

Durability design of prestressed concrete structures

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Abstract: Based on the durability characteristics of prestressed concrete structures, the durability limit states of carbonation and chloride ion attack are defined, respectively. Durability predicting models on the basis of reliability mathematics and stochastic processes are constructed, and the pure theoretical formulae of failure probability of prestressed concrete structures are analyzed. In addition, a simple durability design method for carbonation of structures is put forward. According to the analysis, the durability of prestressed concrete structures is superior to that of traditional structures. The research also indicates that the concrete cover prescribed in the current code (GB 50010—2002) is not adequate. The rational cover thickness should not be less than 35 or 45 mm according to carbonation or chloride ion attack, respectively.

Key words: prestressed concrete structure; limit state; failure probability; durability design

Modern prestressed concrete structures are most popularly applied to civil engineering, hydraulic engineering and communication engineering. Like other structures, durability failure of prestressed concrete structures may occur from long-term corrosion in the aggressive environment. However, the researches on the durability mechanism and durability design have been rare both at home and abroad up to now. Under the support of the National Natural Science Foundation of China, and on the basis of the fruits of reinforced concrete structure durability research, new research has been executed on the durability of prestressed concrete structures.

The research on the durability of concrete structures is currently classified into two categories: one is to analyze microscopically the main factors which influence the durability of a concrete structure, study corrosion mechanisms of concrete and steel, and find effective measures to stop corrosion or retard the process of deterioration; the other is to study macroscopically the degradation process of structure's performance due to material deterioration.

The research on the durability of material engineering is much more extensive. Many research achievements have been made and some of them have been applied to code and standard in some countries^[1–3]. Consequently, the durability of reinforced

concrete structures can be guaranteed in terms of material parameters. However, there are so many factors which influence the durability of concrete structures that it is impossible to precisely describe the decline process of structure's performance; thus it is not practical to make a reliable durability design of a concrete structure. Based on the durability characteristics of prestressed concrete structures and the research achievements on the durability of concrete structures, the new research on durability design of prestressed concrete structures in carbonation and chloride ion aggressive environments will be presented as follows.

1 Analysis Model of Durability Limit State of Prestressed Concrete Structure

The key to durability design of a prestressed concrete structure is to determine the type of corrosion and establish the durability limit state. The durability limit state of a prestressed concrete structure can be expressed as

$$R(t) - S(t) = 0 \quad (1)$$

where $R(t)$ is the performance of stochastic process of structure durability resistance; $S(t)$ is the action of stochastic process imposed externally, such as load, deformation or environmental action, etc. For a prestressed concrete structure, $R(t)$ is composed of two parts and can be written as^[4]

$$R(t) = R_p(t) + R_s(t) \quad (2)$$

where $R_p(t)$ is the performance of stochastic process of the structure being prestressed, and $R_s(t)$ is the performance of stochastic process of the structure being reinforced.

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Because the ratio of $R_p(t)/R(t)$ usually equals 0.55–0.75^[4], the prestressed part bears most of the external loads. The resistance of prestressed structures is greatly dependent on the prestressing tendons, and the carrying capacity of the tendons is greatly dependent on the state of corrosion. Before the initiation of tendon corrosion, the durability resistance of the structure decreases slowly. Nevertheless, once the corrosion of prestressing tendons occurs, the durability resistance and then the load resistance of the structure decrease rapidly and sudden failure may arise. It is difficult to describe $R(t)$ and $S(t)$ exactly, so on the basis of the characteristics of durability failure of a prestressed concrete structure, different limit states are defined according to the corrosive environments, respectively.

1.1 In carbonation environment

Carbon dioxide is the factor that initiates concrete carbonation of prestressed structures. The carbonation limit state is defined as the carbonation reaching the surface of prestressed tendon. Thus we can obtain a carbonation limit state equation:

$$\psi_x(t) = c - x(t) = 0 \quad (3)$$

where c is the thickness of a concrete cover of prestressed steel.

Using $x(t)$ denotes the carbonation depth. In the tensile stress state, $x(t)$ can be written as

$$x(t) = \eta k_{RH} k_T k_{CO_2} k_{wc} (1 + \alpha_{ct} \sigma_{ct} + \beta_{ct} \sigma_{ct}^2 + \gamma_{ct} \sigma_{ct}^3) \sqrt{t} \quad (4)$$

In the compressive stress state, $x(t)$ can be written as

$$x(t) = \eta k_{RH} k_T k_{CO_2} k_{wc} (1 + \alpha_{cp} \sigma_{cp} + \beta_{cp} \sigma_{cp}^2 + \gamma_{cp} \sigma_{cp}^3) \sqrt{t} \quad (5)$$

where σ_{ct} is the tensile stress, and σ_{cp} is the compressive stress.

1.2 In Cl^- aggressive environment

Cl^- attack is the main factor initiating the durability failure of prestressed concrete structures. Therefore the durability limit state is assumed to be when the chloride concentration on the surface of prestressed tendon is equal to the critical threshold. Thus we can obtain a chloride attack limit state equation:

$$\psi_c(t) = C_{cr} - C_{c,t} = 0 \quad (6)$$

where $C_{c,t}$ is Cl^- concentration at the depth c of the concrete cover, i. e., on the surface of prestressing tendons; C_{cr} is the critical concentration that initiates corrosion.

In the state of tensile stress, $C_{c,t}$ can be written as

$$C_{c,t} = C_0 + (C_s - C_0) \cdot$$

$$\left[1 - \operatorname{erf} \left(c / \left(2 \sqrt{\frac{D_{ini}(1 + \xi_{ct} \sigma_{ct} + \eta_{ct} \sigma_{ct}^2 + \zeta_{ct} \sigma_{ct}^3)}{(1+R)(1-m)}} t^{1-m} \right) \right) \right] \quad (7)$$

In the state of compressive stress, $C_{c,t}$ can be written as

$$C_{c,t} = C_0 + (C_s - C_0) \cdot \left[1 - \operatorname{erf} \left(c / \left(2 \sqrt{\frac{D_{ini}(v + \xi_{cp} \sigma_{cp} + \eta_{cp} \sigma_{cp}^2 + \zeta_{cp} \sigma_{cp}^3)}{(1+R)(1-m)}} t^{1-m} \right) \right) \right] \quad (8)$$

where R and m are empirical coefficients, $\operatorname{erf}(\cdot)$ is the error function.

2 Probability Analysis on Durability of Prestressed Concrete Structure

2.1 In carbonation environment

Because $R(t)$, $S(t)$ and $X(t)$ are stochastic processes according to Eqs. (1), (2), (4) and (5), usually their mean values are only available from the mathematical model. According to many research achievements^[5,6], concrete carbonation depth is subjected to normal distribution, so if the carbonation depth equals the thickness c of the concrete cover for prestressing tendon, the durability failure probability of the structure is regarded as 50%. According to practical design principles, the failure probability of structure must be limited to a certain allowable value, i. e.

$$P_f = P\{X(t) > c\} < P_{\max} \quad (9)$$

where P_f is the failure probability and P_{\max} is the maximum allowable durability failure probability. Once the distribution of carbonation depth $X(t)$ is obtained, the durability failure probability can be calculated as

$$P_f = P\{X(t) > c\} = 1 - P\{X(t) < c\} = 1 - \Phi(\beta_X) \quad (10)$$

$$\beta_X = \frac{c - X(t)}{\sigma_X} \quad (11)$$

where

$$\sigma_X = \sqrt{\sum_{i=1}^n \left[\frac{\partial X}{\partial x_i} \sigma(x_i) \right]^2} \quad (12)$$

x_i are the factors influencing carbonation depth.

2.2 In Cl^- aggressive environment

The analysis of Cl^- attack is much more complex than that of carbonation. And the probability distribution of the Cl^- concentration on the surface of prestressing tendon is sophisticated. For the sake of simplicity, we regard the distribution as equivalent to normal distribution. And the mean value, i. e. expected value of the chloride ion concentration, is used in the calculation of failure probability. If $C_{c,t}$ equals C_{cr} , the

durability failure probability is approximately 50%. According to the practical design principle, the failure probability of structure must be limited to a certain allowable value^[7,8], i. e.

$$P_f = P\{C_{c,t} > C_{cr}\} < P_{\max} \quad (13)$$

where P_f is the failure probability and P_{\max} is the maximum allowable durability failure probability.

$$P_f = P\{C_{c,t} > C_{cr}\} = 1 - P\{C_{c,t} < C_{cr}\} = 1 - \Phi(\beta_C) \quad (14)$$

$$\beta_C = (C_{cr} - C_{c,t})/\sigma_C \quad (15)$$

where

$$\sigma_C = \sqrt{\sum_{i=1}^n \left[\frac{\partial C}{\partial x_i} \sigma(x_i) \right]^2} \quad (16)$$

x_i are the factors influencing the chloride ion content.

3 Application of Semi-Theory Durability Design Method

It is impossible to be very accurate regarding the durability design for structures due to the complex interaction of many factors that contribute to a structure's durability. The durability design method is based on the principle of rational estimation. Hence, the degradation degree of the structures due to carbonation or chloride ion attack should be evaluated; and determining the partial differential coefficients of $X(t)$ or $C_{c,t}$ is the simplest method to evaluate the degree. Because research on the chloride ion attack is not self-contained, only the application of the method of the partial differential coefficients on carbonation is presented in the following.

Based on the analyses above, carbonation depends on relative temperature, environmental temperature, CO_2 concentration, water/cement ratio and stress state, etc. The prediction model of carbonation depth can be written as

1) In the tensile stress state

$$X(t) = \eta k_{RH} k_T k_{\text{CO}_2} k_{wc} (1 - 0.112581 \sigma_{ct} + 0.129026 \sigma_{ct}^2 - 0.0126427 \sigma_{ct}^3) \sqrt{t} \quad (17)$$

$$\sigma_X = \sqrt{\left[\frac{\partial X(t)}{\partial k_{RH}} \frac{100 - 2RH}{RH_0(100 - RH_0)} \right]^2 \sigma_{RH}^2 + \left[\frac{\partial X(t)}{\partial k_T} \right]^2 \frac{1}{16T_0^2} \left(\frac{T}{T_0} \right)^{-\frac{3}{2}} \sigma_T^2 + 2.25 \left[\frac{\partial X(t)}{\partial k_{wc}} \right]^2 \frac{A^2 (f_{cu,0}^2 \sigma_{f_{ce}}^2 + f_{ce}^2 \sigma_{f_{cu,0}}^2)}{(f_{cu,0} + ABf_{ce})^4}} \quad (23)$$

4 Example: A Sing-Span Prestressed Concrete Beam

There is a single-span prestressed concrete beam, whose concrete strength is C50, cement strength level is 42.5 and water to cement ratio is 0.40. In the normal service stage, the compressive stress on the top fiber of beam is $\sigma_{cp} = 11.75$ MPa, and the tensile stress on the bottom fiber is $\sigma_{ct} = 3.80$ MPa. According to Eqs. (17),

2) In the compressive stress state

$$X(t) = \eta k_{RH} k_T k_{\text{CO}_2} k_{wc} (1 + 0.00452352 \sigma_{cp} - 0.00354674 \sigma_{cp}^2 + 0.00014676 \sigma_{cp}^3) \sqrt{t} \quad (18)$$

Relative humidity, environmental temperature, and concrete quality are the main factors which should necessarily be considered, influencing the deviation of carbonation depth. Furthermore, CO_2 concentration and relative humidity fluctuate slightly in the same city, but they vary substantially in other cities^[5-7], so the carbonation should be evaluated in different cities, respectively.

Based on the tolerance transfer formula, the standard deviation of carbonation depth can be expressed as

$$\sigma_X = \sqrt{\left[\frac{\partial X(t)}{\partial k_{RH}} \right]^2 \sigma_{k_{RH}}^2 + \left[\frac{\partial X(t)}{\partial k_T} \right]^2 \sigma_{k_T}^2 + \left[\frac{\partial X(t)}{\partial k_{wc}} \right]^2 \sigma_{k_{wc}}^2} \quad (19)$$

$$\text{As } k_{RH} = \frac{RH(100 - RH)}{RH_0(100 - RH_0)}, \text{ then}$$

$$\sigma_{k_{RH}} = \frac{100 - 2RH}{RH_0(100 - RH_0)} \sigma_{RH} \quad (20)$$

$$\text{As } k_T = \sqrt[4]{\frac{T}{T_0}}, \text{ then}$$

$$\sigma_{k_T} = \frac{1}{4T_0} \left(\frac{T}{T_0} \right)^{-\frac{3}{4}} \sigma_T \quad (21)$$

$$\text{As } k_{wc} = 1.23 (1 - 0.8e^{-1.5w/c})^{[8]} \text{ and } w/c = Af_{ce}/(f_{cu,0} + ABf_{ce})^{[9]}, \text{ then}$$

$$\sigma_{k_{wc}} = 1.50 \frac{A(f_{cu,0} \sigma_{f_{ce}} - f_{ce} \sigma_{f_{cu,0}})}{(f_{cu,0} + ABf_{ce})^2} \quad (22)$$

where A and B are regressive coefficients according to large numbers of test data; $f_{cu,0}$ is the designed strength of the concrete; f_{ce} is the effective strength of the cement, $f_{ce} = r_c f_{ce,k}$, herein $f_{ce,k}$ is the standard strength of the cement and r_c is marginal factor of the standard strength of the cement, and $r_c = 1.13$ ^[9].

Substituting Eqs. (20) – (22) into Eq. (19), one can obtain the following equation:

(18) and (23), we calculate the concrete covers required on the top and bottom of the beam in the 32 central cities in China. In order to simplify the calculation, assume the deviation coefficients of the relative humidity and the temperature are 0.05 and 0.04, respectively. The standard error of the designed concrete strength is 6.5 MPa, and that of the standard strength of the cement is 5.5 MPa. Additionally, $A = 0.34$ and $B = 0.30$ ^[10,11]. Assume the designed service life is 50 years. The concrete cover

should satisfy the 95% probability reliability. The calculated results are shown in Tab. 1.

Tab. 1 The concrete cover calculated

City	Annual average relative humidity/%	Annual average temperature/°C	Concrete cover required in different states/mm		
			State free from stress	State of tensile stress	State of compressive stress
Beijing	58	11.8	26.1	45.4	20.9
Harbin	66	3.8	18.3	31.9	14.7
Changchun	64	5.2	20.2	35.2	16.2
Shenyang	63	8.1	22.8	39.8	18.3
Tianjin	61	12.5	25.9	45.1	20.8
Shijiazhuang	63	13.0	25.7	44.8	20.6
Hohhot	55	6.2	22.5	39.2	18.0
Taiyuan	60	9.6	24.4	42.5	19.6
Jinan	57	14.4	27.5	48.0	22.1
Xi'an	71	13.4	23.4	40.8	18.8
Lanzhou	58	9.3	24.5	42.8	19.7
Yinchuan	58	8.7	24.1	42.0	19.4
Xining	55	5.9	22.2	38.7	17.8
Urumchi	58	6.6	22.5	39.2	18.1
Chongqing	80	17.7	21.1	36.7	16.9
Chengdu	83	16.0	19.0	33.1	15.2
Guiyang	77	15.3	21.7	37.9	17.4
Kunming	73	14.6	23.2	40.4	18.6
Lhasa	45	7.5	23.6	41.1	18.9
Zhengzhou	66	14.2	25.5	44.3	20.4
Hefei	76	15.7	22.3	38.9	17.9
Wuhan	79	16.3	21.1	36.8	16.9
Changsha	81	17.2	20.4	35.5	16.4
Nanjing	77	15.3	21.7	37.9	17.4
Shanghai	78	15.8	21.4	37.3	17.2
Hangzhou	79	16.2	21.1	36.7	16.9
Fuzhou	77	19.7	23.2	40.3	18.6
Nanchang	77	17.5	22.5	39.1	18.0
Guangzhou	78	21.8	23.2	40.5	18.6
Nanning	79	21.6	22.7	39.5	18.2
Haikou	85	23.8	19.8	34.4	15.9
Taipei	80	22.3	22.3	38.9	17.9

The calculated results indicate that the concrete cover of the members or the structures in the tensile state is much greater than that in the compressive state. Hence, the stress state in the concrete structure should be considered in the durability design. It should be noted that the thickness of the concrete cover calculated in Tab. 1 is not the final one, and the thickness of cover should also satisfy other durability requirements and service requirements.

5 Conclusion

Structural durability design is drawing more and more civil engineers' attentions. Some durability research achievements have been used in practical applications. However, if we expect the durability design method to gain the same importance as the strength design method, we should have to adhere to the dura-

bility research, and promote and popularize the durability design method. However, it will certainly take time.

For the structural durability design, it is key to make certain which corrosive factors act on the structure, how the factors act on the structure and when some factor becomes a critical one. Wherefore, if we know when the factor becomes critical, we can establish the durability limit state equation. In this paper, based on the durability characteristics of prestressed concrete structures, the durability limit states of carbonation and chloride ion attack are defined, respectively. In addition, a simple durability design method for carbonation of structures is put forward.

A prestressed concrete structure is stressed by jacking beforehand, so the fine cracks in the concrete can close and the concrete is denser. Consequently, the

concrete in service is like an elastic body and its capability against corrosion is evidently strengthened; in other words, the durability of prestressed concrete structures is superior to that of traditional structures.

The research simultaneously indicates that the concrete cover prescribed in the current code^[12] is not adequate. The rational cover thickness should not be less than 35 or 45 mm according to the surrounding environment. It is especially notable for civil engineers that the climate of Northwest and North China is drier than that of Southern China, but the carbonation of structures in the North is potentially more severe than that in the South.

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预应力混凝土结构的耐久性设计方法研究

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摘要: 根据预应力混凝土结构的耐久性特点, 分别定义了结构构件在碳化环境和氯离子侵蚀环境下的耐久性极限状态, 建立了以可靠性数学和随机过程为理论基础的数学模型, 并对耐久性失效概率进行了理论分析. 提出了碳化环境下的预应力混凝土结构构件的耐久性实用设计方法. 计算结果表明: 预应力混凝土结构的耐久性优于传统钢筋混凝土结构; 现行国家规范 (GB 50010—2002) 中规定的混凝土保护层厚度是不足的; 根据碳化和氯离子侵蚀环境的不同, 混凝土保护层厚度应该分别不小于 35 mm 和 45 mm.

关键词: 预应力混凝土结构; 极限状态; 失效概率; 耐久性设计

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