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Linguistic approaches to multiple attribute decision making in uncertain linguistic setting

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Abstract: We study multiple attribute decision-making problems with uncertain linguistic information, in which the preference values take the form of uncertain linguistic variables. We introduce some operational laws of uncertain linguistic variables and a formula for the comparison between two uncertain linguistic variables. We propose two new aggregation operators called extended uncertain linguistic aggregation (EULA) operator and interval linguistic aggregation (ILA) operator, and then develop an EULA operator-based linguistic approach and an ILA operator-based linguistic approach, respectively, to multiple attribute decision making in uncertain linguistic setting. The approaches are straightforward and do not produce any loss of information. Finally, an illustrative example is given to verify the developed approaches and to demonstrate their practicality and effectiveness.

Key words: aggregation; multiple attribute decision making; EULA operator; ILA operator; operational laws

Making decisions with linguistic information is a usual task faced by many decision makers[1,2], and thus, the use of a linguistic approach is necessary[3]. Many approaches have been proposed for aggregating information up to now[4-7]. An ordinal linguistic computational model, which makes direct computations on labels, using the ordinal structure of the linguistic term sets, has been developed in Ref. [4]. An approximate computational model, based on the extension principle, to make computations over the linguistic variables has been developed in Ref. [5]. Herrera and Martínez[6] have developed a fuzzy linguistic representation model, which represents the linguistic information with a pair of values called 2-tuple, composed by a linguistic term and a number. Xu[7] has developed a direct approach to decision making with linguistic preference relations. All of these approaches, however, will fail in dealing with the situations in which the decision information takes the form of uncertain linguistic variables. The aim of this paper is to develop some approaches for overcoming this limitation.

1 Operational Laws of Uncertain Linguistic Variables

Let $S=\{s_i \mid i=0,1,\cdots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i, represents a possible value for a linguistic variable, and it should satisfy the following characteristics[3]: ① The set is ordered: $s_i > s_j$ if $i > j$; ② There is the negation operator: $\operatorname{neg}(s_i)=s_j$ such that $i+j=t$; ③ Max operator: $\max(s_i, s_j)=s_i$ if $s_i \geqslant s_j$; ④ Min operator: $\min(s_i, s_j)=s_i$ if $s_i \leqslant s_j$. For example, S can be defined as

$$S=\{s_0=\text{extremely poor},\ s_1=\text{very poor},\ s_2=\text{poor},\ s_3=\text{medium},\ s_4=\text{good},\ s_5=\text{very good},\ s_6=\text{extremely good}\}$$

To preserve all the given information, we extend the discrete term set S to a continuous term set $\bar{S}=\{s_a \mid s_0 \leqslant s_a \leqslant s_q,\ a\in[0, q]\}$, whose elements also meet all the characteristics above. If $s_a \in S$, then we call s_a the original linguistic term, otherwise, we call s_a the virtual linguistic term, q is a large positive integer. In general, the decision maker uses the original linguistic terms to evaluate attributes and alternatives, and the virtual linguistic terms can only appear in calculation.

Let $\tilde{s}=[s_\alpha, s_\beta]$, where $s_\alpha, s_\beta \in \bar{S}$, s_α and s_β are the lower and the upper limits, respectively, we then call $\tilde{s}$ the uncertain linguistic variable. Let $\tilde{S}$ be the set of all the uncertain linguistic variables.

Consider any three linguistic variables s_λ, s_{λ_1} and s_{λ_2}, and any three uncertain linguistic variables $\tilde{s}=[s_\alpha, s_\beta]$, $\tilde{s}_1=[s_{\alpha_1}, s_{\beta_1}]$ and $\tilde{s}_2=[s_{\alpha_2}, s_{\beta_2}]$, we define their operational laws as follows:

1) $\tilde{s}_1 \oplus \tilde{s}_2=[s_{\alpha_1}, s_{\beta_1}] \oplus [s_{\alpha_2}, s_{\beta_2}]=[s_{\alpha_1} \oplus s_{\alpha_2}, s_{\beta_1} \oplus s_{\beta_2}]=[s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}]$;

2) $\tilde{s}_1 \otimes \tilde{s}_2=[s_{\alpha_1}, s_{\beta_1}] \otimes [s_{\alpha_2}, s_{\beta_2}]=[s_{\alpha_1} \otimes s_{\alpha_2}, s_{\beta_1} \otimes s_{\beta_2}]=[s_{\alpha_1\alpha_2}, s_{\beta_1\beta_2}]$;

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3) $s_\lambda \otimes \tilde{s} = s_\lambda \otimes [s_\alpha, s_\beta] = [s_\lambda \otimes s_\alpha, s_\lambda \otimes s_\beta] = [s_{\lambda\alpha}, s_{\lambda\beta}]$;

4) $\tilde{s}_1 \oplus \tilde{s}_2 = \tilde{s}_2 \oplus \tilde{s}_1$;

5) $\tilde{s} \otimes (\tilde{s}_1 \oplus \tilde{s}_2) = (\tilde{s} \otimes \tilde{s}_1) \oplus (\tilde{s} \otimes \tilde{s}_2)$;

6) $\boldsymbol{s}_\lambda \otimes (\tilde{s}_1 \oplus \tilde{s}_2) = (\boldsymbol{s}_\lambda \otimes \tilde{s}_1) \oplus (\boldsymbol{s}_\lambda \otimes \tilde{s}_2)$;

7) $(\tilde{s}_1 \oplus \tilde{s}_2) \otimes \tilde{s} = (\tilde{s}_1 \otimes \tilde{s}) \oplus (\tilde{s}_2 \otimes \tilde{s})$;

8) $(s_{\lambda_1} \oplus s_{\lambda_2}) \otimes \tilde{s} = (s_{\lambda_1} \otimes \tilde{s}) \oplus (s_{\lambda_2} \otimes \tilde{s})$.

Definition 1 Let $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}]$ be two uncertain linguistic variables, and let $\mathrm{len}(\tilde{s}_1) = \beta_1 - \alpha_1$ and $\mathrm{len}(\tilde{s}_2) = \beta_2 - \alpha_2$, then the degree of possibility of $\tilde{s}_1 \geqslant \tilde{s}_2$ is defined as

$$p(\tilde{s}_1 \geqslant \tilde{s}_2) = \frac{\max\{0,\ \mathrm{len}(\tilde{s}_1) + \mathrm{len}(\tilde{s}_2) - \max(\beta_2 - \alpha_1, 0)\}}{\mathrm{len}(\tilde{s}_1) + \mathrm{len}(\tilde{s}_2)} \tag{1}$$

From definition 1, we can easily get the following results:

1) $0 \leqslant p(\tilde{s}_1 \geqslant \tilde{s}_2) \leqslant 1$, $0 \leqslant p(\tilde{s}_2 \geqslant \tilde{s}_1) \leqslant 1$;

2) $p(\tilde{s}_1 \geqslant \tilde{s}_2) + p(\tilde{s}_2 \geqslant \tilde{s}_1) = 1$. Especially, $p(\tilde{s}_1 \geqslant \tilde{s}_1) = p(\tilde{s}_2 \geqslant \tilde{s}_2) = 1/2$.

2 Linguistic Approaches

Definition 2 Let EULA: $\tilde{S}^n \to \tilde{S}$, if

$$\mathrm{EULA}_{s_\lambda}(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = (s_{\lambda_1} \otimes \tilde{s}_1) \oplus (s_{\lambda_2} \otimes \tilde{s}_2) \oplus \cdots \oplus (s_{\lambda_n} \otimes \tilde{s}_n)$$

where $\boldsymbol{s}_\lambda = \{s_{\lambda_1}, s_{\lambda_2}, \cdots, s_{\lambda_n}\}^{\mathrm{T}}$ is the linguistic weight vector of uncertain linguistic variables $\tilde{s}_i$ $(i = 1, 2, \cdots, n)$, and $s_{\lambda_i} \in S$, $\tilde{s}_i \in \tilde{S}$, $i = 1, 2, \cdots, n$, then EULA is called the extended uncertain linguistic aggregation (EULA) operator.

Definition 3 Let ILA: $\tilde{S}^n \to \tilde{S}$, if

$$\mathrm{ILA}_{\tilde{s}_\lambda}(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n) = (\tilde{s}_{\lambda_1} \otimes \tilde{s}_1) \oplus (\tilde{s}_{\lambda_2} \otimes \tilde{s}_2) \oplus \cdots \oplus (\tilde{s}_{\lambda_n} \otimes \tilde{s}_n)$$

where $\tilde{\boldsymbol{s}}_\lambda = \{\tilde{s}_{\lambda_1}, \tilde{s}_{\lambda_2}, \cdots, \tilde{s}_{\lambda_n}\}^{\mathrm{T}}$ is the uncertain linguistic weight vector of uncertain linguistic variables $\tilde{s}_i$ $(i = 1, 2, \cdots, n)$, and $\tilde{s}_{\lambda_i}$, $\tilde{s}_i \in \tilde{S}$, $i = 1, 2, \cdots, n$, then ILA is called the interval linguistic aggregation (ILA) operator.

In the following, we shall develop an EULA operator-based linguistic approach and an ILA operator-based linguistic approach, respectively, to multiple attribute decision making in an uncertain linguistic setting.

Case 1 For the multiple attribute decision-making problems, in which the attribute weights take the form of linguistic variables, and the preference values take the form of uncertain linguistic variables, we shall develop a linguistic approach based on the EULA operator as follows.

Step 1 Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete set of alternatives, $U = \{u_1, u_2, \cdots, u_m\}$ be the set of attributes, and $\boldsymbol{s}_\lambda = \{s_{\lambda_1}, s_{\lambda_2}, \cdots, s_{\lambda_m}\}^{\mathrm{T}}$ be the linguistic weight vector of the attributes u_i $(i = 1, 2, \cdots, m)$, where $s_{\lambda_i} \in S$, $i = 1, 2, \cdots, m$. Let $\tilde{\boldsymbol{A}} = (\tilde{a}_{ij})_{m \times n}$ be the decision matrix, where $\tilde{a}_{ij} \in \tilde{S}$ is a preference value, which takes the form of uncertain linguistic variable, given by the decision maker, for the alternative $x_j \in X$ with respect to the attribute $u_i \in U$.

Step 2 Utilize the decision information given in matrix $\tilde{\boldsymbol{A}}$, and the EULA operator

$$\tilde{a}_j = \mathrm{EULA}_{s_\lambda}(\tilde{a}_{1j}, \tilde{a}_{2j}, \cdots, \tilde{a}_{mj}) = (s_{\lambda_1} \otimes \tilde{a}_{1j}) \oplus (s_{\lambda_2} \otimes \tilde{a}_{2j}) \oplus \cdots \oplus (s_{\lambda_m} \otimes \tilde{a}_{mj}) \qquad j = 1, 2, \cdots, n$$

to derive the overall preference value $\tilde{a}_j$ of the alternative x_j.

Step 3 To rank these collective overall preference values $\tilde{a}_j$ $(j = 1, 2, \cdots, n)$, we first compare each $\tilde{a}_j$ with all the $\tilde{a}_j$ $(j = 1, 2, \cdots, n)$ by using Eq.(1). For simplicity, we let $p_{ij} = p(\tilde{a}_i \geqslant \tilde{a}_j)$, then we develop a complementary matrix[2] as $\boldsymbol{P} = (p_{ij})_{n \times n}$, where $p_{ij} \geqslant 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 1/2$, $i, j = 1, 2, \cdots, n$. Summing all the elements in each line of matrix $\boldsymbol{P}$, we have $p_i = \sum_{j=1}^{n} p_{ij}$, $i = 1, 2, \cdots, n$. Then we rank the overall preference values $\tilde{a}_j$ $(j = 1, 2, \cdots, n)$ in descending order in accordance with the values of p_i $(i = 1, 2, \cdots, n)$.

Step 4 Rank all the alternatives x_j $(j = 1, 2, \cdots, n)$ and select the best one(s) in accordance with the collective overall preference values $\tilde{a}_j$ $(j = 1, 2, \cdots, n)$.

Case 2 For the multiple attribute decision-making problems, in which all the attribute weights and the preference values take the form of uncertain linguistic variables, we shall develop a linguistic approach based on the ILA operator as follows.

Step 1 Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete set of alternatives, $U = \{u_1, u_2, \cdots, u_m\}$ be the set of attributes, and $\tilde{\boldsymbol{s}}_\lambda = \{\tilde{s}_{\lambda_1}, \tilde{s}_{\lambda_2}, \cdots, \tilde{s}_{\lambda_m}\}^{\mathrm{T}}$ be the uncertain linguistic weight vector of attributes, where $\tilde{s}_{\lambda_i} \in \tilde{S}$, $i = 1, 2, \cdots, m$. Let $\tilde{\boldsymbol{A}} = (\tilde{a}_{ij})_{m \times n}$ be the decision matrix, where $\tilde{a}_{ij} \in \tilde{S}$ is a preference value, which takes the form of an uncertain linguistic variable, given by the decision maker, for the alternative $x_j \in X$ with respect to the attribute $u_i \in U$.

Step 2 Utilize the decision information given in matrix $\tilde{\boldsymbol{A}}$, and the ILA operator

$$\tilde{a}_j = \mathrm{ILA}_{\tilde{s}_\lambda}(\tilde{a}_{1j}, \tilde{a}_{2j} \cdots, \tilde{a}_{mj}) = (\tilde{s}_{\lambda_1} \otimes \tilde{a}_{1j}) \oplus (\tilde{s}_{\lambda_2} \otimes$$

$\tilde{a}_{2j}) \oplus \cdots \oplus (\tilde{s}_{\lambda_m} \otimes \tilde{a}_{mj}) \qquad j=1, 2, \cdots, n$

to derive the overall preference value $\tilde{a}_j$ of the alternative x_j.

Step 3 See case 1.

Step 4 See case 1.

3 Illustrative Example

Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from Ref. [3]). There is a panel with five possible alternatives to invest the money: ① x_1 is a car company; ② x_2 is a food company; ③ x_3 is a computer company; ④ x_4 is an arms company; ⑤ x_5 is a TV company. The investment company must make a decision according to the following four attributes: ① u_1 is the risk analysis; ② u_2 is the growth analysis; ③ u_3 is the social-political impact analysis; ④ u_4 is the environmental impact analysis. The five possible alternatives $x_j(j=1, 2, \cdots, 5)$ are to be evaluated using the linguistic term set

$S = \{s_0$ = extremely poor, s_1 = very poor, s_2 = poor, s_3 = medium, s_4 = good, s_5 = very good, s_6 = extremely good$\}$

by the decision maker under the above four attributes, as listed in Tab.1.

Tab.1 Decision matrix $\tilde{\boldsymbol{A}}$

u_i	x_1	x_2	x_3	x_4	x_5
u_1	$[s_2, s_3]$	$[s_1, s_2]$	$[s_2, s_4]$	$[s_4, s_5]$	$[s_3, s_4]$
u_2	$[s_3, s_4]$	$[s_4, s_5]$	$[s_1, s_3]$	$[s_4, s_5]$	$[s_4, s_5]$
u_3	$[s_1, s_2]$	$[s_3, s_4]$	$[s_4, s_5]$	$[s_2, s_3]$	$[s_4, s_5]$
u_4	$[s_4, s_5]$	$[s_3, s_4]$	$[s_0, s_1]$	$[s_2, s_3]$	$[s_3, s_4]$

Case 1′ Suppose that the linguistic weight vector of four attributes $u_i(i=1, 2, 3, 4)$ is $\boldsymbol{s}_\lambda = \{s_3, s_1, s_2, s_4\}^{\mathrm{T}}$, then we utilize the linguistic approach developed in case 1 to get the most desirable alternative(s). The following steps are involved.

Step 1 Utilize the decision information given in matrix $\tilde{\boldsymbol{A}}$, and the EULA operator

$$\tilde{a}_j = \mathrm{EULA}_{s_\lambda}(\tilde{a}_{1j}, \tilde{a}_{2j}, \tilde{a}_{3j}, \tilde{a}_{4j}) = (s_{\lambda_1} \otimes \tilde{a}_{1j}) \oplus (s_{\lambda_2} \otimes \tilde{a}_{2j}) \oplus (s_{\lambda_3} \otimes \tilde{a}_{3j}) \oplus (s_{\lambda_4} \otimes \tilde{a}_{4j}) \qquad j=1, 2, \cdots, 5$$

to derive the overall preference value $\tilde{a}_j$ of the alternative x_j:

$\tilde{a}_1 = [s_{27}, s_{37}]$, $\tilde{a}_2 = [s_{25}, s_{35}]$, $\tilde{a}_3 = [s_{15}, s_{29}]$

$\tilde{a}_4 = [s_{28}, s_{38}]$, $\tilde{a}_5 = [s_{33}, s_{43}]$

Step 2 To rank these overall preference values $\tilde{a}_j(j=1, 2, \cdots, 5)$, we first compare each $\tilde{a}_j$ with all the $\tilde{a}_j(j=1, 2, \cdots, 5)$ by using Eq.(1), and then develop a complementary matrix:

$$\boldsymbol{P} = \begin{bmatrix} 0.500 & 0.600 & 0.917 & 0.450 & 0.200 \\ 0.400 & 0.500 & 0.833 & 0.350 & 0.100 \\ 0.083 & 0.167 & 0.500 & 0.042 & 0.000 \\ 0.550 & 0.650 & 0.958 & 0.500 & 0.250 \\ 0.800 & 0.900 & 1.000 & 0.750 & 0.500 \end{bmatrix}$$

Summing all the elements in each line of matrix $\boldsymbol{P}$, we have

$p_1 = 2.667$, $p_2 = 2.183$, $p_3 = 0.792$

$p_4 = 2.908$, $p_5 = 4.250$

Then we rank the overall preference values $\tilde{a}_j(j=1, 2, \cdots, 5)$ in descending order in accordance with the values of $p_i(i=1, 2, \cdots, 5)$: $\tilde{a}_5 > \tilde{a}_4 > \tilde{a}_1 > \tilde{a}_2 > \tilde{a}_3$.

Step 3 Rank all the alternatives $x_j(j=1,2, \cdots, 5)$ in accordance with the overall preference values $\tilde{a}_j$ $(j=1,2, \cdots, 5)$: $x_5 \succ x_4 \succ x_1 \succ x_2 \succ x_3$, and thus the most desirable alternative is x_5.

Case 2′ Suppose that the uncertain linguistic weight vector of five attributes $x_j(j=1, 2, \cdots, 5)$ is

$$\tilde{\boldsymbol{s}}_\lambda = \{[s_2, s_3], [s_1, s_2], [s_0, s_1], [s_3, s_4]\}^{\mathrm{T}}$$

then we utilize the linguistic approach developed in case 2 to get the most desirable alternative(s). The following steps are involved.

Step 1 Utilize the decision information given in matrix $\tilde{\boldsymbol{A}}$, and the EULA operator

$$\tilde{a}_j = \mathrm{EULA}_{\tilde{s}_\lambda}(\tilde{a}_{1j}, \tilde{a}_{2j}, \tilde{a}_{3j}, \tilde{a}_{4j}) = (\tilde{s}_{\lambda_1} \otimes \tilde{a}_{1j}) \oplus (\tilde{s}_{\lambda_2} \otimes \tilde{a}_{2j}) \oplus (\tilde{s}_{\lambda_3} \otimes \tilde{a}_{3j}) \oplus (\tilde{s}_{\lambda_4} \otimes \tilde{a}_{4j}) \qquad j=1, 2, \cdots, 5$$

to derive the overall preference value $\tilde{a}_j$ of the alternative x_j:

$\tilde{a}_1 = [s_{19}, s_{39}]$, $\tilde{a}_2 = [s_{15}, s_{36}]$, $\tilde{a}_3 = [s_5, s_{27}]$

$\tilde{a}_4 = [s_{18}, s_{40}]$, $\tilde{a}_5 = [s_{19}, s_{43}]$

Step 2 To rank these overall preference values $\tilde{a}_j(j=1, 2, \cdots, 5)$, we first compare each $\tilde{a}_j$ with all the $\tilde{a}_j(j=1, 2, \cdots, 5)$ by using Eq.(1), and then develop a complementary matrix:

$$\boldsymbol{P} = \begin{bmatrix} 0.500 & 0.585 & 0.810 & 0.500 & 0.455 \\ 0.415 & 0.500 & 0.721 & 0.419 & 0.378 \\ 0.190 & 0.279 & 0.500 & 0.205 & 0.174 \\ 0.500 & 0.581 & 0.795 & 0.500 & 0.457 \\ 0.545 & 0.622 & 0.826 & 0.543 & 0.500 \end{bmatrix}$$

Summing all the elements in each line of matrix $\boldsymbol{P}$, we have

$p_1 = 2.850$, $p_2 = 2.433$, $p_3 = 1.348$

$p_4 = 2.833$, $p_5 = 3.036$

Then we rank the overall preference values $\tilde{a}_j(j=1, 2, \cdots, 5)$ in descending order in accordance with the values of $p_i(i=1, 2, \cdots, 5)$: $\tilde{a}_5 > \tilde{a}_1 > \tilde{a}_4 > \tilde{a}_2 > \tilde{a}_3$.

Step 3 Rank all the alternatives $x_j(j=1,2, \cdots, 5)$ in accordance with the overall preference values $\tilde{a}_j$ $(j=1, 2, \cdots, 5)$: $x_5 \succ x_1 \succ x_4 \succ x_2 \succ x_3$, and thus the most desirable alternative is x_5.

4 Conclusion

In this paper, we have introduced some operational laws of uncertain linguistic variables, and proposed two new aggregation operators called EULA operator and ILA operator. We have developed two linguistic approaches to multiple attribute decision making in an uncertain linguistic setting. The approaches are straightforward and do not produce any loss of information. An illustrative example has been given to verify the developed approaches and to demonstrate their feasibility and practicality.

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不确定语言环境下的多属性决策方法

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摘要: 研究了偏好值以不确定语言变量形式给出的多属性决策问题. 介绍了不确定语言变量的运算法则, 给出了不确定语言变量之间两两比较的可能度公式, 提出了2种新的数据信息集成算子拓展的不确定语言集成(EULA)算子和区间语言集成(ILA)算子, 并且分别提出了基于EULA算子和基于ILA算子的不确定语言环境下的多属性决策方法. 这2种方法不仅简洁、易懂, 而且在运算过程中不会丢失任何决策信息. 最后, 通过算例对2种方法的实用性和有效性进行了说明.

关键词: 集成; 多属性决策方法; EULA算子; ILA算子; 运算法则

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