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New approach of Kalman filter to nonlinear system

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Abstract: In order to achieve higher accuracy in nonlinear/non-Gaussian state estimation, this paper proposes a new unscented Kalman filter (UKF). It uses a deterministic sampling approach. We choose the unscented transformation (UT) scaling parameters $\alpha = 0.85, \beta = 2, l = 0$ to construct $2n + 1$ sigma points. These sigma points completely capture the mean and covariance of the Gaussian random variables of the nonlinear system $\boldsymbol{Y}_i = F(\boldsymbol{X}_i)$. Simulation results show that the posterior mean and covariance of the sigma points can achieve the accuracy of the third-order Taylor series expansion after having propagated through the true nonlinear system $\boldsymbol{Y}_i = F(\boldsymbol{X}_i)$. Extended Kalman filter (EKF) only can achieve the first-order accuracy. The computational complexity of UKF is the same level as that of EKF. UKF can yield better performance and higher accuracy than EKF.

Key words: nonlinear; extended Kalman filter; unscented Kalman filter; unscented transformation

Extended Kalman filter (EKF) is probably the most common and popular tool to deal with nonlinear estimation problems. It is based on and limited by model linearization and Gaussian hypothesis. EKF may cause more errors for the nonlinear system while estimating system state and its variance. Moreover, the linearization may lead to divergence of filtering process. This paper introduces a new approach to optimal nonlinear filter named unscented Kalman filter (UKF). The main feature of UKF is a deterministic sampling approach. This method uses a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the variable. When the sample points propagate through the nonlinear system, the sample points can accurately capture the posterior mean and covariance to the third-order Taylor series expansion. EKF only can obtain the accuracy up to the first order.

1 Extended Kalman Filter

Extended Kalman filter has become a standard technique used in nonlinear estimation. It includes state estimation of a nonlinear dynamic system, parameters estimation for nonlinear system identification and dual estimation where both states and parameters are estimated simultaneously.

The equations of the general nonlinear system can be described as follows[1]:

$$\boldsymbol{X}_k = f(\boldsymbol{X}_{k-1}, \boldsymbol{W}_{k-1}, k-1) \tag{1}$$

$$\boldsymbol{Z}_k = h(\boldsymbol{X}_k, \boldsymbol{V}_k, k) \tag{2}$$

where $\boldsymbol{X}_k$ is the unobserved state of the system; $\boldsymbol{Z}_k$ is the observed measurement signal; $f(\cdot)$ is n dimensional nonlinear function of the state $\boldsymbol{X}_k$; $h(\cdot)$ is m dimensional nonlinear function; $\boldsymbol{W}_k (k \in \mathbf{N})$ is process noise sequences, $E[\boldsymbol{W}_k] = 0$, $\mathrm{cov}[\boldsymbol{W}_k, \boldsymbol{W}_j] = E[\boldsymbol{W}_k \boldsymbol{W}_j^{\mathrm{T}}] = \boldsymbol{Q}_k \delta_{kj}$, $\boldsymbol{Q}_k$ is process noise sequences covariance matrix; $\boldsymbol{V}_k (k \in \mathbf{N})$ is measurement noise sequences, $E[\boldsymbol{V}_k] = 0$, $\mathrm{cov}[\boldsymbol{V}_k, \boldsymbol{V}_j] = E[\boldsymbol{V}_k \boldsymbol{V}_j^{\mathrm{T}}] = \boldsymbol{R}_k \delta_{kj}$, $\boldsymbol{R}_k$ is process noise sequences covariance matrix.

Kalman filter assumes that the posterior density at every time step is Gaussian and parameterized by a mean and covariance. If the posterior probability density function $p(\boldsymbol{X}_{k-1} \mid \boldsymbol{Z}_{1:k-1})$ approximates Gaussian and the assumptions below hold: ① $\boldsymbol{W}_{k-1}$ and $\boldsymbol{V}_k$ are drawn from the Gaussian distributions of the known parameters; ② $f(\boldsymbol{X}_{k-1}, \boldsymbol{W}_{k-1})$ is a known nonlinear function of $\boldsymbol{X}_{k-1}$ and $\boldsymbol{W}_{k-1}$; ③ $h(\boldsymbol{X}_k, \boldsymbol{V}_k)$ is a known nonlinear function of $\boldsymbol{X}_k$ and $\boldsymbol{V}_k$. It can be proved that $p(\boldsymbol{X}_k \mid \boldsymbol{Z}_{1:k})$ is also Gaussian distribution. Eqs. (1) and (2) can be rewritten as

$$\boldsymbol{X}_k = \boldsymbol{\Phi}_{k,k-1} \boldsymbol{X}_{k-1} + \boldsymbol{W}_{k-1} \tag{3}$$

$$\boldsymbol{Z}_k = \boldsymbol{H}_k \boldsymbol{X}_k + \boldsymbol{V}_k \tag{4}$$

where $\boldsymbol{\Phi}_{k,k-1}$ is the local linearization of nonlinear function $f_k(\cdot)$, clearly, $\boldsymbol{\Phi}_{k,k-1}$ is the state transition matrix for the step from k to $k+1$; $\boldsymbol{H}_k$ is the local lin-

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earization of nonlinear function $h_k(\cdot)$; $\boldsymbol{W}_{k-1}$ and $\boldsymbol{V}_k$ have zero mean and are statistically independent. The covariances of $\boldsymbol{W}_{k-1}$ and $\boldsymbol{V}_k$ are $\boldsymbol{Q}_{k-1}$ and $\boldsymbol{R}_k$, respectively. The system matrix $\boldsymbol{\Phi}_{k,k-1}$ and measurement matrix $\boldsymbol{H}_k$ are allowed to be time variant.

EKF is based on approximation. Eqs. (1) and (2) are a nonlinear system. Because a local linearization of the equations may be a sufficient description of the nonlinearity, Eqs. (1) and (2) can be linearized in the form of Eqs. (3) and (4).

The EKF algorithm can be described as follows:

$$\hat{\boldsymbol{X}}_{k|k-1} = \boldsymbol{\Phi}_{k,k-1}\hat{\boldsymbol{X}}_{k-1} \tag{5}$$

$$\hat{\boldsymbol{X}}_k = \hat{\boldsymbol{X}}_{k|k-1} + \delta\hat{\boldsymbol{X}}_k \tag{6}$$

$$\delta\hat{\boldsymbol{X}}_k = \boldsymbol{K}_k[\boldsymbol{Z}_k - h(\hat{\boldsymbol{X}}_{k|k-1}, k)] \tag{7}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^{\mathrm{T}}[\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_k]^{-1} \tag{8}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{\Phi}_{k,k-1}\boldsymbol{P}_{k-1}\boldsymbol{\Phi}_{k,k-1}^{\mathrm{T}} + \boldsymbol{Q}_{k-1} \tag{9}$$

$$\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{k|k-1}(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)^{\mathrm{T}} + \boldsymbol{K}_k\boldsymbol{R}_k\boldsymbol{K}_k^{\mathrm{T}} \tag{10}$$

where $\hat{\boldsymbol{X}}_k$ is updated state estimate, $\boldsymbol{K}_k$ is Kalman gain matrix that can yield an updated state estimate, and $\boldsymbol{P}_k$ is the error covariance matrix associated with the updated state estimate.

Since EKF utilizes the first term in a Taylor expansion of the nonlinear function, the estimation result is sub-optimal, not optimal. A higher order EKF that retains further terms in the Taylor expansion can show a better performance, but the additional complexity has limited its widespread use.

2 Unscented Kalman Filter

UKF was first proposed by Julier and Uhlmann[2-4]. A central and vital operation performed in the Kalman filter is the propagation of a Gaussian random variable through the system dynamics. In EKF, the state distribution is approximated by a Gaussian random variable. It is propagated analytically through the first-order linearization of the nonlinear system. It can introduce large errors in the true posterior mean and covariance of the transformed Gaussian random variable, which may lead to sub-optimal performance or sometimes divergence. UKF deals with this problem by using a deterministic sampling approach. The state distribution is again approximated by a Gaussian random variable, but is now represented using a minimal set of carefully chosen sample points[4]. These sample points completely capture the true mean and covariance of the Gaussian random variable. When the sample points propagate through the true nonlinear system, the sample points can accurately capture the posterior mean and covariance to the third-order Taylor series expansion for any nonlinearity. EKF only can achieve the first-order accuracy. Remarkably, the computational complexity of UKF is the same lever as that of EKF.

2.1 Unscented transformation

The kernel of UKF is unscented transformation (UT)[5]. UT is a method for calculating the statistics of a random variable that undergoes a nonlinear transformation. Consider propagating a random variable $\boldsymbol{X}$ (n dimensional) through a nonlinear system $\boldsymbol{Y} = F(\boldsymbol{X})$. We assume that $\boldsymbol{X}$ has the mean $\bar{\boldsymbol{X}}$ and the covariance $\boldsymbol{P}_x$, and calculate the statistics of $\boldsymbol{Y}$. We can form a matrix of $2n+1$ sigma vectors (with corresponding weights) according to the following equations:

$$\boldsymbol{X}_0 = \bar{\boldsymbol{X}} \tag{11}$$

$$\boldsymbol{X}_i = \bar{\boldsymbol{X}} + (\sqrt{(n+\lambda)\boldsymbol{P}_x})_i \qquad i = 1, 2, \ldots, n \tag{12}$$

$$\boldsymbol{X}_i = \bar{\boldsymbol{X}} - (\sqrt{(n+\lambda)\boldsymbol{P}_x})_i \qquad i = n+1, n+2, \ldots, 2n \tag{13}$$

where $(\sqrt{(n+\lambda)\boldsymbol{P}_x})_i$ is the i-th column of the matrix square root, λ is defined by

$$\lambda = \alpha^2(n+l) - n \tag{14}$$

α determines the spread of the sigma points around $\bar{\boldsymbol{X}}$, and is usually set to a small positive value, l is a secondary scaling parameter which is usually set to 0.

Once the sigma point vector is computed, we perform the prediction step by propagating each column through the nonlinear function.

$$\boldsymbol{Y}_i = F(\boldsymbol{X}_i) \qquad i = 1, 2, \ldots, 2n \tag{15}$$

The mean and covariance of $\boldsymbol{Y}$ are approximated using the mean and covariance of the weighted posterior sigma points.

$$\bar{\boldsymbol{Y}} = \sum_{i=0}^{2n} \omega_i^{\mathrm{m}}\boldsymbol{Y}_i \tag{16}$$

$$\boldsymbol{P}_y = \sum_{i=0}^{2n} \omega_i^{\mathrm{c}}(\boldsymbol{Y}_i - \bar{\boldsymbol{Y}})(\boldsymbol{Y}_i - \bar{\boldsymbol{Y}})^{\mathrm{T}} \tag{17}$$

$$\boldsymbol{P}_{xy} = \sum_{i=0}^{2n} \omega_i^{\mathrm{c}}(\boldsymbol{X}_i - \bar{\boldsymbol{X}})(\boldsymbol{Y}_i - \bar{\boldsymbol{Y}})^{\mathrm{T}} \tag{18}$$

where

$$\omega_0^{\mathrm{m}} = \frac{\lambda}{n+\lambda} \tag{19}$$

$$\omega_0^{\mathrm{c}} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \tag{20}$$

$$\omega_i^{\mathrm{m}} = \omega_i^{\mathrm{c}} = \frac{0.5}{n+\lambda} \qquad i = 1, 2, \ldots, 2n \tag{21}$$

ω_i^{m} is the first-order statistical weight, ω_i^{c} is the second-order statistical weight, β is used to incorporate the prior knowledge of the distribution of $\boldsymbol{X}$ (for Gaussian distributions, $\beta = 2$ is optimal).

Note that $2n + 1$ sample points need to be carefully chosen for unscented transformation. The method differs substantially from general Monte-Carlo sampling methods that require more sample points in an attempt to propagate an accurate distribution of the state[6, 7]. If the nonlinear system has Gaussian white noise, the approach taken with the UT results in approximations that are accurate to the third-order. For non-Gaussian inputs, approximations are accurate to at least the second-order. The accuracy of the third and higher order moments is determined by the choice of α and β.

2.2 Unscented Kalman filter

UKF is a straightforward extension of UT to the estimation in Eqs. (5) to (8). The UKF algorithm is given below[8-10]:

① Initialization

$$\hat{\boldsymbol{X}}_0 = E[\boldsymbol{X}_0] \tag{22}$$

$$\boldsymbol{P}_0 = E[(\boldsymbol{X}_0 - \hat{\boldsymbol{X}}_0)(\boldsymbol{X}_0 - \hat{\boldsymbol{X}}_0)^{\mathrm{T}}] \tag{23}$$

Augmented state vectors

$$\boldsymbol{X}^{\mathrm{a}} = [\boldsymbol{X}^{\mathrm{T}} \quad \boldsymbol{X}^{\mathrm{T}} \quad \boldsymbol{V}^{\mathrm{T}}] \tag{24}$$

$$\hat{\boldsymbol{X}}_0^{\mathrm{a}} = E[\boldsymbol{X}_0^{\mathrm{a}}] = [\hat{\boldsymbol{X}}_0^{\mathrm{T}} \quad \boldsymbol{0} \quad \boldsymbol{0}]^{\mathrm{T}} \tag{25}$$

$$\boldsymbol{P}_0^{\mathrm{a}} = E[(\boldsymbol{X}_0^{\mathrm{a}} - \hat{\boldsymbol{X}}_0^{\mathrm{a}})(\boldsymbol{X}_0^{\mathrm{a}} - \hat{\boldsymbol{X}}_0^{\mathrm{a}})^{\mathrm{T}}] = \begin{bmatrix} \boldsymbol{P}_0 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{R}_0 \end{bmatrix} \tag{26}$$

② Calculating sigma points

$$\boldsymbol{\chi}_{0,k-1}^{\mathrm{a}} = \hat{\boldsymbol{X}}_{k-1}^{\mathrm{a}}, \quad \omega_0 = \frac{k}{n+k} \tag{27}$$

$$\boldsymbol{\chi}_{i,k-1}^{\mathrm{a}} = \hat{\boldsymbol{X}}_{k-1}^{\mathrm{a}} + (\sqrt{(n+\lambda)\boldsymbol{P}_{k-1}^{\mathrm{a}}})_i, \ \omega_i = \frac{1}{2n+2k} \quad i = 1, 2, \ldots, n \tag{28}$$

$$\boldsymbol{\chi}_{i,k-1}^{\mathrm{a}} = \hat{\boldsymbol{X}}_{k-1}^{\mathrm{a}} - (\sqrt{(n+\lambda)\boldsymbol{P}_{k-1}^{\mathrm{a}}})_i, \ \omega_i = \frac{1}{2n+2k} \quad i = n+1, n+2, \ldots, 2n \tag{29}$$

③ Time update

$$\boldsymbol{\chi}_{i,k|k-1}^{\mathrm{a}} = f(\boldsymbol{\chi}_{i,k-1}^{\mathrm{a}}, k-1) \tag{30}$$

$$\hat{\boldsymbol{X}}_{k|k-1} = \sum_{i=0}^{2n} \omega_i^{\mathrm{m}} \boldsymbol{\chi}_{i,k|k-1}^{\mathrm{a}} \tag{31}$$

$$\boldsymbol{P}_{k|k-1} = \sum_{i=0}^{2n} \omega_i^{\mathrm{c}} (\boldsymbol{\chi}_{i,k|k-1}^{\mathrm{a}} - \hat{\boldsymbol{X}}_{k|k-1}) \cdot (\boldsymbol{\chi}_{i,k|k-1}^{\mathrm{a}} - \hat{\boldsymbol{X}}_{k|k-1})^{\mathrm{T}} + \boldsymbol{Q}_k \tag{32}$$

$$\boldsymbol{Z}_{i,k|k-1} = h(\boldsymbol{\chi}_{i,k|k-1}, k-1) \tag{33}$$

$$\hat{\boldsymbol{Z}}_{k|k-1} = \sum_{i=0}^{2n} \omega_i^{\mathrm{m}} \boldsymbol{Z}_{i,k|k-1} \tag{34}$$

④ Measurement update

$$\boldsymbol{P}_{k|k-1}^{zz} = \sum_{i=0}^{2n} \omega_i^{\mathrm{c}} (\boldsymbol{Z}_{i,k|k-1} - \hat{\boldsymbol{Z}}_{k|k-1}) \cdot (\boldsymbol{Z}_{i,k|k-1} - \hat{\boldsymbol{Z}}_{k|k-1})^{\mathrm{T}} + \boldsymbol{R}_k \tag{35}$$

$$\boldsymbol{P}_{k|k-1}^{xz} = \sum_{i=0}^{2n} \omega_i^{\mathrm{c}} (\boldsymbol{\chi}_{i,k|k-1} - \hat{\boldsymbol{X}}_{k|k-1}) \cdot (\boldsymbol{Z}_{i,k|k-1} - \hat{\boldsymbol{Z}}_{k|k-1})^{\mathrm{T}} \tag{36}$$

With the transformed function $\hat{\boldsymbol{Z}}_{k|k-1}$, we compute the posterior state estimate:

$$\hat{\boldsymbol{X}}_k = \hat{\boldsymbol{X}}_{k|k-1} + \boldsymbol{K}_k(\boldsymbol{Z}_k - \hat{\boldsymbol{Z}}_{k|k-1}) \tag{37}$$

Finally we calculate the posterior estimate of the error covariance:

$$\boldsymbol{P}_k = \boldsymbol{P}_{k|k-1}^{xz} - \boldsymbol{K}_k \boldsymbol{P}_{k|k-1}^{zz} \boldsymbol{K}_k^{\mathrm{T}} \tag{38}$$

where the Kalman gain is

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}^{xz} (\boldsymbol{P}_{k|k-1}^{zz})^{-1} \tag{39}$$

No explicit calculation of Jacobians or Hessians is necessary to implement the UKF algorithm. Furthermore, the overall numbers of computations are the same order as that of EKF.

3 Application and Results

We consider the following equations as an illustrative example:

$$\boldsymbol{X}_k = 0.5\boldsymbol{X}_{k-1} + \frac{25\boldsymbol{X}_{k-1}}{1 + \boldsymbol{X}_{k-1}^2} + 8\cos(1.2(k-1)) + \boldsymbol{W}_{k-1} \tag{40}$$

$$\boldsymbol{Z}_k = \frac{\boldsymbol{X}_k^2}{20} + \boldsymbol{V}_k \tag{41}$$

where $\boldsymbol{W}_{k-1}$ and $\boldsymbol{V}_k$ are the zero-mean Gaussian white noise. The covariances of $\boldsymbol{W}_{k-1}$ and $\boldsymbol{V}_k$ are Q_k and R_k, respectively. $Q_k = 10$ and $R_k = 1$. The example is nonlinear both in the system equations and the measurement equations. The initial state is taken to be $X_0 = 0.1$. The constant in UT $\alpha = 0.85, \beta = 2$. The secondary scaling parameter $l = 0$. Fig. 1 shows an 80 step realization of Eq. (40). Fig. 2 is the observation of the nonlinear system.

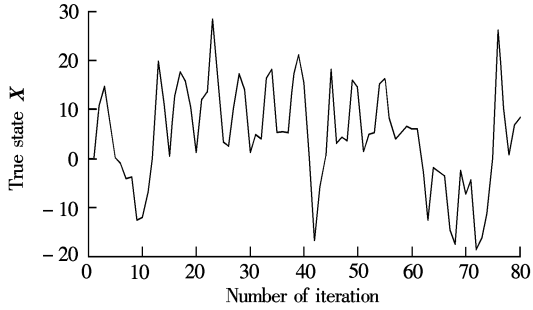

Fig. 1 True state $\boldsymbol{X}$

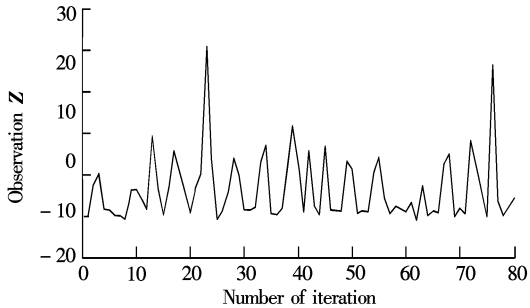

Fig. 2 Observation of the state X

In order to qualitatively gauge performance and discuss resulting issues, we consider using the traditional measure of performance: the root mean-squared error (RMSE). The simulation results of the mean of the RMSE over 100 independent runs are depicted in Tab. 1.

Tab. 1 Mean of RMSE and execution time over 100 independent runs

Algorithms	Mean of RMSE	Execution time/s
EKF	22.920 5	0.043 2
UKF	10.982 0	0.157 9

Fig. 3 shows the state estimations generated from a single run with the different filtering methods. The estimation of the UKF algorithm is closer to the true state than that of EKF. The mean of the RMSE over 100 independent runs also indicates that the EKF algorithm has the less accurate performance than the UKF algorithm.

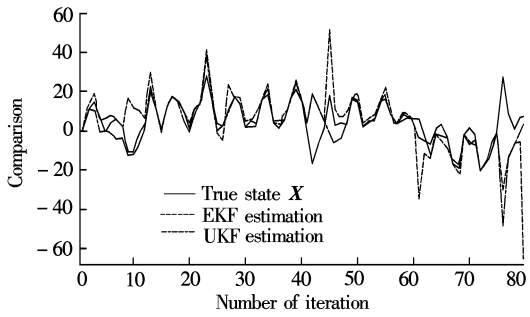

Fig. 3 Comparison between the state estimations of EKF and UKF

4 Conclusion

In this paper we propose the principle of EKF and UKF, describe the EKF and UKF state estimation algorithms for nonlinear systems. Finally, we compare the performance of EKF with UKF. By virtue of the unscented transformation, UKF acts on the premise that it is easier to approximate a Gaussian distribution, instead of linearization using Jacobian matrices as EKF. This algorithm has two great advantages over EKF. First, it is able to predict the state of the system more accurately. Secondly, it is much less difficult to implement. The benefits of the algorithm are demonstrated in the above example. UKF is a powerful nonlinear estimation method and has been shown to be a superior alternative to EKF in a variety of applications including state estimation.

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非线性系统卡尔曼滤波新方法

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摘要: 为了在非线性、非高斯系统估计中获得更好的精度, 提出一种新的 unscented 卡尔曼滤波(UKF). 采用确定性采样方法, 通过选择 unscented 变换中的参数 $\alpha = 0.85$, $\beta = 2$ 和 $l = 0$, 确定出 $2n+1$ 个 σ 点, 使这些 σ 点完全符合非线性系统 $\boldsymbol{Y}_i = F(\boldsymbol{X}_i)$ 的高斯随机变量的均值和方差. 仿真结果表明: σ 点通过实际的非线性系统 $\boldsymbol{Y}_i = F(\boldsymbol{X}_i)$ 传递后, 其后验均值和协方差可以达到泰勒展开式的三阶精度, 广义卡尔曼滤波(EKF) 只能达到一阶精度. 该 UKF 滤波与 EKF 算法复杂度相近, 但具有比 EKF 更好的估计精度.

关键词: 非线性; 广义卡尔曼滤波; unscented 卡尔曼滤波; unscented 变换
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