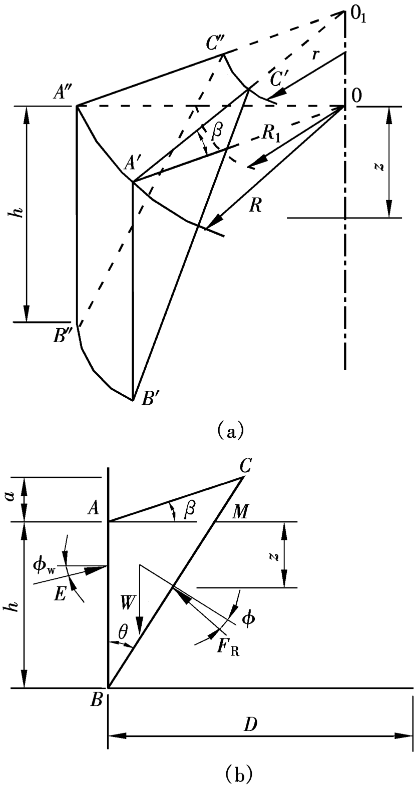

Fig.1 Sliding wedge unit of bulk materials. (a) Sliding wedge element; (b) Free body diagram of sliding wedge

The gravity of sliding wedge unit is

$$W = \gamma V \tag{2}$$

where γ is the density of the bulk materials.

According to the equilibrium condition, the relationship between W, the gravity of sliding block and E, the force from the wall with unit width is

$$E = \frac{W\cos(\theta+\phi)}{\sin(\theta+\phi+\phi_w)} \tag{3}$$

where ϕ is the internal friction angle of the bulk materials, and ϕ_w is the frictional angle between the silo wall and the bulk materials. Substituting Eqs. (1) and (2) into Eq.(3), we obtain

$$E = \frac{\cos(\theta+\phi)}{\sin(\theta+\phi+\phi_w)}\gamma\Big[\Big(\frac{1}{2}\tan\theta - \frac{1}{3}\delta\tan^2\theta\Big)h^2 + \Big(\frac{1}{2} - \frac{2\delta}{3}\tan\theta - \frac{\delta}{3}\frac{\sin\theta\sin\beta}{\cos(\theta+\beta)}\tan\theta\Big)\cdot \frac{h^2\sin\theta\sin\beta}{\cos(\theta+\beta)}\tan\theta\Big] \tag{4}$$

Differentiating Eq. (4) with $h = z$, the part of stress along horizontal at depth z can be caculated as

$$p_z = \gamma\frac{\cos(\theta+\phi)}{\sin(\theta+\phi+\phi_w)}\Big[\frac{1}{2}z\Big(2 - \frac{z}{R}\tan\theta\Big)\tan\theta + z\Big(1 - \frac{z}{R}\tan\theta - \frac{z}{2R}\frac{\sin\theta\sin\beta}{\cos(\theta+\beta)}\tan\theta\Big)\cdot \frac{\sin\theta\sin\beta}{\cos(\theta+\beta)}\tan\theta\Big]\cos\phi_w \tag{5}$$

The rupture angle θ_{cr} is determined by the condition that $E(\theta)$ reaches its maximum, namely

$$\frac{dE}{d\theta}\Big|_{\theta=\theta_{cr}} = 0$$

Only when the plane of rupture does not cut the central axis of the curvy wall, may this kind of failure occur. According to the special curvy wall whose projection is circular on the plane, the above condition is $\theta_{cr} \leqslant \theta_0$.

To the bulk materials with a top pile which is shown in Fig.2, θ_0 is determined by

$$\cot\theta_0 = \frac{h}{R} + \tan\beta \tag{6}$$

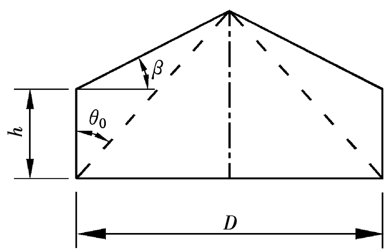

Fig.2 Bulk materials with a top pile

In this case, the pressure can be computed by Eq. (5), which is distributed as a curve (two power parabola).

2 Calculation Examples and Analyses

Example 1 $h = 11.4$ m, $D = 30$ m, $\phi = 25°$, $\gamma = 7.88$ kN/m^3(wheat), $\phi_w = 21.8°$(the silo wall is made from reinforced concrete). To the conical pile at the top, β is the dead angle of bulk materials, and $\beta = 25°$. Generally, $\beta \leqslant \phi$, i.e. β may be equal to or less than ϕ slightly.

E-θ graph is shown in Fig.3 from Eq.(4). From Fig.3, we obtain $\theta_{cr} = 32.5°$. Eq.(6) gives $\theta_0 = 39.93°$. Obviously, $\theta_{cr} < \theta_0$.

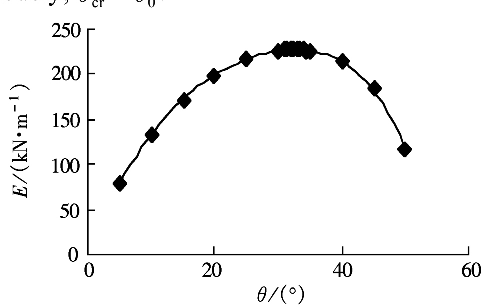

Fig.3 E-θ graph of calculation example 1

The results of wall pressure p_z computed by Eq. (5) are shown in Tab.1. The data from the code[2], the Coulomb formula and full-sized silo experiment are also given. Fig.4 is obtained from the data of Tab.1.

Tab.1 Wall pressure p_z of bulk materials with a conical pile kPa

No.	z/m	Measured value[9]	This paper	Coulomb formula	Code[2]	This paper ($\phi_w = 0°$)
8	1.1	9.070	4.571 9	7.668	10.98	5.736 9
7	2.6	14.202	10.299 8	18.125	15.78	12.924 3
6	4.1	17.341	15.475 5	28.651	20.61	19.418 9
5	5.6	21.262	20.115 8	39.317	25.5	25.241 4
4	7.2	25.211	24.099 1	49.843	30.33	30.239 8
3	8.7	29.324	27.510 7	60.440	35.2	34.520 7
2	10.1	30.904	30.138 8	70.270	39.71	37.818 5
1	11.1	35.778	31.718 1	77.380	42.97	39.800 2

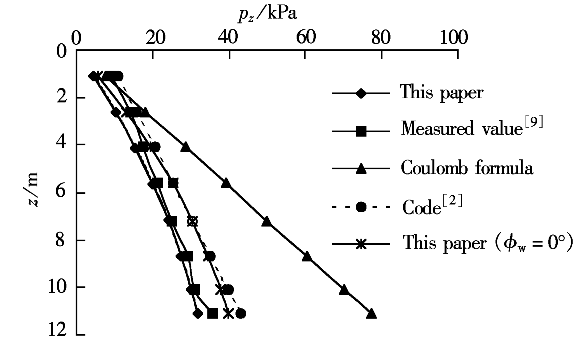

Fig.4 Wall pressure p_z of bulk materials with a conical pile

Example 2 $h = 13.77$ m, $D = 30$ m, $\phi = 25°$, $\gamma = 7.85$ kN/m^3 (wheat), and $\phi_w = 21.8°$ (the silo wall is made from reinforced concrete). The top of the stored bulk materials is a horizontal plane ($\beta = 0°$).

Calculating with the same method above, we obtain $\theta_{cr} = 27°$ and $\theta_0 = 47.47°$.

Obviously, $\theta_{cr} < \theta_0$.

The results of wall pressure p_z are shown in Tab.2. Fig.5 is obtained from the data of Tab.2.

Tab.2 Wall pressure p_z of bulk materials when its top is a horizontal plane kPa

No.	z/m	Measured value[9]	This paper	Coulomb formula	Rankine formula	This paper ($\phi_w = 0°$)
8	2.8	7.377	7.976	7.745	8.825	10.468
7	4.8	12.530	13.371	13.476	15.357	17.550
6	6.3	18.808	16.998	17.614	20.072	22.310
5	7.8	20.821	20.490	21.864	24.914	26.892
4	9.3	24.691	23.723	26.086	29.725	31.136
3	10.9	28.915	26.780	30.391	34.632	35.148
2	12.4	32.612	29.558	34.641	39.474	38.795
1	13.4	35.972	31.224	37.381	42.597	40.981

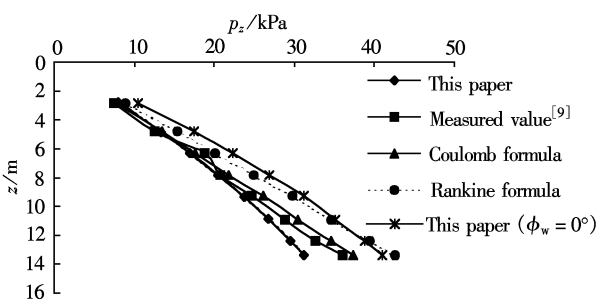

Fig.5 Wall pressure p_z of bulk materials when its top is a horizontal plane

Analyses of calculation examples are as follows:

1) When the top of the stored bulk materials is a horizontal plane, the calculated results from the Coulomb formula accord with real measured values; but when the top is a conical pile, the calculated results do not. When z is smaller, the calculated results underestimate real measured values; when z is larger, the calculated results overestimate real measured values considerably.

2) Whether the top of the stored bulk materials is a horizontal plane or a conical pile, the wall pressure distribution as a parabola in this paper accords with real measured values. If the friction between the silo wall and the bulk materials is considered, the calculated results underestimate real measured values slightly; if not, the condition is opposite. The real measured values are between two conditions and not far from them.

3) The formula for squat silos in the code has the above shortcomings, so its results are only referenced and not discussed again.

4) Although the friction between the silo wall and the bulk materials is not considered, the wall pressure distributes nonlinearly.

If the friction is considered, the calculated results underestimate real measured values slightly. That is because, the wall pressure of a curvy retaining wall is active pressure, while the wall pressure in the squat silo may be static pressure, and yet the two pressures are not far from each other.

3 Conclusions

1) When the interface between the retaining wall and the bulk materials is erect and smooth, and the top plane of the bulk materials behind the retaining wall is horizontal, the calculated results by the Rankine active earth pressure formula are the same as those by the Coulomb formula. But when the friction between the silo wall and the bulk materials is considered or the bulk materials with a top pile, the Rankine formula is not applicable.

2) The formula for squat silos in the code does not meet the stress state of the intersection of the top surface with the silo wall, and its calculated results overestimate real measured values. With the increase of the height of bulk materials, this deviation may become greater and greater.

3) When the top of the stored bulk materials is a horizontal plane and the diameter of the silo is large, the Coulomb formula is applicable; but when the top is a conical pile, the Coulomb formula is inapplicable. Although the top is a horizontal plane, the Coulomb formula may overestimate the real pressure, when the diameter of the silo is not large enough or the height of stored bulk materials is large enough.

4) Whether the top of the stored bulk materials is a horizontal plane or a conical pile, and whether the diameter of the silo is large or not, the wall pressure distribution law as a parabola is suitable. Considering the safety of silos and simplicity, the friction between the silo wall and the bulk materials may not be considered when calculating the horizontal wall pressure in squat silos.

When the ratio of height to diameter is larger, the formula in this paper needs further tests and verifications because of test data's absence.

References

[1] Ooi J Y, Chen J F, Lohnes R A, et al. Prediction of static wall pressure in coal silos [J]. *Construction and Building*, **1996, 10**(2): 109 − 116.

[2] GBJ77—85 Code for design of reinforced concrete silos [S]. Beijing: China Architecture & Building Press, 1990. (in Chinese)

[3] South China University of Technology, et al. *Groundwork and foundation*. 3rd Ed[M]. Beijing: China Architecture & Building Press, 1998. 134 − 141. (in Chinese)

[4] Shao Xing, Yuan Fang, Cui Yuanrui. Analyses of wall pressure in grain squat silo [J]. *Special Structures*, **2003, 20**(2): 44 − 45. (in Chinese)

[5] Yuan Fang, Shao Xing, Cui Yuanrui. A practical method for wall pressure calculation of big diameter squat silos [J]. *Special Structures*, **2003, 20**(2): 16 − 19. (in Chinese)

[6] Yuan Fang, Shao Xing, Wang Lumin, et al. A method for calculating lateral pressures on squat silo walls [J]. *Engineering Mechanics*, **2004, 21** (3): 86 − 100. (in Chinese)

[7] Yuan Fang, Wang Fuming, Shao Xing, et al. Formulae for computing bulk-solid pressure in squat silos [A]. In:Niu Ditao, Ru Jiping, eds. *Proc of the 8th International Symposium on Structural Engineering for Young Experts* [C]. Beijing: Science Press, 2004. 202 − 207.

[8] Yuan Fang. Analysis of bulk-solid pressures on curve walls and its engineering application [D]. Dalian: School of Civil and Hydraulic Engineering, Dalian University of Technology, 2004. (in Chinese)

[9] Li Xingzhao. Measurements and analyses of wall pressure in grain squat silo with large diameter [D]. Zhengzhou: Civil Engineering Department, Henan University of Technology, 2002. (in Chinese)

浅圆仓侧压力分析

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摘要: Rankine 理论和 Coulomb 理论均不适合用于浅圆仓的侧压力计算. 基于此, 本文研究了浅圆仓散料侧压力的实际分布规律及计算方法. 根据极限平衡理论, 首先得到散粒体作用在单位周长的仓壁上的合压力, 然后通过严格的数学推导, 得到浅圆仓侧压力的分布规律. 结果表明, 本文结果与实仓试验结果相符, 说明本文公式不论对平堆还是锥堆都比较适合, 而 Rankine 理论和 Coulomb 理论用于浅圆仓均有缺陷.

关键词: 曲线墙; 浅圆仓; 散体物料; 侧压力; 破裂面

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