Journal of Southeast University (English Edition) Vol. 20 No. 4 Dec. 2004 ISSN 1003—7985

Performance analysis of coded OFDM systems using a novel class of LDPC codes

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Abstract: A new method of constructing regular low-density parity-check (LDPC) codes is proposed. And the novel class of LDPC codes is applied in a coded orthogonal frequency division multiplexing (OFDM) system. This method extends the class of LDPC codes which can be constructed from shifted identity matrices. The method can avoid short cycles in Tanner graphs with simple inequation in the construction of shifting identity matrices, which makes the girth of Tanner graphs 8. Because of the quasi-cyclic structure and the inherent block configuration of parity-check matrices, the encoders and the decoders are practically feasible. They are linear-time encodable and decodable. The LDPC codes proposed have various code rates, ranging from low to high. They perform excellently with iterative decoding and demonstrate better performance than other regular LDPC codes in OFDM systems.

Key words: orthogonal frequency division multiplexing (OFDM); low-density parity-check (LDPC) codes; permutation matrix; construction; iterative decoding

Since Turbo codes were proposed in 1993[1], the study on channel codes closing the Shannon limit has become a new focus. Especially, in Ref. [2], MacKay and Neal rediscovered low-density parity-check codes, which were first proposed by Gallager[3,4]. Compared with Turbo codes, low-density parity-check (LDPC) codes with large block sizes have similar performances. There are two key distinctions between them. The first is that the latter are much easier to decode than the former. The second is that, for LDPC codes, the error floor cannot be detected at the bit error rate (BER) area, where it can be done for Turbo codes. These properties make LDPC codes worth considering for use in wideband systems.

For future mobile communication systems, high data rates through a wireless environment must be supported. As data rates increase, the effect of fading or multi-path in the channel becomes prominent. To solve this problem, some researchers have proposed orthogonal frequency division multiplexing (OFDM) systems, which use multiple orthogonal subcarriers to transmit large blocks of data.

1 A Novel Class of LDPC Codes

LDPC codes are represented by the Tanner graph[5], which is a bipartite graph with variable nodes and check nodes. If Tanner graph of the code is free of cycles, the sum-product decoding algorithm can provide the optimal decoding. However, it is very difficult or impossible to construct a bipartite graph without cycles for a deterministic block size. It is natural to maximize their length to reduce the effect, when these cycles exist in the graphs.

Some improvements have been made to Gallager's original random construction method to avoid short cycles. Researchers[6-8] gave some random or systematic methods to produce some LDPC codes. These constructions provide regular codes with Tanner graphs free of cycles of length 4 (4-cycles).

A novel class of LDPC codes is constructed in this paper using the block configuration of the parity check matrix in Ref. [8]. We propose two propositions to guarantee the girth of the Tanner graph 8 and 10. The number m of rows in the permutation matrix may be arbitrary. It is not necessary for the row weight and the column weight to be the prime factors of $m-1$.

Let m denote an arbitrary positive integer and we can generate an $m \times m$ identity matrix. If a regular (j, k) LDPC code needs to be constructed, we will generate such identity matrices as jk and circularly shift them to different places to get jk permutation matrices. The parity check matrix $\boldsymbol{H}$ of this LDPC code is made up of these permutation matrices and the code length is mk. We can obtain a $j \times k$ matrix $\boldsymbol{P}$, which is composed of some integers modulo m. Every sub-matrix $\boldsymbol{I}(S)$ in $\boldsymbol{H}$, denotes an $m \times m$ permutation matrix, which is obtained by circularly shifting an m

Received 2004-04-12.

Foundation item: Key Project of the National Natural Science Foundation of China (No. 60496311).

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$\times m$ identity matrix to the right by S places.

$$\boldsymbol{P} = \begin{bmatrix} S_{00} & S_{01} & S_{02} & \cdots & S_{0,k-1} \\ S_{10} & S_{11} & S_{12} & \cdots & S_{1,k-1} \\ \vdots & \vdots & \vdots & & \vdots \\ S_{j-1,0} & S_{j-1,1} & S_{j-1,2} & \cdots & S_{j-1,k-1} \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{I}(S_{00}) & \boldsymbol{I}(S_{01}) & \boldsymbol{I}(S_{02}) & \cdots & \boldsymbol{I}(S_{0,k-1}) \\ \boldsymbol{I}(S_{10}) & \boldsymbol{I}(S_{11}) & \boldsymbol{I}(S_{12}) & \cdots & \boldsymbol{I}(S_{1,k-1}) \\ \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{I}(S_{j-1,0}) & \boldsymbol{I}(S_{j-1,1}) & \boldsymbol{I}(S_{j-1,2}) & \cdots & \boldsymbol{I}(S_{j-1,k-1}) \end{bmatrix}$$

There is a single "1" element in every row or every column in a permutation sub-matrix, so a 4-cycle is possibly and only possibly produced among 4 sub-matrices. And the four corresponding symbols in $\boldsymbol{H}$ lie in two columns and two rows, which are illustrated in Fig.1. Similarly, a 6-cycle is possibly and only possibly produced among 6 sub-matrices. The six corresponding symbols in $\boldsymbol{H}$ lie in three columns, two in each column, and they lie in three rows, two in each row, which are illustrated in Fig.2. Assuming the element (r, c) in sub-matrix $\boldsymbol{I}(S)$ to be "1", we can easily get $r = (c - S) \bmod m$ and $c = (r + S) \bmod m$, where r and c are the sequence numbers in the sub-matrix $\boldsymbol{I}(S)$.

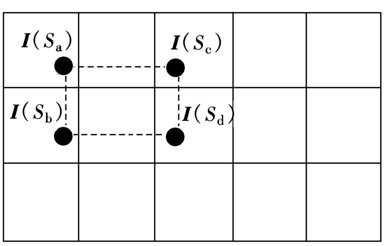

Fig.1 Four elements generating a 4-cycle

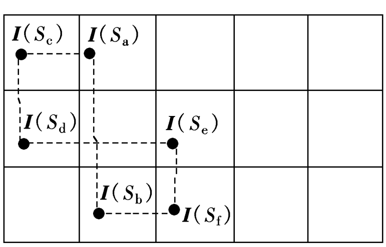

Fig.2 Six elements generating a 6-cycle

Proposition 1 If $(S_b - S_a) \bmod m \neq (S_d - S_c) \bmod m$ comes into existence, 4-cycles do not exist among the sub-matrices $\boldsymbol{I}(S_a)$, $\boldsymbol{I}(S_b)$, $\boldsymbol{I}(S_c)$ and $\boldsymbol{I}(S_d)$ in Fig.1.

Proposition 2 If $(S_b - S_a) \bmod m \neq (S_d - S_c + S_f - S_e) \bmod m$ comes into existence, 6-cycles do not exist among the six sub-matrices in Fig.2.

Propositions 1 and 2 can be used to construct new LDPC codes with girth 8. The basic steps may be given as follows.

Step 1 Generate k random integers modulo m as the values of S_{00}, S_{01}, $\cdots$, $S_{0,k-1}$ and $j-1$ random integers modulo m as the values of S_{10}, S_{20}, $\cdots$, $S_{j-1,0}$ in matrix $\boldsymbol{P}$.

Step 2 Randomly generate other elements in matrix $\boldsymbol{P}$ and check whether proposition 1 and proposition 2 are satisfied or not, if not, repeat the procedure or go back to step 1.

Step 3 Expand from matrix $\boldsymbol{P}$ to the corresponding matrix $\boldsymbol{H}$, the parity-check matrix of an LDPC code without cycles of length 4 and 6.

2 LDPC Coded OFDM System

The concept of using parallel data transmission and frequency division multiplexing was published in the mid 1960s[9,10]. Since the 1970s, the discrete Fourier transform (DFT) has been applied to parallel data transmission systems as part of the modulation and demodulation processes[11]. OFDM is a modulation technique, which separates a wideband signal into many narrowband signals to transmit. In frequency spectrum, these narrowband signals are overlapping and orthogonal to each other. If these subbands are narrow enough, they will suffer from flat fading, which makes the equalizer of the receiver be easily implemented. To avoid inter-carrier inference (ICI), sub-band signals through a multi-path channel need to keep orthogonal. Generally, cyclic prefix (CP) is added before transmitted data to delete ISI[12]. In this paper, an OFDM system implemented by an FFT transformation with CP is illuminated in Fig.3.

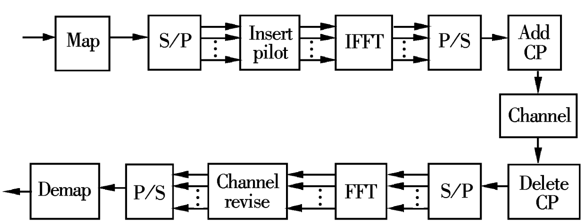

Fig.3 FFT-based OFDM system

In a multi-path fading channel, some sub-carriers of OFDM may be completely lost because of the deep fades. Error-correcting codes will be adopted to improve the system's performance. As a kind of block codes nearing the Shannon limit, LDPC codes will be operated. It is hoped that errors caused by fading on a small number of sub-carriers will be corrected by the LDPC decoder.

The performance of LDPC codes in OFDM systems has been examined[13,14] in the context of frequency and spatial diversity. In this paper, the emphasis is how to construct LDPC codes adapted to an OFDM system. The system model is shown in Fig.4.

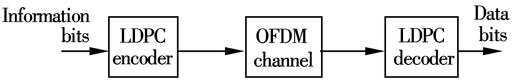

Fig.4 LDPC coded OFDM system model

At the transmitter, information bits are encoded with the LDPC encoder. Then codewords are sent to the OFDM channel, a generalized channel, to be dealt with. First the symbol mapper is used to modulate codewords into BPSK, QPSK and other such symbols. After serial-to-parallel conversion, the sub-channel modulation is implemented by using an inverse fast Fourier transform (IFFT). The CP is inserted for the purpose of eliminating the ISI on a frequency-selective fading channel. The reasons to use a cyclic prefix for the guard interval are[12] as follows: ① To maintain the receiver carrier synchronization; ② Cyclic convolution can still be applied between the OFDM signal and the channel response to model the transmission system.

At the receiver, the channel reviser modifies receiving subchannel data with some pilot channel data. In our simulation, the comb pilot pattern arrangement is considered. Frequency offset is supposed to be estimated accurately.

3 Simulation and Performance

Sum-product decoding algorithm, or belief propagation algorithm, has been described well in Refs.[6, 7, 15]. In the simulation, it is adopted and the maximum number of iterations is set to 200. The maximum error frame number is 50 at every signal-noise-ratio (SNR).

The number of OFDM subcarriers is set to 64 while BPSK modulation schemes are evaluated for each subcarrier. In the simulation, one frame in this system consists of 17 and 21 OFDM symbols, respectively. There are 48 data sub-carriers and 16 pilot sub-carriers in every symbol. For the multipath Rayleigh channel, the path number is 6 and the according delays are 0, T_c, $2T_c$, $3T_c$, $4T_c$ and $5T_c$, where T_c is chip length, equal to 2×10^{-8} s. The gains are 1.0, 0.5, 0.33, 0.25, 0.167 and 0.143. The carrier frequency is 2.86 GHz and the velocity of mobile terminator is 60 km/h. Finite-length regular LDPC codes are constructed by the new method which is introduced in section 1. They are named SIM-LDPC codes, since they are constructed based on shifted identity matrices. In simulation, MacKay codes come from Ref. [16], which are described in Ref. [6].

Fig.5 shows the block/frame error rate of SIM-LDPC codes and MacKay codes of length 816 and code rate 1/2. It can be seen that our codes outperform MacKay codes in coded OFDM system. Especially, to obtain a BER of 10^{-6}, the SNR of SIM816 codes is 0.5 dB lower than that of MacKay816. To obtain a frame error rate (FER) of 10^{-3}, SNR of the former is 0.7 dB lower than that of the latter.

Fig.6 shows the block/frame error rate of SIM-LDPC codes and MacKay codes of length 1 008 and code rate 1/2. Similarly, it can be seen that SIM1008 codes perform better than MacKay1008 codes in the coded OFDM system.

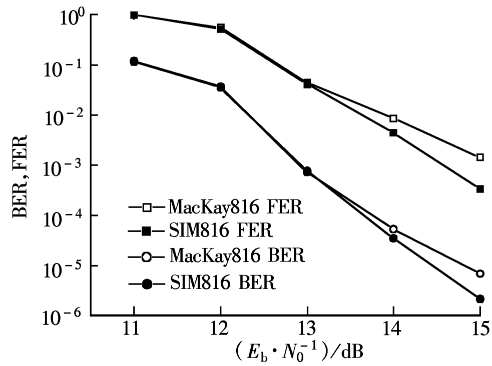

Fig.5 Performance of LDPC codes of length 816 with rate 1/2

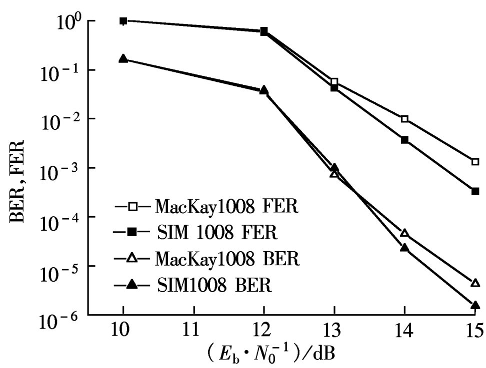

Fig.6 Performance of LDPC codes of length 1 008 with rate 1/2

4 Conclusion

In this paper, a novel class of LDPC codes is constructed and their performance in a coded OFDM system is examined. The construction is clearly an effective realization of LDPC codes with very good performance. Additionally, the encoder is easy to implement since newly-built LDPC codes are quasi-cyclic codes.

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利用一种新 LDPC 码的编码 OFDM 系统性能分析

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摘要: 提出了一种新的构造 LDPC 码的方法, 并把用该方法产生的 LDPC 码应用到编码正交频分复用(OFDM)系统中进行了研究. 该方法拓展了单位阵移位构造的 LDPC 码, 它利用简单的不等式可确保在置换单位阵构造中不会产生 Tanner 图中的短圈, 使得产生的 LDPC 码的 Tanner 图最小圈长为 8. 由于该类码是准循环码以及其校验矩阵所固有的分层结构, 编码器和解码器都易于实现, 它们分别是线性可编和线性可译的. 所提出的 LDPC 码码率范围较大, 可以灵活选取. 利用迭代解码进行计算机仿真, 发现新的 LDPC 码比其他的规则 LDPC 码在编码 OFDM 系统中具有更好的性能.

关键词: 正交频分复用(OFDM); 低密度奇偶校验(LDPC)码; 置换矩阵; 构造; 迭代解码

中图分类号: TN929.533