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Calculation of torque and speed of induction machines under rotor winding faults

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Abstract: Based on the multi-loop method, the rotating torque and speed of the induction machine are analyzed. The fluctuating components of the torque and speed caused by rotor winding faults are studied. The models for calculating the fluctuating components are put forward. Simulation and computation results show that the rotor winding faults will cause electromagnetic torque and rotating speed to fluctuate; and fluctuating frequencies are the same and their magnitude will increase with the rise of the severity of the faults. The load inertia affects the torque and speed fluctuation, with the increase of inertia, the fluctuation of the torque will rise, while the corresponding speed fluctuation will obviously decline.

Key words: induction machine; rotor winding fault; torque; speed; fluctuating

During steady operation, if the load torque is constant and there is no external disturbance, the electromagnetic torque of an induction machine should be steady, equalizing the sum of the load torque and the no-load torque. For healthy machine, theoretically, this torque should be a constant. However, for an induction machine with rotor winding faults, the asymmetry of the rotor current will cause stator winding to generate additional current component with frequency of $(1-2s)f_1$ [1,2]. Because of the influence of this additional current component, stator current will change periodically[3]. The stator current (The magnitude of which changes cyclically) counteracts the rotor, generating a torque with a frequency two times the slip frequency, and the rotor speed fluctuating with the fluctuation frequency two times the slip frequency. This paper analyzes the impact of the rotor winding fault on the machine's current, torque and speed and the relationships among them. This paper also analyzes the fluctuating components of the torque and speed caused by rotor winding faults.

1 Mathematical Model

Based on the multi-loop method[4], any two neighboring bars and the end-ring connecting them form a loop. Assuming that the rotor has n slots and there is no fault, the nodal number of rotor is $2n$, the branch number is $3n$, then the independent loop number is $3n-(2n-1)=n+1$, and the individual loop current is $i_1, i_2, \ldots, i_n, i_e$ (i_e is the end-ring loop current). Assuming that the three phase windings in the stator are symmetrical and there is no fault, for simplification, stator windings are represented by the phase winding and only have three loops, loop currents are i_a, i_b and i_c.

According to the above assumptions, the pre-fault machine's voltage equation and flux equation can be obtained:

$$\boldsymbol{U} = p\boldsymbol{\Psi} + \boldsymbol{RI} \tag{1}$$

$$\boldsymbol{\Psi} = \boldsymbol{MI} \tag{2}$$

where $\boldsymbol{U}$, $\boldsymbol{I}$ and $\boldsymbol{\Psi}$ are $(n+4)\times 1$ column vectors; $\boldsymbol{R}$ and $\boldsymbol{M}$ are $(n+4)\times(n+4)$ square matrices. Expand Eqs. (1) and (2), we have

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} = p\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \\ \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \\ \Psi_e \end{bmatrix} + \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & r_r & -r_b & \cdots & -r_b & r_e \\ 0 & 0 & 0 & -r_b & r_r & \cdots & 0 & r_e \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & -r_b & 0 & \cdots & r_r & r_e \\ 0 & 0 & 0 & r_e & r_e & \cdots & r_e & nr_e \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_1 \\ i_2 \\ \vdots \\ i_n \\ i_e \end{bmatrix} \tag{3}$$

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where

$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \\ \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \\ \Psi_e \end{bmatrix} = \begin{bmatrix} L_{aa} & M_{ab} & M_{ac} & M_{a1} & M_{a2} & \cdots & M_{an} & M_{ae} \\ M_{ba} & L_{bb} & M_{bc} & M_{b1} & M_{b2} & \cdots & M_{bn} & M_{be} \\ M_{ca} & M_{cb} & L_{cc} & M_{c1} & M_{c2} & \cdots & M_{cn} & M_{ce} \\ M_{1a} & M_{1b} & M_{1c} & L_{11} & M_{12} & \cdots & M_{1n} & M_{1e} \\ M_{2a} & M_{2b} & M_{2c} & M_{21} & L_{22} & \cdots & M_{2n} & M_{2e} \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ M_{na} & M_{nb} & M_{nc} & M_{n1} & M_{n2} & \cdots & L_{nn} & M_{ne} \\ M_{ea} & M_{eb} & M_{ec} & M_{e1} & M_{e2} & \cdots & M_{en} & L_{ee} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_1 \\ i_2 \\ \vdots \\ i_n \\ i_e \end{bmatrix} \tag{4}$$

r_b and r_e are the rotor bar resistance and the rotor end-ring resistance, respectively; $r_r = 2(r_b + r_c)$ is the rotor loop resistance; r_s is the phase resistance of stator winding; $\boldsymbol{\Psi}$, $\boldsymbol{L}$ and $\boldsymbol{M}$ are the flux, self-inductance and mutual inductance, respectively; subscripts a, b and c denote stators A, B and C phase winding; subscripts 1, 2,…, n denote the rotor bar loop; subscript e denotes the rotor end-ring loop.

Transform the voltage equation and flux equation into state equation:

$$\frac{d\boldsymbol{I}}{dt} = \boldsymbol{M}^{-1}\left[\boldsymbol{U} - \frac{d\boldsymbol{M}}{dt}\boldsymbol{I} - \boldsymbol{R}\boldsymbol{I}\right] \tag{5}$$

Using single stator and single rotor loop parameters represents machine's electromagnetic torque, and the rotor operation equation is an effective method for the transient analysis of machine's fault. The rotor operation equation is

$$J\frac{d\omega}{dt} + T_L = T_E,\ \omega = \frac{d\theta_m}{dt}$$

where J is the inertia, T_E is the electromagnetic torque, and T_L is the load torque.

The general magnetic energy is

$$W_m = \frac{1}{2}\boldsymbol{I}^T\boldsymbol{M}\boldsymbol{I} = \frac{1}{2}\boldsymbol{I}_S^T\boldsymbol{M}_{SS}\boldsymbol{I}_S + \frac{1}{2}\boldsymbol{I}_S^T\boldsymbol{M}_{SR}\boldsymbol{I}_R + \frac{1}{2}\boldsymbol{I}_R^T\boldsymbol{M}_{RS}\boldsymbol{I}_S + \frac{1}{2}\boldsymbol{I}_R^T\boldsymbol{M}_{RR}\boldsymbol{I}_R \tag{6}$$

For a linear system, the magnetic coenergy equals the magnetic energy that the machine stores. Thus, the electromagnetic torque is

$$T_E = \frac{1}{2}\left(\boldsymbol{I}_S^T\frac{\partial \boldsymbol{M}_{SR}}{\partial\theta_m}\boldsymbol{I}_R + \boldsymbol{I}_R^T\frac{\partial \boldsymbol{M}_{RS}}{\partial\theta_m}\boldsymbol{I}_S\right) = P\boldsymbol{I}_S^T\frac{\partial \boldsymbol{L}_{SR}}{\partial\theta_m}\boldsymbol{I}_R \tag{7}$$

where P denotes the number of pole-pairs, and θ_m denotes the rotor displacement in mechanical radians, and θ_R denotes the rotor displacement in electrical radians. The stator voltage $\boldsymbol{U}_S = \{u_a, u_b, u_c\}^T$, the stator current $\boldsymbol{I}_S = \{i_a, i_b, i_c\}^T$, and the rotor current $\boldsymbol{I}_R = \{i_1, i_2, \ldots, i_n, i_e\}^T$.

If the higher harmonic component of the mutual inductance between the stator and rotor is not considered, the torques can be listed as follows:

$$T_E = \frac{PM_{RS}}{\omega}\{i_a, i_b, i_c\}\frac{d}{dt}\begin{bmatrix} \cos[(1-s)\omega_0 t + \varphi] & \cos[(1-s)\omega_0 t + 2\varphi] & \cdots & \cos[(1-s)\omega_0 t + n\varphi] & 0 \\ \cos\left[(1-s)\omega_0 t - \frac{2}{3}\pi + \varphi\right] & \cos\left[(1-s)\omega_0 t - \frac{2}{3}\pi + 2\varphi\right] & \cdots & \cos\left[(1-s)\omega_0 t - \frac{2}{3}\pi + n\varphi\right] & 0 \\ \cos\left[(1-s)\omega_0 t + \frac{2}{3}\pi + \varphi\right] & \cos\left[(1-s)\omega_0 t + \frac{2}{3}\pi + 2\varphi\right] & \cdots & \cos\left[(1-s)\omega_0 t + \frac{2}{3}\pi + n\varphi\right] & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \\ i_e \end{bmatrix} = T_{Ave} + T_{Pul}$$

where T_{Ave} is the mean torque, and T_{Pul} is the pulsating torque.

2 Analysis Method

After rotor winding fault, in different instances, the mutual inductance between the stator and the rotor is not a constant. The individual loop's inductance in different instances during a certain time interval is calculated firstly, by modifying Eqs. (3) and (4), getting the corresponding circuit equation for post-fault machine[5], and then the machine's current, speed and torque in different instances during a time internal can be acquired.

The four graphs in Fig. 1 are the time domain curves for the electromagnetic torque and rotation speed, respectively (Machine has run into steady operation). Figs. 1(a) and (c) correspond to no fault; Figs. 1(b) and (d) correspond to the rotor with two broken bars.

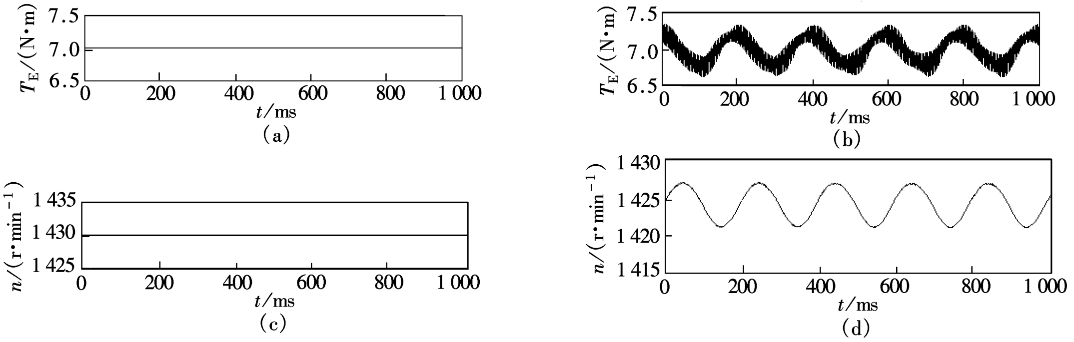

Fig. 1 Waveform of the torque and speed. (a) Torque(no fault); (b) Torque(rotor with two broken bars); (c) Speed (no fault); (d) Speed(rotor with two broken bars)

From Fig. 1, we can draw the following conclusions: ① For a healthy machine, the torque and speed are constants (straight line) without external disturbance; ② After rotor winding fault, the electromagnetic torque exhibits obvious fluctuation, its frequency is $2sf_1$; ③ The fluctuating frequency of the torque and speed is the same, which equals the frequency of the fluctuating envelope line of the stator current; ④ After the rotor broken bar fault, not only will the speed of the machine fluctuate, but also its average value will drop.

Since the speed fluctuation and torque fluctuation are related, the main frequency component of their fluctuation always varies in the same frequency, and their fluctuating frequencies equal the differentiation between the stator current fundamental frequency and corresponding fault characteristics frequency.

$$f_{Pul} = f_1 - f_{2s} = 2sf_1 \tag{8}$$

3 Relation between Load Inertia and Fluctuation of Electromagnetic Torque and Rotating Speed

With rotor winding faults, what is the impact of the change of load inertia on the speed and electromagnetic torque? The inertia of the simulating machine is 2. 1 g·m^2. Now assume that there are two broken bars, the simulation is done under various load inertias for the corresponding electromagnetic torques and rotating speeds.

Fig. 2 shows the time domain curves for electromagnetic torque and rotating speed of a Y90S-4 small-type induction machine with two rotor bars broken, when the load inertia is 4. 2 g·m^2 and 84 g·m^2, respectively. Fig. 3 is the corresponding spectrum graph. It can be concluded that with the increase of load inertia, the fluctuation of the torque will rise, but the fluctuation of speed will obviously decline.

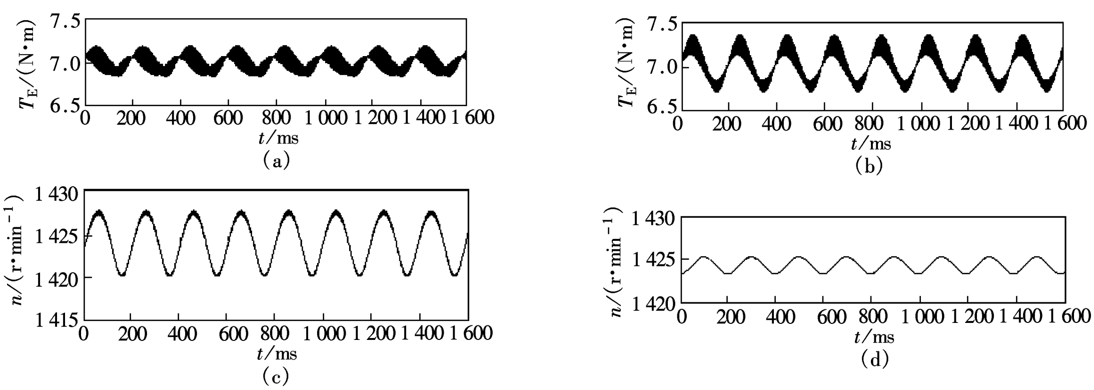

Fig. 2 Influence of inertia on torque and speed with broken bars. (a) Torque(Load inertia is 4. 2 g·m^2); (b) Torque(Load inertia is 84 g·m^2); (c) Speed(Load inertia is 4. 2 g·m^2); (d) Speed(Load inertia is 84 g·m^2)

From Figs. 3(a) and (b), it can be seen that as for the magnitude of torque fluctuation, the torque fluctuation under the inertia of 84 g·m^2 is 36. 61 dB smaller than the steady average torque, while the torque fluctuation under the inertia of 4. 2 g·m^2 is 44. 56 dB smaller than the steady average torque, the former is 7. 95 dB larger than the latter (i. e. the former is 3. 5 times the latter).

From Figs. 3(c) and (d), it can be seen that the speed fluctuation under the inertia of 84 g·m^2 is 70. 69 dB smaller than the steady average speed, while the speed fluctuation under the inertia of 4. 2 g·m^2 is 59. 34 dB smaller than the steady average speed. The former is 11. 35 dB smaller than the latter (i. e. the former is 1/4. 7 times

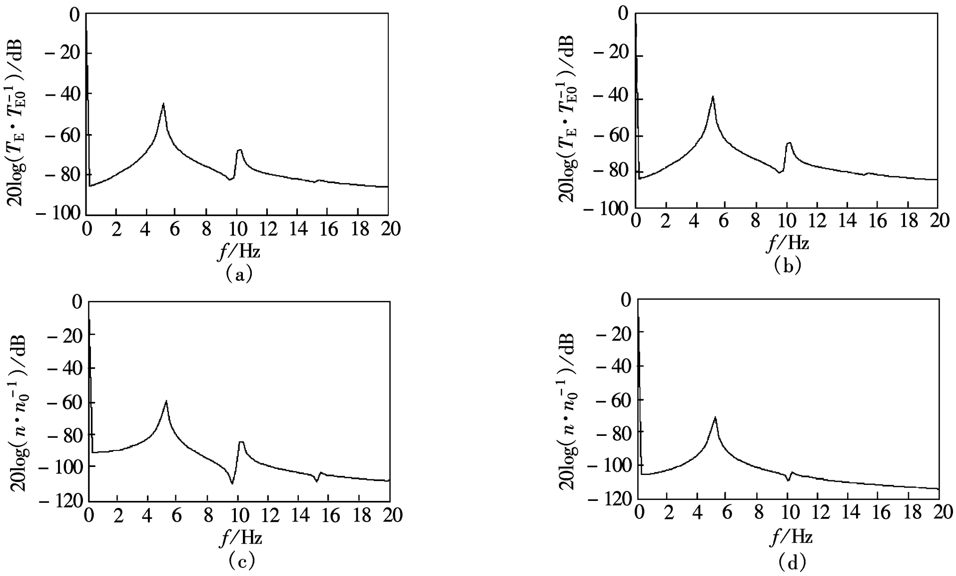

Fig. 3 Influence of the inertia on torque and speed with broken bars. (a) Torque (Load inertia is 4.2 $g \cdot m^2$); (b) Torque(Load inertia is 84 $g \cdot m^2$); (c) Speed(Load inertia is 4.2 $g \cdot m^2$); (d) Speed(Load inertia is 84 $g \cdot m^2$)

the latter).

From the above, it can be seen that load inertia has a big impact on the machine's torque and speed fluctuations. With the increase of inertia, the fluctuation of the torque will rise, while the fluctuation of the speed will decline dramatically.

4 Conclusions

This paper studies the inherent mutual relationships between the cage rotor winding fault and machine's stator current, torque and speed:

1) Rotor winding faults will cause current, torque and speed to fluctuate. The frequency of fluctuating components is in direct proportion to the slip rate.

2) Load inertia has a big impact on the machine's torque and speed fluctuations. With the increase of inertia, the fluctuation of the torque will rise, while the fluctuation of the speed will decline dramatically.

3) It is possible to quantitatively analyze the magnitude of torque and speed fluctuation, which are caused by rotor winding faults and the impact extent of inertia.

4) Using the fluctuation component of torque and speed to diagnose rotor winding fault can make up for the shortcomings of the current spectrum method and be complementary to the latter.

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转子绕组故障感应电机转矩和转速的计算

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摘要: 基于多回路方法, 对异步电机定子电流、转矩、转速进行了分析, 对转子绕组故障引起的电机转矩、转速的波动进行了深入研究, 提出了波动计算模型. 仿真与计算结果表明: 转子绕组故障引起电机转矩、转速波动; 波动分量的大小、频率可定量计算; 转矩与转速波动的频率相等, 波动的幅值随转子绕组故障严重程度增加而增加; 负载转动惯量对电机转矩、转速有较大的影响, 负载转动惯量增加, 转矩波动分量的幅值增加而转速波动分量的幅值减小.

关键词: 异步电机; 转子绕组故障; 转矩; 转速; 波动
中图分类号: TM343; TM306