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Stability analysis of a concrete gravity dam and its foundation

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Abstract: The stability of dams and their foundations is an important problem to which dam engineers have paid close attention over the years. This paper presents two methods to analyze the stability of a gravity dam and its foundation. The direct analysis method is based on a rigid limit equilibrium method which regards both dam and the rock foundation as undeformable rigid bodies. In this method, the safety factor of potential sliding surfaces was computed directly. The second method, the indirect analysis method, is based on elasto-plastic theory and employs nonlinear finite element method (FEM) in the analysis of stresses and deformation in the dam and its foundation. The determination of the safety degree of the structure was based on the convergence and abrupt the change criterion. The results obtained show that structures' constituent material behavior plays an active role in the failure of engineered structures in addition to the imposed load.

Key words: stability; gravity dam; safety factor; rigid limit equilibrium theory; elasto-plastic theory; nonlinear finite element method

Evaluation of the stability of concrete gravity dams and their foundations is always a major design consideration because of the catastrophic consequences of dam failure, which may involve a considerable amount of direct and indirect losses. Stability problems in dam engineering are becoming increasingly complex and sophisticated, demanding the use of efficient and accurate methods that are precise and, if possible, exhaustive. Considerable research work has been done in the field; engineers and designers have built many experimental and practical dams to enhance a dam's stability[1]. There are many hazards associated with dams; various methods have been employed for their safety evaluation. Du, et al. used the nonlinear finite element to study the effect of a perimentral joint on the stress state of the Xiaowan dam[1]. The strength reduction technique has become an efficient tool to evaluate the safety degree of structures; this method was applied by some researchers to study stability problems in engineering[2]. The failure of the Meihua dam was studied by overload method using the nonlinear finite element model, the various failure processes of the dam were simulated and the failure pattern coincided with the actual failure of the dam[3]. In this study, the nonlinear finite element method that accounts for the elastic-plastic material behavior and the strength reduction techniques are employed to evaluate the safety degree of a concrete gravity dam. These methods are complemented by the traditional dam stability analysis method — limit equilibrium method.

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1 Limit Equilibrium Approach

The limit equilibrium approach in stability problems follows the conventional soil mechanics logic in defining the limit equilibrium safety factor K as the ratio of shear strength to mean applied shear stress across the plane of failure.

1) Single sliding surface

$$K=\frac{F'[(\sum W+G_1)\cos\alpha-H\sin\alpha-U_1]+C_1A_1}{(\sum W+G_1)\sin\alpha+H\cos\alpha}\tag{1}$$

2) Double sliding surfaces

$$\left.\begin{aligned}K_{AB}&=\frac{F'_1[(\sum W+G_1)\cos\alpha-H\sin\alpha-Q\sin(\phi-\alpha)-U_3\sin\alpha-U_1]+C_1A_1}{(\sum W+G_1)\sin\alpha+H\cos\alpha-Q\cos(\phi-\alpha)-U_3\cos\alpha}\\K_{BC}&=\frac{F'_2[(\sum V+G_2)\cos\beta+Q\sin(\alpha+\beta)+U_3\sin\beta-U_2]+C_2A_2}{Q\cos(\alpha+\beta)-(\sum V+G_2)\sin\beta+U_3\cos\beta}\end{aligned}\right\}\tag{2}$$

where F'_1 and F'_2 are the frictional coefficients; $\sum W$ is the sum of the weight of gravity dam; G_1 is the self weight of sliding block ABC; G_2 is the self weight of sliding block BCD; H is the water height at the upstream face of the dam; U_1 is the uplift pressure at face AB; C_1 is the cohesion parameter at face AB; C_2 is the cohesion parameter at face BC; A_1 is the length of the sliding face AB; α is the angle of inclination of the sliding face AB; A_2 is the length of sliding face BC; Q is the reaction between the two sliding blocks; ϕ is the angle which Q makes with the horizontal; β is the angle of inclination of sliding face BC; U_2 is the uplift pressure at face BC; V is the vertical water pressure at the downstream face of the dam; K_{AB} and K_{BC} are the safety factors for sliding faces AB and BC; U_3 is the uplift pressure at face BD.

The advantages of this method are as follows: computation is simple and the meaning is quite clear; furthermore, this method has been in use by dam engineers over the years and it is well known in the field of dam engineering for solving dam's stability problem. This method only considers the balance condition of the system under consideration, and does not consider the deformation compatibility conditions and the constitutive relations of the material under investigation.

2 Elasto-Plastic Modeling of Material Behavior

It is reasonable to assume that the recent development of numerical methods in general and of the finite element method (FEM) in particular permits solutions to be obtained for any rationally conceived constitutive model of material behavior[4].

In order to formulate a theory which models elasto-plastic material deformation, three requirements are of paramount importance[5].

- An explicit relationship between stress and strain must be formulated to describe material behavior under elastic conditions, that is, before the on-set of plastic deformation.
- A yield criterion which indicates the stress level at the point where plastic flow commences.
- Stress-strain relationship must be developed for post-yield behavior; deformation is made up of both elastic and plastic components. Only the essential expressions of this formulation are provided in this paper.

The relationship between stress and strain before the plastic yielding begins is represented as follows:

$$\sigma_{ij} = D^{e}_{ijkl}\varepsilon_{kl} \tag{3}$$

where σ_{ij} and ε_{kl} are the stress and strain components, D^{e}_{ijkl} is the tensor of elastic constants.

2.1 Yielding criterion for solid elements

It is accepted as an experimental fact that yielding can occur only if stress σ satisfies the general criterion.

$$F = (\boldsymbol{\sigma},\ k) = 0 \tag{4}$$

where k is a hardening parameter. In this formulation, for rocks and concrete only, yielding is considered as a failure pattern and the condition of failure is

$$F = \alpha I_1 + \sqrt{J_2} - \frac{\sigma}{\sqrt{3}} \tag{5}$$

where $I_1 = \sigma_{ij}$, $J_2 = \frac{1}{2}\sigma'_{ij}\sigma'_{ij}$.

Analysis of linear Mohr Coulomb material based on the constitutive description is available in computer software MARC 2000 through the isotropic model definition option.

The factors α and σ are related to c and ϕ as follows:

$$c = \frac{\sigma}{3(1-12\alpha^2)^{\frac{1}{2}}}, \quad \sin\phi = \frac{3c}{(1-3\alpha^2)^{\frac{1}{2}}}$$

where c and ϕ denote the cohesion and the internal friction angle in the material, respectively.

2.2 Total stress-strain relationships

During the infinitesimal increment of stress, changes in strain are assumed to be divisible into elastic and plastic parts:

$$\partial\boldsymbol{\varepsilon} = \partial\boldsymbol{\varepsilon}_{\mathrm{e}} + \partial\boldsymbol{\varepsilon}_{\mathrm{p}} \tag{6}$$

The elastic strain increments are related to stress increments by a symmetric matrix of constants $\boldsymbol{D}_{\mathrm{e}}$, known as the elasticity matrix; where $\partial\boldsymbol{\varepsilon}_{\mathrm{p}}$ denotes the increment of plastic strain. This complete elasto-plastic incremental stress-strain relationship can be represented accordingly as

$$\partial\boldsymbol{\sigma} = \boldsymbol{D}_{\mathrm{ep}}\partial\boldsymbol{\varepsilon} \tag{7}$$

where $\boldsymbol{D}_{\mathrm{ep}}$ is the elasto-plastic matrix.

$$\boldsymbol{D}_{\mathrm{ep}} = \left[\boldsymbol{D}_{\mathrm{e}} - \frac{\boldsymbol{D}_{\mathrm{e}}\boldsymbol{a}\boldsymbol{a}^{\mathrm{T}}\boldsymbol{D}_{\mathrm{e}}}{A + \boldsymbol{a}^{\mathrm{T}}\boldsymbol{D}_{\mathrm{e}}\boldsymbol{a}}\right] \tag{8}$$

where $\boldsymbol{a}$, $\boldsymbol{D}$ and A are plastic flow, elasticity matrix and hardening parameters, respectively.

2.3 Finite element formulation

The finite element procedure of analysis considered the dam as assemblage of elements interconnected at nodes. For elastic condition,

$$\boldsymbol{\sigma} = \boldsymbol{D}_{\mathrm{e}}\boldsymbol{B}\boldsymbol{\delta} \tag{9}$$

where $\boldsymbol{B}$ is the typical sub-matrix with the first derivative of shape function in respect to the global coordinate. The stress is related to the strain as

$$\boldsymbol{\sigma} = \boldsymbol{D}_{\mathrm{ep}}\boldsymbol{B}\boldsymbol{\delta} \tag{10}$$

3 Geometrical Model

The gravity dam considered in this analysis is 138 m high and 45 m wide at the dam top, 150 m wide at the base on a faulted rock foundation; the reservoir elevation is 135 m at the up-stream face of the dam.

The study was carried out by using the limit equilibrium model (Fig. 1) and finite element model (Fig. 2) to access the safety factors of the concrete gravity dam. The dam system was discretized into 1 050 isoparametric 1131-noded 2-D elements. The dam body was modeled with 209 elements; while the remaining elements were used for the foundation, which is composed of three different materials viz. the competent rock (735 elements), the fault (28 elements) and the less competent rock (78 elements). The material's parameters considered in the analysis are shown in Tab.1.

Tab.1 Materials adopted in the FEM analysis

Materials	Young's Modulus E/GPa	Poisson's ratio μ	Initial yield stress σ/MPa	Mass density ρ/(kN · m^{-2})	α	Cohesion c/MPa	Frictional angle $\tan\phi$
Rock 1	15.00	0.220	1.894 0	0.026	0.246 0	1.20	1.40
Concrete	17.60	0.167	2.280 0	0.024	0.159 0	0.91	0.57
Fault	3.00	0.250	1.0430	0.026	0.0330	0.35	0.10
Rock 2	7.25	0.250	1.9752	0.026	0.2172	1.00	0.99

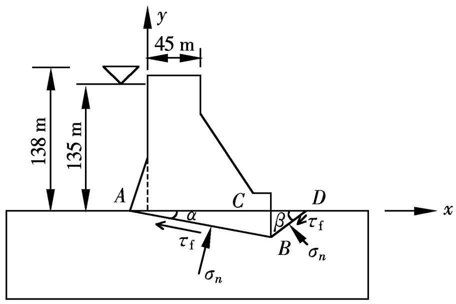

Fig.1 Geometry for limit equilibrium analysis

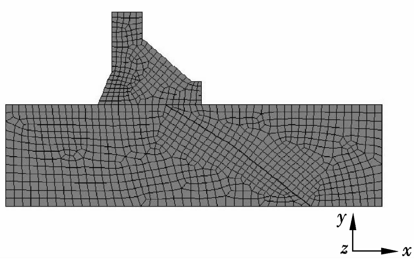

Fig.2 Finite element mesh (1 050 elements)

4 Safety Factor Assessment

The principle behind the shear strength reduction techniques in nonlinear finite element analysis that accounts for dam system materials elastic-plastic response for 2-D case was employed. The shear strength parameters C and $\tan\phi$, and frictional angle are reduced incrementally by a reduction factor k, for $k = 1, 2, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2$. The reduced shear strength parameters and the mechanical properties (Young's modulus E and Poisson ratio μ of the dam's foundation) are used as input data in the computer software MARC 2000 to evaluate the displacement and stress regime.

The Young's modulus E is increasingly reduced by the same value of k; this corresponds to reducing the hardness of the dam's foundation materials. The reduced E values with other parameters, c and $\tan\phi$ (unchanged) are used as input data for the same computer software MARC 2000.

The above numerical procedure of analysis was complimented by an analytical method — the limit equilibrium by using Eqs.(1) and (2).

A computer program coded in Fortran language was used to estimate k under the assumption of two potential sliding faces as indicated above.

5 Results and Discussion

5.1 Results for the scenarios of shear strength reduction and E values

Results can be drawn from the displacement curves as follows:

1) Turning points of all curves are the critical points; the corresponding value of $k = 2.6$ from each graph is considered to be the limit safety factor. See Figs.3(a) and (b).

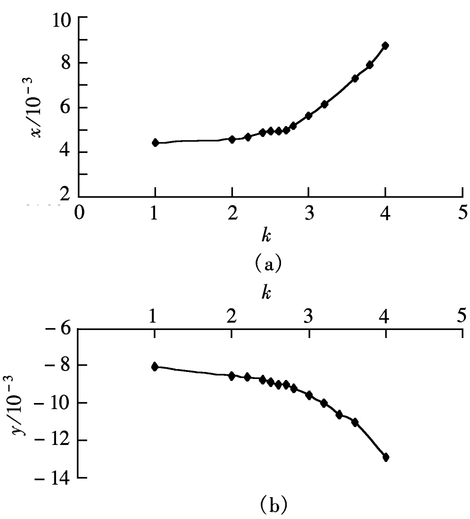

Fig.3 Displacement curves. (a) x direction at node 16; (b) y direction at node 3

2) With an increase in the values of the reduction factor, the displacement of the dam increases correspondingly; above 2.6, the displacement increases rapidly, as can be observed from the displacement curves in Figs.3(a) and (b).

3) The displacement of the dam top is very small before the critical point and the displacement increases rapidly after the value of $k > 2.6$. This indicates that yielding failure of the dam is closely related to the critical value of k.

4) There is a linear relationship between Young's modulus E and displacement. As E values are reduced, the displacement increases correspondingly as shown in Figs.4(a) and (b).

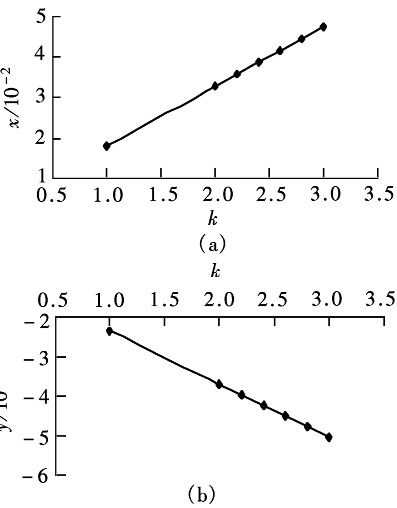

Fig.4 Displacement curves for sensitivity analysis. (a) x direction at node 6; (b) y direction at node 5

5) In order to realize the failure process of the dam, Fig.5 shows the process of yielding at $k = 1.0$ and 2.2, and Fig.6 shows the yielding process at $k =$ 2.6 and 2.7. Yielding starts at dam-foundation interface and spreads to foundation materials (fault) with lower strength.

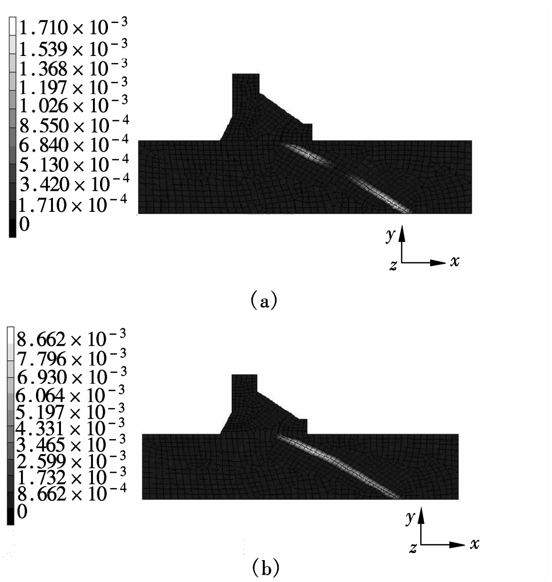

Fig.5 The plastic strain when the safety factor k is 1 or 2.2. (a) $k = 1.0$; (b) $k = 2.2$

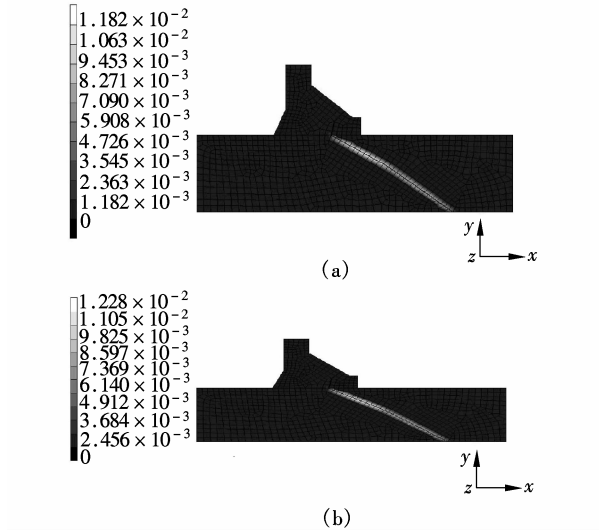

Fig.6 The plastic strain when the safety factor k is 2.6 or 2.7. (a) $k = 2.6$; (b) $k = 2.7$

5.2 Results of limit equilibrium method

Using this method, the estimated value of k is always higher as compared with FEM. In this study, the value of $k = 3.168$ for two sliding faces, with angle of inclination $\alpha = 18.53$ and $\beta = 22$.

A sensitive analysis carried out with the values of $\beta = 20, 22, 25, 27.5, 30, 35$ and constant value of $\alpha = 18.53$, and angle alpha α varying as 15°, 18.53°, 20°, 22° and 25°, with constant $\beta = 22°$, shows that the safety factor k increases as the depth of sliding faces increases. See Figs.4(a) and (b).

6 Conclusions

1) Using FEM, the estimated safety factor against failure is 2.6. For the limit equilibrium method, the safety factor k estimated is 3.168 for two potential sliding surfaces under the same reservoir elevation of 135 m. The class difference between the safety factors computed shows that, FEM is a better technique for safety factor evaluation in stability analysis problems than the limit equilibrium method.

2) The adopted technique of analysis and the various assumptions made on material behavior proved to be adequate and robust in representing a real practical situation of stability problems in dams.

3) The use of finite element software (MARC 2000) for stability problems is quite efficient and computationally adequate.

4) From this study, it can be deduced that in addition to the imposed load a structure's constituent material behavior plays an active role in the failure of engineered structures. Therefore, stability analysis requires adequate prediction.

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大坝与坝基稳定性分析研究

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摘要: 大坝与坝基的稳定一直是工程师密切关注的一个安全问题. 论文提出 2 种分析重力坝和坝基稳定的方法: 一种是基于刚体极限平衡原理的直接分析方法, 它将坝体和岩基作为不变形的刚体, 利用该方法可直接计算出可能滑动面的安全因数; 另一种是基于弹塑性理论的间接分析方法, 它采用非线性有限元方法分析大坝和坝基的应力和变形. 根据收敛和突变准则来确定稳定安全度.结果表明:工程建筑物的破坏不仅仅是由于施加荷载的原因,同时也与组成材料的性质有关.

关键词: 稳定性; 重力坝; 安全因数; 刚体极限平衡原理; 弹塑性理论; 非线性有限元方法

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