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# Edge span of $L(d, 1)$-labeling on some graphs

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**Abstract:** Given a graph $G$ and a positive integer $d$, an $L(d, 1)$-labeling of $G$ is a function $f$ that assigns to each vertex of $G$ a non-negative integer such that $|f(u) - f(v)| \geqslant d$ if $d_G(u, v) = 1$; $|f(u) - f(v)| \geqslant 1$ if $d_G(u, v) = 2$. The $L(d, 1)$-labeling number of $G$, $\lambda_d(G)$ is the minimum range span of labels over all such labelings, which is motivated by the channel assignment problem. We consider the question of finding the minimum edge span $\beta_d(G)$ of this labeling. Several classes of graphs such as cycles, trees, complete $k$-partite graphs, chordal graphs including triangular lattice and square lattice which are important to a telecommunication problem are studied, and exact values are given.

**Key words:** $L(d, 1)$-labeling; edge span; triangular lattice; square lattice; choral graphs; $r$-path

## 1 Background

The $L(2, 1)$-labeling is introduced by Griggs and Yeh[1], as a variation of the channel assignment problem in radio system which was first formulated as a graph coloring problem by Hale[2]. Suppose that a number of transmitters or stations are given. We ought to assign a channel to each of the given transmitters or stations in order to avoid interference. To reduce interference, any two "close" transmitters must receive different channels, and any two "very close" transmitters must receive channels that are at least two apart. We can construct an interference graph for this problem so that each transmitter or station is represented by a vertex on $R^2$. There is an edge between two "very close" transmitters or stations, and we define "close" if the corresponding vertices are of distance two.

Given a graph $G = (V, E)$, an $L(2, 1)$-labeling of $G$ is a non-negative integral function such that $|f(x) - f(y)| \geqslant 2$, if $\{x, y\} \in E$ and $|f(x) - f(y)| \geqslant 1$, if $d(x, y) = 2$. There are many papers concerning the $L(2, 1)$-labeling number[3 - 10]. Griggs and Yeh[1] proved that $\lambda(G) \leqslant \Delta^2 + 2\Delta$ and conjectured that $\lambda(G) \leqslant \Delta^2$; Chang and Kuo[3] improved the bound to $\lambda(G) \leqslant \Delta^2 + \Delta$. In this paper, we consider a more general problem $L(d, 1)$-labeling. The $L(d, 1)$-labeling also has been extensively studied in many papers. The $L(d, 1)$-labeling for chordal graphs was investigated in Ref. [4]. It proved that $\lambda_d(G) \leqslant (\Delta + 2d - 1)^2/4$ for any chordal graphs with the maximum degree $\Delta$. At the same time, it also has proved that $\lambda_d(G) \leqslant \Delta^2 + (d-1)\Delta$, where $\Delta$ is the maximum degree of the graph. But in this paper we consider another parameter of the $L(d, 1)$-labeling. Given an $L(d, 1)$-labeling $f$ on $G$, we define the edge span of $f$, denoted by $\beta_d(G, f) = \max\{|f(x) - f(y)|: \{x, y\} \in E(G)\}$. The edge span of $L(d, 1)$-labeling on $G$, denoted by $\beta_d(G)$, is $\min \beta_d(G, f)$, where the minimum runs over all $L(d, 1)$-labelings $f$ on $G$. For this parameter, Yeh[5] studied $\beta_2(G)$ for some graphs. In section 2, we give some more general results $\beta_d(G)$ on cycle, tree, complete multipartite graph, $r$-path as well as the triangular lattice and the square lattice. The latter two graphs are motivated by the design of planar regions for cellular phone networks.

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## 2 Main Results

It is clear that $\beta_d(G) \leqslant \lambda_d(G)$ for any graph $G$. If $G$ is a complete graph, then $\beta_d(G) = \lambda_d(G)$. If $G$ is a path $P_n$ where $n \geqslant 2$, then $\beta_d(G) = d$. If $H$ is a subgraph of $G$, then $\beta_d(H) \leqslant \beta_d(G)$. In fact, $\beta_d(G)$ might be far less than $\lambda_d(G)$, we can find it from the following results.

**Theorem 1** Let $C_n$ be a cycle of order $n \geqslant 3$, $d \in \mathbf{N}$, then $\beta_d(C_3) = 2d$, $\beta_1(C_n) = 2$, $\beta_2(C_n) = 3$, $\beta_d(C_n) = d + 1$, where $n \geqslant 4$, $d \geqslant 3$ and $n \neq 5, 7, \ldots, 2d - 1$.

**Proof** We can easily see that $\beta_d(C_3) = 2d$, then we assume $n \geqslant 4$. Let $V(C_n) = \{v_0, v_1, \ldots, v_{n-1}\}$, where $v_i$ is adjacent to $v_{i+1}$, and $v_{n-1}$ is adjacent to $v_0$ where $i = 0, 1, \ldots, n-1$.

**Case 1** When $n = 2k$, $k \geqslant 2$.

First because all the labels are non-negative, there must be a vertex labeled 0, without loss of generality, say, $v_0$. By the definition of the $L(d,1)$-labeling, either the label of $v_1$ or the label of $v_{n-1}$ is at least $d+1$. Then $\beta_d(C_n)\geqslant d+1$ with $n\geqslant 4$. Secondly, label $v_0, v_1,\dots, v_{k-1}$ with $0, d, 2d,\dots,(k-1)d$ respectively and label $v_{n-1}, v_{n-2},\dots, v_k$ with $d+1, 2d+1,\dots, kd+1$ respectively. Clearly, this labeling is an $L(d,1)$-labeling, and $\beta_d(C_n)\leqslant d+1$. Then $\beta_d(G)=d+1$ for $n=2k, k\geqslant 2$.

**Case 2** When $n=2k+1, k\geqslant 2$.

From above, clearly $\beta_d(C_n)\geqslant d+1$.

When $d=1$, we can label $v_0, v_1,\dots, v_{k+1}$ with $0, 1, 3, 5,\dots, 2k-1$, respectively. Label $v_{n-1}, v_{n-2},\dots, v_{k+2}$ with $2k, 2(k-1),\dots, 4, 2$, respectively. Clearly this labeling is an $L(1,1)$-labeling with edge span $d+1=2$. Therefore $\beta_1(G)=d+1=2$.

When $d=2$, we can label $v_0, v_1,\dots, v_{k+1}$ with $0, 2, 4,\dots, 2k, 2(k+1)$ respectively. Label $v_{n-1}, v_{n-2},\dots, v_{k+2}$ with $3, 5,\dots, 2k-3, 2k-1$ respectively. Clearly this labeling is an $L(2,1)$-labeling with edge span $d+1=3$. Therefore $\beta_2(G)=d+1=3$.

When $d\geqslant 3$, let $t=(n-(2d+1))/2$ where $n\geqslant 2d+1$. We can label $v_0, v_1, v_2,\dots, v_{d+t+1}$ with $0, d, 2d, 3d,\dots, td, (t+1)d, (t+2)d,\dots,(d+t+1)d$ respectively. Label $v_{d+t+2}, v_{d+t+3},\dots, v_{2d+t}, v_{2d+t+1}$ with $(d-1)(d+1)+td, (d-2)(d+1)+td,\dots, 2(d+1)+td, (d+1)+td$, respectively. Label $v_{2d+t+2}, v_{2d+t+3},\dots, v_{2d+2t-1}, v_{2d+2t}, v_{2d+2t+1}$ with $(d+1)+(t-1)d, (d+1)+(t-2)d,\dots,(d+1)+2d, (d+1)+d, d+1$ respectively. Clearly this labeling is an $L(d,1)$-labeling with edge span $d+1$. Therefore when $n\neq 5, 7,\dots, 2d-1, \beta_d(G)=d+1$.

From the above, we have not given any results about $C_n$ where $n=5, 7,\dots, 2d-1$ and $d\geqslant 3$. The following theorem gives their upper and lower boundaries.

**Theorem 2** Let $C_{2k+1}$ be a cycle of order $2k+1$, where $2\leqslant k\leqslant d-1$, then $d+1\leqslant\beta_d(C_{2k+1})\leqslant\min\left\{\left\lceil\frac{3}{2}d\right\rceil, d+2+\left\lfloor\frac{d-k}{2}\right\rfloor\right\}$.

**Proof** First we can label $v_0, v_1,\dots, v_k, v_{k+1}$ with $0, d, 2d,\dots, kd, (k+1)d$, respectively. Label $v_{n-1}, v_{n-2},\dots, v_{k+2}$ with $\left\lceil\frac{3}{2}d\right\rceil, \left\lceil\frac{5}{2}d\right\rceil,\dots,\left\lceil\left(k-\frac{1}{2}\right)d\right\rceil$, respectively. Clearly, this labeling is an $L(d,1)$-labeling with edge span $\left\lceil\frac{3}{2}d\right\rceil$. Secondly we can label $v_0, v_1, v_2, v_3,\dots, v_{2k-2}, v_{2k-1}, v_{2k}$ with

$$d, 0, d+\left\lfloor\frac{d-k}{2}\right\rfloor+1, \left\lfloor\frac{d-k}{2}\right\rfloor+1,\dots, 2d-\left\lfloor\frac{d-k}{2}\right\rfloor-2, d-\left\lfloor\frac{d-k}{2}\right\rfloor-2, 2d \qquad (1)$$

or

$$d, 0, d+\left\lfloor\frac{d-k}{2}\right\rfloor+2, \left\lfloor\frac{d-k}{2}\right\rfloor+2,\dots, 2d-\left\lfloor\frac{d-k}{2}\right\rfloor-2, d-\left\lfloor\frac{d-k}{2}\right\rfloor-2, 2d \qquad (2)$$

When $(d-k)\equiv 0 \pmod 2$, we label $C_{2k+1}$ with (1). When $(d-k)\equiv 1 \pmod 2$, we label $C_{2k+1}$ with (2). Clearly this labeling is an $L(d,1)$-labeling with edge span $d+\left\lfloor\frac{d-k}{2}\right\rfloor+2$. Therefore $d+1\leqslant\beta_d(C_{2k+1})\leqslant\min\left\{\left\lceil\frac{3}{2}\right\rceil d, d+2+\left\lfloor\frac{d-k}{2}\right\rfloor\right\}$.

**Theorem 3** Let $T$ be a tree with the maximum degree $\Delta$, then $\beta_d(T)=\lceil\Delta/2\rceil+d-1$.

**Proof** Let $m=\lceil\Delta/2\rceil+d-1$. Since $\Delta$ is the maximum degree of $T$, $K_{1,\Delta}$ is a subtree of $T$. Let $V(K_{1,\Delta})=\{v_0, v_1,\dots, v_\Delta\}$ where $v_i, i=1,2,\dots,\Delta$ are leaves of $K_{1,\Delta}$. We label $v_0$ with $\lceil\Delta/2\rceil+d-1$ and other vertices labeled with $0, 1,\dots,\lceil\Delta/2\rceil-1, \lceil\Delta/2\rceil+2d-1,\dots,\Delta+2d-2$. We can easily see that the $L(d,1)$ edge span of $K_{1,\Delta}$ is equal to $m$. So $\beta_d(T)\geqslant\beta_d(K_{1,\Delta})=m$. On the other hand, we can give it the labeling as follows.

For any vertex of $T$, we can visit it from root $v_0$ labeled $x$ by a breadth-first search. For a visited vertex $v$, if $v$ is labeled by $y$ but its children are unlabeled. Since $v$ has at most $\Delta-1$ children which are unlabeled, we can label them with $\{y-d, y+d, y-d-1, y+d+1,\dots, y-m, y+m\}$.

We can continue as above till all interval vertices are visited. Every labeling adds the same large positive integer $n$ such that every labeling is positive. Clearly, this is an $L(d,1)$-labeling with edge span $m$, i. e. $\beta_d(T)\leqslant m$. Therefore, $\beta_d(T)=m$.

**Theorem 4** Let $K=K_{n_1,n_2,\dots,n_k}$ be a complete $k$-partite graph, where $n_1\geqslant n_2\geqslant n_3\geqslant\dots\geqslant n_k$, and $n_1-n_2\geqslant d-1$. Then $\beta_d(K)=\lceil n_1/2\rceil+n_2+n_3+\dots+n_k+(k-1)(d-1)-1$.

**Proof** Let $t=\lceil n_1/2\rceil+n_2+n_3+\dots+n_k+(k-1)(d-1)-1$. Suppose $V(K)=V_1\cup V_2\cup\dots\cup V_k\cup V_{k+1}$, where $V_1\cup V_{k+1}, V_2,\dots, V_k$ are partite sets of $K$, where $|V_1|=\lceil n_1/2\rceil$, $|V_{k+1}|=\lfloor n_1/2\rfloor$ and $|V_i|=n_i, i=2,3,\dots,k$. Then we can label the vertices in each $V_j$ with consecutive integers such that the minimum label in $V_1$ is 0, and the minimum label of $V_j$ is the maximum label of $V_{j-1}$ plus $d$. Clearly, this is an $L(d,1)$-labeling with edge span $t$, thus $\beta_d(K)\leqslant t$.

On the other hand, let $f$ be an $L(d,1)$-labeling, we shall prove that $\beta_d(K,f)\geqslant t$. Let $v\in V_r$ and $u\in V_s$ be such that $f(v)=0$ and $p=f(u)=\max_{\omega\in V_i}f(\omega)$ where $i$

$=1,2,\dots,k+1$. Then

$$p \geqslant \lambda_d(K) = \sum_{i=1}^{k} n_i + (k-1)(d-1) - 1 = t + \left\lfloor \frac{n_1}{2} \right\rfloor \geqslant t$$

If $r \neq s$, then $\beta_d(K,f) = p \geqslant t$. So we assume that $r=s$. Let $z$ and $y$ be the maximum and the minimum value of $f$ on all vertices but not in $V_r$.

$$z - y \geqslant \sum_{i=1}^{k} n_i - n_r + (k-2)(d-1) - 1 = t + \left\lfloor \frac{n_1}{2} \right\rfloor - n_r - (d-1)$$

and

$$(z-0)+(p-y) \geqslant 2t + 2\left\lfloor \frac{n_1}{2} \right\rfloor - n_r - (d-1) \geqslant 2t-1$$

Therefore, either $z-0 \geqslant t$ or $p-y \geqslant t$, which implies that $\beta_d(K,f) \geqslant t$.

The vectors $\boldsymbol{\varepsilon}_1 = \{1,0\}$ and $\boldsymbol{\varepsilon}_2 = \{1/2, \sqrt{3}/2\}$ in the Euclidean plane. Then the triangular lattice $\Delta$ is defined by $\Lambda_\Delta$ s. t. $V(\Delta) = \Lambda_\Delta = \{i\boldsymbol{\varepsilon}_1 + j\boldsymbol{\varepsilon}_2 : i,j \in \mathbf{Z}\}$, $E(\Delta) = \{uv: u, v \in \Lambda_\Delta, d_E(u,v) = 1\}$. The triangular lattice is important to the radio engineer, since, if the area of coverage of each transmitter is a disk of fixed radius $r$ centered on the transmitter site, then placing those sites at the vertices of a regular triangular lattice (with adjacent sites a distance $r\sqrt{3}$ apart) covers the whole plane with the smallest possible transmitter density. There is a subgraph of $\Delta$, denote $\Delta_m$ which is the subgraph induced by $\{(i,j): -m \leqslant i \leqslant 0, 0 \leqslant j \leqslant m$ and $0 \leqslant i+j \leqslant m$ for $m \geqslant 2\}$ in $\Delta$.

**Theorem 5** $\beta_d(\Delta) = 2d+1$.

**Proof** $V(\Delta) = \{i\boldsymbol{\varepsilon}_1 + j\boldsymbol{\varepsilon}_2 : i,j \in \mathbf{Z}\}$, then for convenience we can use $(i,j)$ to represent a vertex $v = i\boldsymbol{\varepsilon}_1 + j\boldsymbol{\varepsilon}_2$ in $\Delta$. Let $f: V(\Delta) \to \mathbf{Z}$ where $f(i,j) = -(d+1)i + dj$, then this is an $L(d,1)$-labeling of $\Delta$: for any two vertices $(i_1,j_1),(i_2,j_2) \in V(\Delta)$.

1) If $d((i_1,j_1),(i_2,j_2)) = 1$ in $\Delta$, then $i_1 = i_2$ and $|j_1 - j_2| = 1$, or $j_1 = j_2$ and $|i_1 - i_2| = 1$, or $(i_1-i_2)(j_1-j_2) = -1$. For each case, $|f(i_1,j_1) - f(i_2,j_2)| \geqslant d$.

2) If $d((i_1,j_1),(i_2,j_2)) = 2$, then $|i_1 - i_2| = 2$ and $j_1 = j_2$, $|j_1 - j_2| = 2$ and $i_1 = i_2$ or $|i_1 - i_2| = 2$, $|j_1 - j_2| = 2$ and $(i_1-i_2)(j_1-j_2) = -4$. We can also find that in each case $|f(i_1,j_1) - f(i_2,j_2)| \geqslant 1$. Therefore, $f$ is an $L(d,1)$-labeling with edge span $2d+1$.

On the other hand, let $W$ be the subgraph of $\Delta_m$ (see Fig. 1). It is easy to see that $\beta_d(W) \geqslant 2d+1$, then $\beta_d \geqslant \beta_d(W) \geqslant 2d+1$. Then $\beta_d(\Delta) = 2d+1$.

The square lattice $\square$ is defined by $V_\square = \Lambda(\square) = \mathbf{Z}^2$ where $\mathbf{Z}$ is the set of integers, $E(\square) = \{uv: d_\square(u,v) = 1\}$ where $d_\square(u,v)$ is the Euclidean distance between $u$ and $v$. In fact, the square lattice is the product of two paths. Given $n$, $m$, $\square_{n,m}$ is the subgraph of $\square$ and is induced by $\{(i,j): 0 \leqslant i \leqslant n, 0 \leqslant j \leqslant m\}$. When $n = m = 4$, we can see Fig. 2.

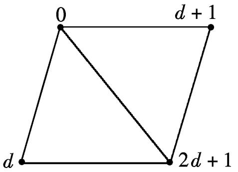

**Fig. 1** Subgraph of $\Delta_m$

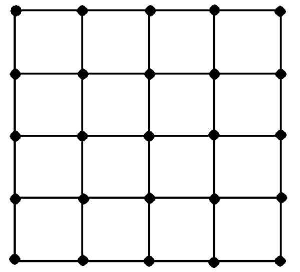
**Fig. 2** Subgraph of $\square$

**Theorem 6** $\beta_d(\square) = \beta_d(\square_{n,m}) = d+1, n \geqslant m \geqslant 1$.

Since $\square_{1,1}$ is isomorphic to $C_4$, by theorem 1, $\beta_d(\square_{1,1}) = \beta_d(C_4) = d+1$. Since $\square_{1,1}$ is a subgraph of $\square_{n,m}$, hence $\beta_d(\square) \geqslant \beta_d(\square_{n,m}) \geqslant \beta_d(\square_{1,1}) = d+1$. On the other hand, let $f: V(\square) \to \mathbf{Z}$ with $f(i,j) = di + (d+1)j$, where $f(i,j)$ stands for $f((i,j))$. For any two vertices $(i_1,j_1),(i_2,j_2) \in V(\square)$:

1) If $d((i_1,j_1),(i_2,j_2)) = 1$, then $|i_1 - i_2| = 1$ and $j_1 = j_2$, or $i_1 = i_2$ and $|j_1 - j_2| = 1$. In each case, $|f(i_1,j_1) - f(i_2,j_2)|$ is either $d$ or $d+1$.

2) If $d((i_1,j_1),(i_2,j_2)) = 2$, then $|i_1 - i_2| = 1 = |j_1 - j_2|$, or $|i_1 - i_2| = 2$ and $j_1 = j_2$, or $i_1 = i_2$ and $|j_1 - j_2| = 2$. In each case, $|f(i_1,j_1) - f(i_2,j_2)|$ is never 0.

We can find that $f$ is an $L(d,1)$-labeling with the edge span $d+1$, then $d+1 \leqslant \beta_d(\square_{n,m}) \leqslant \beta_d(\square) \leqslant d+1$. Then we have $\beta_d(\square_{n,m}) = \beta_d(\square) = d+1$.

An $r$-path is a graph with vertices $v_1, v_2, \dots, v_n$, where $n > r$, such that $v_i, v_{i+1}, \dots, v_{i+r}$ induce a clique for $i = 1,2,\dots,n-r$, denoted by $P_n^r$. By the definition, we can easily find that the chromatic number and the order of the maximum clique of $P_n^r$ are both equal to $r+1$. For choral graphs, i. e. graphs do not contain induced circuits with more than three vertices. Clearly $P_n^r$ is a special choral graph.

**Theorem 7** Let $G$ be an $r$-path with $n$ vertices and $d$ a positive integer with $d \neq 1$, where $n \geqslant r+2$. Then $\lambda_d(G) \leqslant rd + t$ where $t = \lceil n/(r+1) \rceil - 1$.

**Proof** Let $m = (r+1)\lceil n/(r+1) \rceil$, clearly $m \geqslant n$, then $\lambda_d(P_m^r) \geqslant \lambda_d(P_n^r)$. Now we can label the vertices of $P_m^r$ with the sequence: $t, d+t, \dots, rd+t, t-1, d+t-1, \dots, rd+t-1, \dots, 0, d, \dots, rd$.

Clearly this labeling is an $L(d,1)$-labeling with the edge span $rd+t$, then $\lambda_d(P_n^r)\leqslant rd+t$.

**Remark** When $r+2\leqslant n\leqslant 2r+2$, $\lambda_d(P_n^r)=rd+1$; when $n\leqslant r+1$, $\lambda_d(P_n^r)=(n-1)d$. This result can be found in Ref. [4].

**Corollary** Let $G$ be an $r$-path with $n$ vertices and $d$ a positive integer with $d\neq 1$. When $n\leqslant r+1$, $\beta_d(G)=(n-1)d$; when $n\geqslant r+2$, $\beta_d(P_n^r)=rd+1$.

**Proof** Since $n\leqslant r+1$, $P_n^r$ is a complete graph with order $n$, then $\beta_d(P_n^r)=(n-1)d$. By theorem 6, we can easily find $\beta_d(P_n^r)\leqslant rd+1$. On the other hand, since $\lambda_d(P_{r+1}^r)=rd+1$, there must be two vertices $v_i$ and $v_j$ labeled with 0, $rd+1$, respectively and $v_iv_j\in E(P_{r+2}^r)$. Then $\beta_d(\{P_n^r\})\geqslant\beta_d(P_{r+2}^r)\geqslant rd+1$.

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# 图的 $L(d,1)$-标号的边跨度


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**摘要**: 对于给定图 $G$ 顶点集上一个非负整数函数 $f$, 满足: 若 $d_G(u,v)=1$, $|f(u)-f(v)|\geqslant d$; 若 $d_G(u,v)=2$, $|f(u)-f(v)|\geqslant 1$. 称 $f$ 为 $L(2,1)$-标号. 这是由频道分配问题抽象出来的数学模型. 本文主要研究该标号问题的一个参数, 即边跨度, 记作 $\beta_d(G)=\min\limits_f\max\{|f(u)-f(v)|: \forall u\in V(G)\}$, 即对于所有正常的 $L(d,1)$-标号, 使得相邻顶点标号之差的最大值达到最小. 本文主要讨论了圈 $C_n$、树 $T$、$k$-部完全图、正三角形网格、正四边形网格以及弦图等图类的边跨度, 并给出了确切的数值.



**关键词**: $L(d,1)$-标号; 边跨度; 正三角形网格; 正四边形网格; 弦图; $r$-路

**中图分类号**: O157.5