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Study on the tradeoff between interpretability and precision in fuzzy modeling

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Abstract: An approach to identifying fuzzy models considering both interpretability and precision is proposed. Firstly, interpretability issues about fuzzy models are analyzed. Then, a heuristic strategy is used to select input variables by increasing the number of input variables, and the Gustafson-Kessel fuzzy clustering algorithm, combined with the least square method, is used to identify the fuzzy model. Subsequently, an interpretability measure is described by the product of the number of input variables and the number of rules, while precision is weighted by root mean square error, and the selection objective function concerning interpretability and precision is defined. Given the maximum and minimum number of input variables and rules, a set of fuzzy models is constructed. Finally, the optimal fuzzy model is selected by the objective function, and is optimized by a genetic algorithm to achieve a good tradeoff between interpretability and precision. The performance of the proposed method is illustrated by the well-known Box-Jenkins gas furnace benchmark; the results demonstrate its validity.

Key words: fuzzy modeling; precision; interpretability; fuzzy clustering

In recent years, fuzzy systems have become an active research area. Compared to mathematical models and pure neural networks, fuzzy systems possess some distinctive advantages including the facility for explicit knowledge representation in the form of if-then rules, the mechanism of reasoning in human understandable terms, and the ability of approximating complicated nonlinear functions with simple models.

Several fuzzy modeling methods have been proposed including fuzzy clustering algorithm[1], neuro-fuzzy systems[2] and genetic rules generation[3]. However all these technologies only focus on model precision that simply fit the data with the highest possible accuracy, neglecting interpretability of the obtained fuzzy model, which is considered as a primary merit of fuzzy systems and is the most prominent feature that distinguishes fuzzy systems from many other models[4].

In this paper, we develop an approach to identifying a fuzzy model considering both interpretability and precision. The precision is weighted by root mean square error, while the interpretability is measured using the product of the number of input variables and the number of rules, which are the two most important factors concerning interpretability.

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1 Preliminaries

1.1 Takagi-Sugeno fuzzy model

The Takagi-Sugeno (TS) fuzzy model[5] was proposed in an effort to develop a systematic approach to generating a fuzzy model from a given input-output data set. A typical fuzzy rule of the model has the form:

R_i: If x_1 is $A_{i,1}$ and $\cdots$ and x_n is $A_{i,n}$,

$$\text{then } y_i = a_{i0} + a_{i1}x_1 + \cdots + a_{in}x_n \tag{1}$$

where x_j are the input variables, A_{ij} are fuzzy sets defined on the universe of discourse of the input variables, and y_i are outputs of rules.

The output of the TS fuzzy model is computed using the normalized fuzzy mean formula:

$$y(k) = \sum_{i=1}^{c} p_i(x) y_i \tag{2}$$

where p_i is the normalized firing strength of the i-th rule:

$$P_i(x) = \frac{\prod_{j=1}^{n} A_{ij}(x_j)}{\sum_{i=1}^{c}\prod_{j=1}^{n} A_{ij}(x_j)} \tag{3}$$

1.2 Interpretability issues about fuzzy model

Interpretability is a subjective property, and there is no formal definition of it. Nevertheless several aspects are believed to be essential, such as[6]:

1) The number of variables: a highly multi-dimensional fuzzy model is difficult to interpret. The model should use as few variables as possible.

2) The number of rules: a fuzzy model with a large rule base is less interpretable than a fuzzy system containing only a few rules. Experientially, the number of fuzzy rules of an interpretable model is no more than ten, which is determined by the human intellect.

3) Characteristics of membership functions: convexity and normality are two principle aspects which are satisfied naturally for most widely used membership functions, e. g., the Gaussian function, triangle function. The fuzzy partition of all input variables should be complete to prevent unpredictable system outputs. Fuzzy sets should be distinguishing, thus meaningful to assign linguistic terms to a fuzzy system. Usually, a minimum/maximum degree of overlapping between fuzzy sets must be enforced.

4) Completeness, consistency and compactness of fuzzy rules: for each effective input variable combination, there must be at least one fuzzy rule being fired, i. e. fuzzy rules cover the whole input space. The fuzzy rules in the rule base should be consistent. There must be no rule whose antecedent is a subset of another rule, and no rule may appear more than once in the rule base.

2 Fuzzy Modeling with Tradeoff between Precision and Interpretability

2.1 Input variable selection

The influence of different input variables to output is not equal. Some variables decide the output, while some variables can be neglected in the process of modeling. Input variable selection is a process of choosing a small subset of input variables from a large set of input variable candidates.

Assume the data are divided into part A and part B, which are used for modeling and testing, respectively; the input variable selection criterion (IVSC) is defined as follows[7]:

$$I_{\text{VSC}} = \frac{\alpha}{2}\left(\sqrt{\frac{1}{N_A}\sum_{i=1}^{N_A}(y_i^A - y_i^{AB})^2} + \sqrt{\frac{1}{N_B}\sum_{i=1}^{N_B}(y_i^B - y_i^{BA})^2}\right) + \frac{\beta}{2}\left(\sqrt{\frac{1}{N_A}\sum_{i=1}^{N_A}(y_i^A - y_i^{AA})^2} + \sqrt{\frac{1}{N_B}\sum_{i=1}^{N_B}(y_i^B - y_i^{BB})^2}\right) \quad (4)$$

where α and β are the weights of validation error and modeling error, N_A and N_B are the sizes of A and B, y_i^A and y_i^B are the actual outputs for A and B, y_i^{AB} is the output of the model trained on B tested with A, y_i^{BA} is the output of the model trained on A tested with B, y_i^{AA} is the output of the model trained on A tested with A, y_i^{BB} is the output of the model trained on B tested with B.

The process of input variable selection is described as follows: firstly, a fuzzy model is constructed for every input variable, and its corresponding IVSC is calculated. The input variable with the minimum IVSC is selected. We denote $S(1)$ for the set containing this variable. After selecting the most important variable, each of the remaining $n-1$ input variables is added to $S(1)$ orderly. For every two input variables, we compute their IVSC, and select the most important two variables. We denote $S(2)$ for the set containing these two variables. Next, we fix these two input variables and add the remaining $n-2$ input variables in order. In the same way, we can get set $S(3)$ including the three most important input variables. This process continues iteratively until either of the following conditions is satisfied.

Condition 1 The IVSC of $S(k+1)$ is not larger than that of $S(k)$.

Condition 2 The element number of subset $S(k)$ reaches the prespecified number of input variables.

Condition 3 The IVSC of $S(k)$ meets our predefined goal.

In all of the above cases, $S(k)$ is selected as the final input variable set.

2.2 Fuzzy modeling based on a fuzzy clustering algorithm

Fuzzy clustering is a well-recognized paradigm for constructing a fuzzy model. Numerous fuzzy clustering algorithms have been developed. Fuzzy C-means algorithm (FCM) proposed by Bezdek[8] is the base algorithm from the set of fuzzy clustering algorithms using the objective function and it has many modified versions. The Gustafson-Kessel[9] (GK) algorithm is an extension of FCM, whereas its clusters are ellipsoids and have different sizes in any dimension.

The objective function of GK algorithm is described as follows:

$$J(\boldsymbol{Z};\boldsymbol{U},\boldsymbol{V}) = \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m D_{ik}^2 \quad (5)$$

where $\boldsymbol{Z}$ is the set of data, $\boldsymbol{U} = [\mu_{ik}]$ is the fuzzy partition matrix, $\boldsymbol{V} = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_c\}^{\mathrm{T}}$ is the set of centers of the clusters, c is the number of clusters, N is the number of data, m is the fuzzy coefficient fuzziness, μ_{ik} is the membership degree between the i-

th cluster and k-th data, which satisfies conditions:

$$\mu_{ik} \in [0, 1], \quad \sum_{i=1}^{C} \mu_{ik} = 1, 0 < \sum_{k=1}^{N} \mu_{ik} < N \tag{6}$$

The norm of distance between the i-th cluster and k-th data is

$$D_{ik}^2 = \|\boldsymbol{z}_k - \boldsymbol{v}_i\|_{\boldsymbol{A}_i}^2 = (\boldsymbol{z}_k - \boldsymbol{v}_i)^{\mathrm{T}} \boldsymbol{A}_i (\boldsymbol{z}_k - \boldsymbol{v}_i) \tag{7}$$

where

$$\boldsymbol{A}_i = (\rho \det(\boldsymbol{F}_i))^{\frac{1}{n}} \boldsymbol{F}_i^{-1} \tag{8}$$

$$\rho = \det(\boldsymbol{A}_i) \tag{9}$$

$\boldsymbol{F}_i$ is the fuzzy covariance matrix of the i-th cluster,

$$\boldsymbol{F}_i = \frac{\sum_{k=1}^{N} (\mu_{ik})^m (\boldsymbol{z}_k - \boldsymbol{v}_i)(\boldsymbol{z}_k - \boldsymbol{v}_i)^{\mathrm{T}}}{\sum_{k=1}^{N} (\mu_{ik})^m} \tag{10}$$

The Lagrange multiplier is used to optimize the objective function (5) and the minimum of ($\boldsymbol{U}$, $\boldsymbol{V}$) is calculated as

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{c} (D_{ik}/D_{jk})^{\frac{2}{m-1}}} \tag{11}$$

$$\boldsymbol{v}_i = \frac{\sum_{k=1}^{N} (\mu_{ik})^m \boldsymbol{z}_k}{\sum_{k=1}^{N} (\mu_{ik})^m} \tag{12}$$

Given the input variable $\boldsymbol{X}$, output $\boldsymbol{y}$ and weight matrix $\boldsymbol{W}$.

$$\boldsymbol{X} = \{x_1, x_2, \cdots, x_N\}$$

$$\boldsymbol{y} = \{y_1, y_2, \cdots, y_N\}$$

$$\boldsymbol{W}_i = \mathrm{diag}(\mu_{i1}, \mu_{i2}, \cdots, \mu_{iN}) \tag{13}$$

Appending a unitary column to $\boldsymbol{X}$ gives the extended matrix:

$$\boldsymbol{X}_{\mathrm{e}} = [\boldsymbol{X} \quad 1] \tag{14}$$

Then the consequent parameters are obtained:

$$\boldsymbol{\theta}_i = (\boldsymbol{X}_{\mathrm{e}}^{\mathrm{T}} \boldsymbol{W}_i \boldsymbol{X}_{\mathrm{e}})^{-1} \boldsymbol{X}_{\mathrm{e}}^{\mathrm{T}} \boldsymbol{W}_i \boldsymbol{y} \tag{15}$$

The procedure of constructing the initial fuzzy model is summarized as follows:

① Choose the number of fuzzy rules and fuzzy coefficient fuzziness, and the stop criterion $\varepsilon > 0$;

② Generate the matrix $\boldsymbol{U}$ with the membership randomly, $\boldsymbol{U}$ must satisfy condition (6);

③ Compute the centers of the clusters using (12) and fuzzy covariance matrix by (10);

④ Calculate norm of distance utilizing (7);

⑤ Update the partition matrix $\boldsymbol{U}$ using (11);

⑥ Stop if $\| \boldsymbol{U}^{(l)} - \boldsymbol{U}^{(l-1)} \| \leqslant \varepsilon$, else go to ③;

⑦ Compute the consequence parameters of the fuzzy model using (15).

2.3 Optimal fuzzy model selection

Different from some fuzzy modeling techniques considering only accuracy, our method adopts both precision and interpretability. As shown in section 1, the numbers of input variables and fuzzy rules are the two most important factors about interpretability, so the measure of interpretability is simply described as follows:

$$I = cn \tag{16}$$

where c is the number of rules, i.e. the number of clusters, n is the number of input variables. The less I is, the more interpretable the fuzzy model is.

The root mean square error (RMSE) is adopted as precision criterion:

$$R = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (\hat{y}_k - y_k)^2} \tag{17}$$

where N is the number of data, $\hat{y}_k$ is the output of fuzzy model, y_k is the output of real system. The smaller R is, the more accurate the fuzzy model is.

Assume $n_{\max}$ and $n_{\min}$ are the maximum and minimum number of input variables, $c_{\max}$ and $c_{\min}$ are the maximum and minimum number of fuzzy rules, we can build $(n_{\max} - n_{\min} + 1)(c_{\max} - c_{\min} + 1)$ fuzzy models. The simplest fuzzy model utilizing $n_{\min}$ input variables and $c_{\min}$ fuzzy rules is the most interpretable, while its error is usually the largest. The most complicated fuzzy model containing $n_{\max}$ input variables and $c_{\max}$ fuzzy rules is most precise possibly, while is difficult to interpret. The proper fuzzy model considering both precision and interpretability is selected based on the objective function:

$$J_i = \lambda \frac{R_i - R_{\min}}{R_{\max} - R_{\min}} + \rho \frac{I_i - I_{\min}}{I_{\max} - I_{\min}} \tag{18}$$

where R_i is the RMSE of the i-th model, I_i is the interpretability measure of the i-th model, $R_{\max}$ and $R_{\min}$ are the maximum and minimum RMSE of models, $I_{\max}$ and $I_{\min}$ are the maximum and minimum interpretability of models, λ and γ are weighted coefficients satisfying $\lambda + \gamma = 1$. The model with the least J_i is chosen as the optimal fuzzy model.

The proposed optimal fuzzy model algorithm can be summarized as follows:

① Given the maximum and minimum number of input variables: $n_{\max}$ and $n_{\min}$, and the maximum and miniimum number of rules: $c_{\max}$ and $c_{\min}$;

② For $i = n_{\max}, \cdots, n_{\min}$;

③ For $j = c_{\max}, \cdots, c_{\min}$;

④ Compute the objective function J_i;

⑤ Go to step ③ until $j = c_{\max}$;

⑥ Go to step ② until $i = n_{\max}$.

The optimal fuzzy model with minimun objective function value gives a good tradeoff between precision and interpretability.

2.4 Genetic algorithm optimization

In order to improve the precision of the obtained fuzzy model, while preserving its interpretability, a constrained real coded genetic algorithm (GA)[10] is applied to optimize the parameters of the fuzzy model simultaneously.

The optimization is subject to search space constraints. Premise parameters are limited to change in a range of $\pm\tau\%$ of the corresponding input domain around their initial values in order to preserve the distinguishability of the fuzzy sets of the fuzzy model. Consequent parameters are restricted to vary $\pm\rho\%$ of the initial range of the corresponding consequent parameters for the sake of maintaining the interpretability of the local fuzzy model.

3 Example

In order to illustrate the performance of the proposed method, the well-known Box-Jenkins gas furnace benchmark is demonstrated in this section. It consists of 296 input-output measurements of gas flow rate $u(t)$ (input) and $y(t)$ (output).

We choose $u(k-1)$, $\cdots$, $u(k-4)$, $y(k-1)$, $\cdots$, $y(k-4)$ as input variable candidates. The result of input variable selection is shown in Fig. 1. Considering the simplicity, we can find that $S(3)$ will be an optimal subset since adding variables cannot improve the performance sharply.

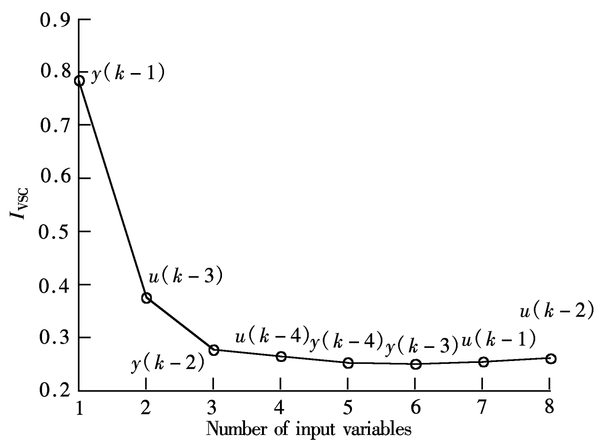

Fig.1 Input variable ordering and selection for gas furnace

Choose $c_{\min}=2$, $c_{\max}=5$, $n_{\min}=1$, $n_{\max}=6$, calculate the objective functions of fuzzy models, and the results are shown in Tab.1.

From Tab. 1, we can find that the model with three inputs and two rules reaches the minimum value of objective function , and is selected as the optimal fuzzy model.

Tab.1 Objective function values of models

Number of input	Number of rules			
	2	3	4	5
1	0.899 9	1.025 8	1.033 7	1.107 1
2	0.302 5	0.367 4	0.431 3	0.499 5
3	0.193 0	0.262 1	0.405 9	0.517 0
4	0.242 3	0.382 9	0.519 5	0.667 6
5	0.294 9	0.471 5	0.645 8	0.821 4
6	0.360 2	0.574 3	0.787 3	1.000 9

After GA optimization, the final TS fuzzy model is described as follows:

R_1: If $y(k-1)$ is A_{11} and $y(k-2)$ is A_{12} and $u(k-3)$ is A_{13}, then $y^1 = 1.408\,2y(k-1) - 0.560\,3y(k-2) - 0.391\,3u(k-3) + 8.200\,8$

R_2: If $y(k-1)$ is A_{21} and $y(k-2)$ is A_{22} and $u(k-3)$ is A_{23}, then $y^1 = 1.294\,5y(k-1) - 0.484\,2y(k-2) - 0.585\,6u(k-3) + 10.095\,6$

where A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, A_{23} are shown in Fig.2.

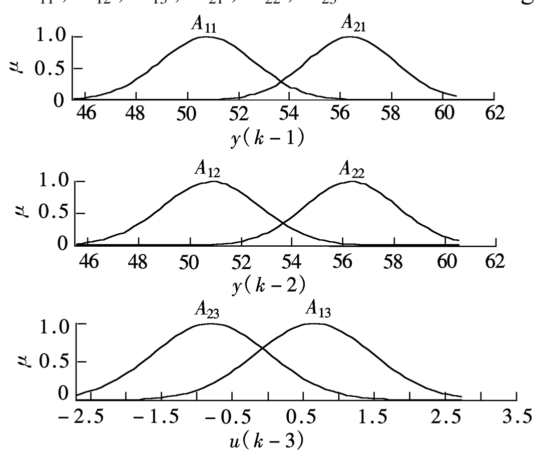

Fig.2 Membership functions of fuzzy model

The comparison of the measured output and the output of the fuzzy model is shown in Fig.3. The root mean square error of model is 0.064 3, which indicates that the obtained fuzzy model fits the system properly.

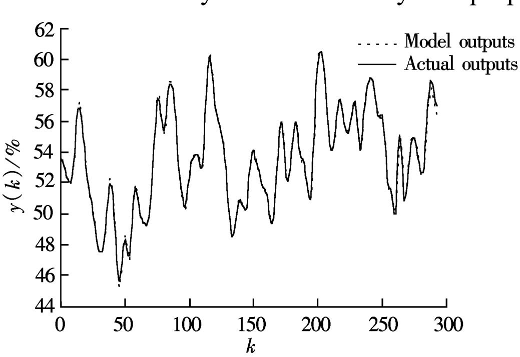

Fig.3 Comparison of the actual outputs and model outputs

4 Conclusion

In this paper, an approach to identifying a fuzzy model considering both precision and interpretability is presented. An input variable selection algorithm is proposed to select the most influenced variable. A fuzzy clustering algorithm, combined with the least square method, is used to identify a fuzzy model. An objective function is defined, integrating precision and interpretability, and applied to select the optimal fuzzy model. A genetic algorithm is used to optimize the fuzzy model to improve its accuracy. The approach is applied to the Box-Jenkins gas furnace benchmark, and the result demonstrates its validity.

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一种模糊建模方法的研究: 精确性与解释性的折衷

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摘要: 提出一种同时考虑解释性和精确性的模糊建模方法. 首先分析影响模糊模型解释性的主要因素, 然后利用启发式搜索策略实现输入变量选择, 利用模糊聚类算法和最小二乘辨识模糊模型. 随后以输入变量数目和模糊规则数目的乘积衡量可解释性, 以均方误差衡量精确性, 并据此定义模型选择目标函数. 最后给定最大最小的输入变量数目和规则数目, 辨识得到一组模糊模型, 利用模型选择目标函数, 选择最优的模糊模型, 并采用遗传算法进行优化, 达到解释性与精确性的折衷. 煤气炉仿真例子验证了该方法的有效性.

关键词: 模糊建模; 精确性; 解释性; 模糊聚类

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