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Three-dimensional calculation of roadway earthwork volume

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Abstract: To solve the inaccuracy problem caused by the two existing methods (average end-area method and prismoidal method) used for the calculation of roadway earthwork volume, this paper puts forward a new concept of the 3-dimensional algorithm that takes all the roadway geometric design procedures as a kind of geometrical operation between the ground model (original terrain model) and the roadway model (designed model) under certain constraints, and then presents a complete 3-dimensional algorithm of roadway earthwork volume as well as its executable computer program. The algorithm benefits from the re-triangulation technique of constrained delaunay triangulation (CDT), which can yield a true volume value theoretically. Through a number of practical tests covering varied intervals between adjacent cross sections, it is proven to possess a higher accuracy compared with that of traditional methods. All the work involved in this paper indicates that the 3-dimensional calculation of roadway earthwork volume is feasible, more accurate and should have further application in practice.

Key words: earthwork volume; digital terrain model (DTM); constrained delaunay triangulation (CDT); roadway design; calculation

The average end-area method and prismoidal method are widely used in the calculation of roadway earthwork volume. A volume is calculated by an average-area or an adjusted area of multiplying distance between the two sections in these two methods. However, the core of the calculations is 2-dimensional and theoretically defective because two adjacent sections do not change linearly along its central line of roadway. Therefore, to get a more accurate result of earthwork volume, some adjustments, such as the special treatment at curved roadway and transition sections, were used[1-4]. Meanwhile, to simplify the calculation, the dispose of ground line as unilateral linear or bi-directional linear straight line was employed[5].

Although there are several 3-dimensional calculation methods, typically using triangle-based or grid-based digital terrain model, for calculating earthwork volume mainly at site engineering[6-8], there has not been a 3-dimensional calculation method for roadway engineering. It is partly because a roadway is a long strip-shape with many more restrictions than that of a site leveling in a DTM-based 3-dimensional calculation, for example.

However, with the help of a digital terrain model (DTM) and its re-triangulation techniques, this paper puts forward a concept in which all results of roadway geometrical design can be regarded as geometrical algorithms between ground models (original terrain model) and roadway models (designed model) under certain restrictions, presenting a complete 3-dimensional algorithm of roadway earthwork volume and furthermore reaching some positive conclusions after a series of tests was accomplished.

1 Principle of 3-Dimensional Calculation of Roadway Earthwork Volume

According to the theories of mesh generation in computer science, any complicated surface can be presented and simulated by triangular meshes. Therefore, the calculation of earthwork volume among a whole area can be divided into the volume calculation of each triangle pair consisting of the original terrain triangle and designed surface triangle, which share the same vertices on a horizontal plane in only four types as shown in Fig. 1. It is easy to calculate the fill or/and cut volume in a single triangle pair as well as the whole earthwork volume. Then, the key of the 3-dimensional algorithm turns into how to organize the triangle pairs of a desired area.

For applications of a 3-dimensional algorithm in fields such as real estate and land record surveying, they are relatively easier because only boundary lines are taken as constraints and constrained delaunay triangulation (CDT) is easily executed to form triangle pairs. However, for the applications in roadway engineering, it is much more complicated. The following steps show how to get the triangle pairs for roadway engineering by re-triangulation technique (see Fig. 2).

① At first, the roadway model should be triangu-

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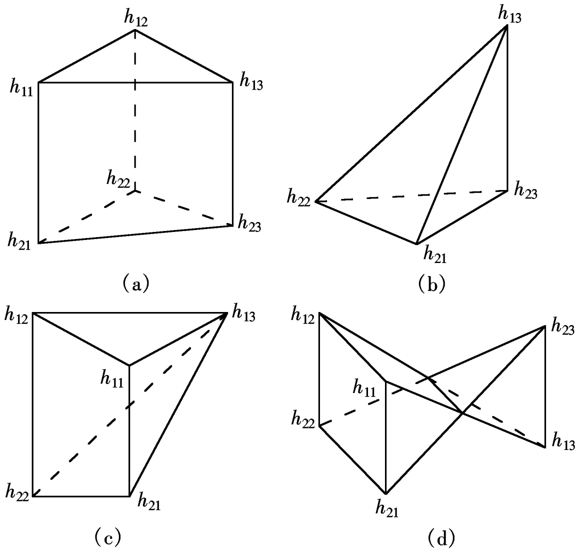

Fig. 1 Four types of triangle pair with three coincided vertices on a horizontal plane

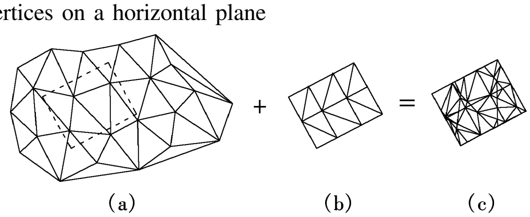

Fig. 2 Re-triangulation of DTM within road boundary. (a) TIN of original terrain, dashed line showing road boundary; (b) Triangular network of road surface; (c) Re-triangulation within road boundary

lated with the constrained lines along cross sections.

② On the horizontal plane, triangle meshes of the roadway model are overlaided on those of the original terrain model, the intersected points and boundary points of which are considered as newly added points, meanwhile, all the existing lines (triangular edges), including roadway boundary lines, may be divided as many new small-segment boundary lines that should be considered as constrained lines.

③ Based on all points and constrained lines created through step ① and step ②, execute CDT.

④ On the vertical plane, elevations of roadway model points can be obtained with their projection on the original terrain model, and elevations of the original terrain model points can also be obtained with their projection on the roadway model. Similarly, elevations of new points produced by intersections can also be obtained with their projection on the two models respectively.

⑤ Triangle pairs with the roadway model and the original terrain model fully identified on the horizontal plane are obtained and their corresponding fill and cut earthwork volumes can be calculated.

⑥ The amount of fill and cut earthwork volumes between each station interval as well as the whole earthwork volume within the roadway boundary can finally be calculated.

Actually, according to the above working flow, the author developed a software program using ObjectARX under AutoCAD to test this 3-dimensional method. The details of the software are omitted for the sake of space.

2 Comparing 3-Dimensional Algorithm with Traditional Average End-Area Method

In order to compare the 3-dimensional method with the traditional average end-area method, five cases are referenced to validate the differences between these two kinds of algorithms, which focus on different intervals between cross sections, that is, 50, 30, 20, 10 and 5 m, respectively. As anticipated, the shorter the interval, the smaller the difference between these two algorithms.

In the test, the original terrain model is described as a plain and small hill and the roadway as a grade-two highway, with 2 × 7. 0 m-wide driving lanes and 2. 5 m-wide shoulders, without a center divider. The route length is 1. 8 km long. According to these two kinds of algorithms, earthwork volumes, classified as cut, fill and sum, of each interval between adjacent cross sections are calculated. For each group of volumes, there are two volumes corresponding to the two methods and then their relative errors can be computed to get three series of them, which can be respectively marked as cut error, fill error, and sum error. As for those cases in which errors exceed 50%, they should be considered as nonpermissible errors caused by defects of the traditional algorithm and the cases should be taken out from the whole sample. The differences between the two methods analyzed by relative error is shown in Tab. 1.

Tab. 1 Error analysis between two methods of earthwork volume calculations

Route length/m	Interval/m	Amount of sample	Average value of cut sample errors/%	Average value of fill sample errors/%	Average value of sum sample errors/%	Proportion of cases in which errors exceed 50%/%
1 800	5	360	2. 45	1. 65	0. 97	1. 11
1 800	10	180	2. 41	2. 14	2. 54	6. 67
1 800	20	90	4. 79	4. 53	6. 26	12. 22
1 800	30	60	7. 07	6. 51	9. 74	16. 67
1 800	50	36	10. 40	12. 15	17. 72	30. 56

In order to describe intuitively, relative errors of different intervals of 5, 10 and 20 m are illustrated in Fig. 3 (Cases within the error range of 5% are just listed for clarity).

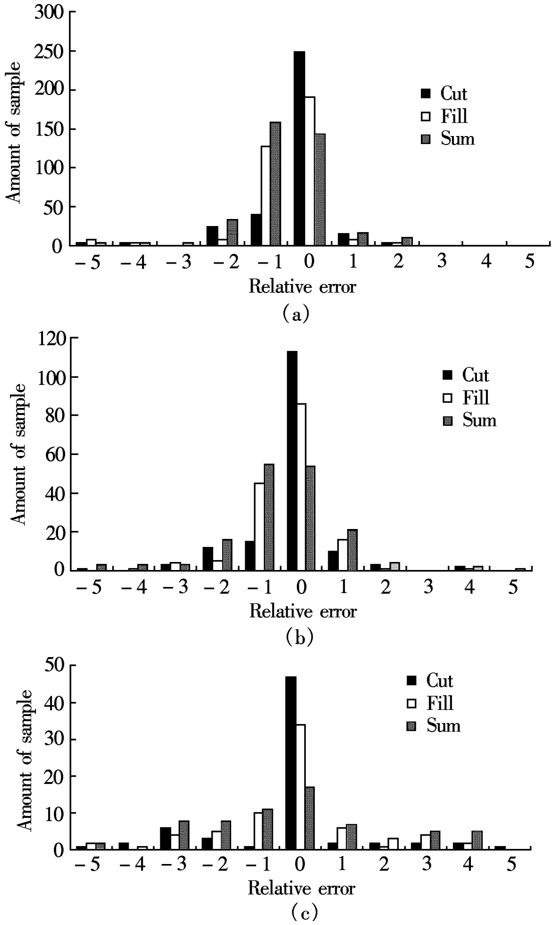

Fig. 3 Error analysis with three different intervals. (a) 5 m; (b) 10 m; (c) 20 m

From Tab. 1 and Fig. 3, we can find the actual results are consistent with the anticipated ones, which indicate that the 3-dimensional algorithm of earthwork volume based on the digital terrain model can be applied in engineering and achieve more accurate results. In order to testify the 3-dimensional algorithm's accuracy, every pair of triangles can be created into meshes under AutoCAD after earthwork volume calculation, and intervals between adjacent stations should be placed on two different layers (for instance, layer "odd" and layer "even"), so we can clearly judge the veracity of the original terrain, designed surface as well as their fill and cut relations by freezing one layer and shading or rendering. As shown in Fig. 4, the roadway model (in darker color) and terrain model (in lighter color) are equally spaced and their 3-dimensional relations (especially in the direction of cross sections) are clear at a glance.

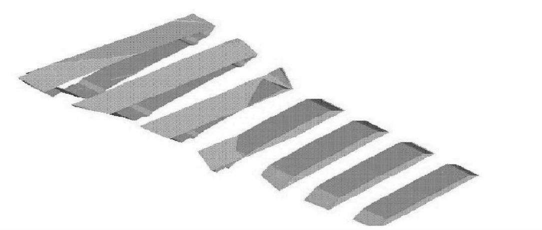

Fig. 4 Both original terrain and road surface created after earthwork volume calculation

3 Conclusions

1) The 3-dimensional algorithm of earthwork volume based on the digital terrain model and re-triangulation technique is proven as theoretically correct and more accurate than that of traditional methods, which can be used in roadway engineering.

2) Furthermore, if there are original terrain data with layered geological information such as soil and stone, the earthwork volumes calculated are also able to be classified to the related layers, which are definitely better than those roughly estimated layer proportions used nowadays.

3) The 3-dimensional algorithm of earthwork volume is also able to be used in site engineering (see Fig. 5), which is easier compared with its use in roadway engineering.

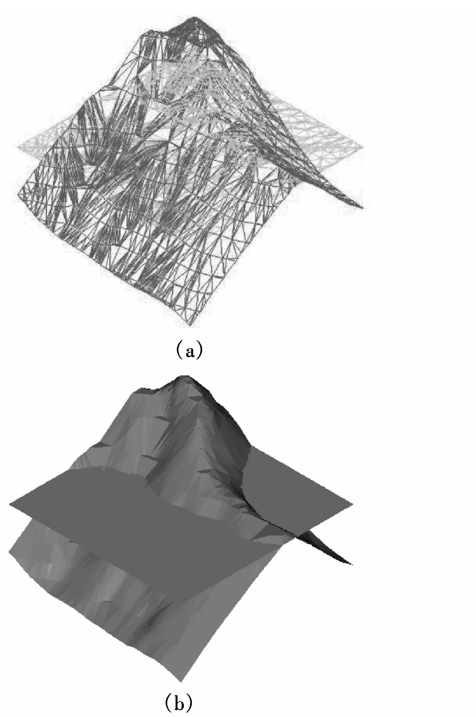

Fig. 5 The 3-dimensional algorithm used in leveling up a site. (a) Triangular meshes consisted by triangle pairs; (b) Rendered view at AutoCAD

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道路土方量的三维计算方法

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摘要: 针对目前广泛使用的道路土方量计算方法平均断面法和棱柱体法计算不准确的缺点, 提出了三维土方量计算算法的概念. 该算法以带约束的狄罗尼三角化(CDT)为技术核心, 认为所有道路几何设计过程都是地面模型和道路(设计)模型进行几何运算的结果. 基于此, 本文设计出相应的算法步骤, 同时完成了相应的软件开发, 使得该三维算法能和传统的方法进行对比. 此外, 结合工程实例, 采用了不同的道路横断面间距对三维计算方法和传统方法的误差进行比较、分析. 结果证明三维算法具有更好的精度, 该算法可用于道路、场地平整等工程土方量计算.

关键词: 土方量; 数字地面模型; 狄罗尼三角化(CDT); 道路设计; 计算

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