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New subspace algorithm for blind channel estimation in OFDM systems

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Abstract: In order to increase the transmission efficiency, a subspace-based algorithm for blind channel estimation using second-order statistics is proposed in orthogonal frequency division multiplexing (OFDM) systems. Because the transmission equation of OFDM systems does not exactly have the desired structure to directly derive a subspace algorithm, the algorithm first divides the OFDM signals into three parts, then, by exploiting the redundancy introduced by the cyclic prefix (CP) in OFDM signals, a new equation with Toeplitz channel matrix is derived. Based on the equation, a new blind subspace algorithm is developed. Toeplitz structure eases the derivation of the subspace algorithm and practical computation. Moreover the algorithm does not change the existing OFDM system, is robust to channel order overdetermination, and the channel zero locations. The performances are demonstrated by simulation results.

Key words: orthogonal frequency division multiplexing (OFDM); blind channel estimation; subspace algorithm

Orthogonal frequency division multiplexing (OFDM) has gained increased interest in the last few years due to its advantages in digital transmissions over frequency-selective fading channels and is now widely adopted and tested in many communication systems[1,2]. However, under the multipath propagation environment, channel estimation is needed for coherent detection in OFDM systems. Usually, channel estimation in OFDM systems is accomplished using training sequences, which results in a reduction of the transmission efficiency. The increasing demand for high data rates makes blind channel estimation methods very attractive because they save bandwidth by avoiding the use of training sequences. It is first shown in Ref. [3] that the redundancy introduced by the cyclic prefix (CP) in the OFDM systems contains sufficient information to accomplish blind channel estimation using second-order statistics only. Based on Ref. [3], a blind algorithm using the transmitted signal cyclostationarity is proposed[4]. But the algorithm has a large estimated channel error. A subspace-based algorithm for blind channel estimation in OFDM systems is proposed in Ref. [5]. The simulations show that the subspace algorithm has a higher accuracy of channel estimation than that of Ref. [4], but because of the complexity of the derivation in Ref. [5], the performances of the algorithm cannot be discussed thoroughly. Further study is needed. Another subspace algorithm which can be used for blind channel estimation in OFDM systems is proposed in Ref. [6]. The algorithm needs a particular filterbank to precode the transmitted OFDM symbol, so the algorithm cannot be used in the traditional OFDM systems directly.

This paper proposes a new subspace algorithm for blind channel estimation in OFDM systems. The algorithm makes use of the inherent redundancy introduced by the CP for channel estimation, and accomplishes blind channel estimation using second-order statistics only.

1 System Description and Notations

A discrete-time OFDM system can be described as follows: The data symbols input to the OFDM system are first grouped into blocks with size N. Then the symbols in each block are modulated into the subcarriers by an inverse discrete Fourier transform (IDFT). The output sequence of the IDFT is called an OFDM symbol and each sample in the k-th OFDM symbol is given as

$$s_i(k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x_n(k)\mathrm{e}^{\frac{\mathrm{j}2\pi in}{N}} \qquad i = 0, 1, \ldots, N-1 \tag{1}$$

where $x_n(k)$ $(n = 0, 1, \ldots, N-1)$ is the data symbol in the k-th data block. For the purpose of avoiding interference from the preceding OFDM symbols and preserving the orthogonality between the subcarriers, a CP, which is the copy of the last M samples of an OFDM symbol, is added at the beginning of each OFDM symbol. Hence the size of the resulting OFDM symbol including the CP is $M + N$. If we denote the k-th OFDM symbol in vector form as $\boldsymbol{s}(k)$, then

$$\boldsymbol{s}(k) = \{s_{N-M}(k), \ldots, s_{N-1}(k), s_0(k), \ldots, s_{N-1}(k)\}^{\mathrm{T}} \tag{2}$$

In this paper, we assume that the transmitted data symbol $x_n(k)$ is independently and identically distributed (i. i. d.), and channel $\boldsymbol{h}$ is a time-invariant L-th-

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order FIR filter. In practice, to estimate the channel order L exactly from the received signals is unrealistic under the additive noise. However, the size of the CP is always equal to or greater than the order of the channel, i. e. $L \leqslant M$, so, in the proposed algorithm, the channel to be estimated is denoted by $\boldsymbol{h} = \{h(0), h(1), \ldots, h(M)\}$. This means that the channel is over-determination, and $h(i) = 0$ for $i > L$. Passing $\boldsymbol{s}(k)$ through $\boldsymbol{h}$, the corresponding received signal is expressed as

$$\boldsymbol{r}(k) = \boldsymbol{H}_0 \boldsymbol{s}(k) + \boldsymbol{H}_1 \boldsymbol{s}(k-1) + \boldsymbol{v}(k) \tag{3}$$

where $\boldsymbol{H}_0, \boldsymbol{H}_1$ are $(N+M) \times (N+M)$ matrices

$$\boldsymbol{H}_0 = \begin{bmatrix} h(0) & & & & \\ \vdots & h(0) & & & \\ h(M) & & \ddots & & \\ & \ddots & & \ddots & \\ & & h(M) & \ldots & h(0) \end{bmatrix} \tag{4}$$

$$\boldsymbol{H}_1 = \begin{bmatrix} 0 & \ldots & 0 & h(M) & \ldots & h(1) \\ & & & & \ddots & \vdots \\ & \ddots & & \ddots & & h(M) \\ & & \ddots & & \ddots & \\ & & & \ddots & & 0 \\ & & & & \ddots & \vdots \\ & & & & & 0 \end{bmatrix} \tag{5}$$

and $\boldsymbol{v}(k)$ is the additive white Gaussian noise (AWGN) vector.

2 Blind Subspace Algorithm

The subspace algorithm for blind channel estimation requires that the transmission equation of the system has a tall and full column rank channel matrix. The typical application of the blind subspace algorithm is in the systems with a multichannel model, e. g. multi-antenna receiving or oversampling to the received signal in the receiver[7, 8]. But OFDM systems do not satisfy the multichannel model, and from Eq. (3), due to the interference from the preceding OFDM symbol $\boldsymbol{H}_1 \boldsymbol{s}(k-1)$, the transmission equation of OFDM systems in Eq. (3) does not exactly have the desired structure to directly derive a subspace algorithm. So, in this section, we first use the redundancy introduced by the CP to address this problem, then a subspace algorithm for blind channel estimation is developed.

Partition $\boldsymbol{r}(k)$, $\boldsymbol{s}(k)$, and $\boldsymbol{v}(k)$ in Eq. (3) into three parts and denote each part by a vector:

$$\left.\begin{aligned} \boldsymbol{r}(k) &= [\boldsymbol{r}_0^{\mathrm{T}}(k) \quad \boldsymbol{r}_1^{\mathrm{T}}(k) \quad \boldsymbol{r}_2^{\mathrm{T}}(k)]^{\mathrm{T}} \\ \boldsymbol{s}(k) &= [\boldsymbol{s}_0^{\mathrm{T}}(k) \quad \boldsymbol{s}_1^{\mathrm{T}}(k) \quad \boldsymbol{s}_2^{\mathrm{T}}(k)]^{\mathrm{T}} \\ \boldsymbol{v}(k) &= [\boldsymbol{v}_0^{\mathrm{T}}(k) \quad \boldsymbol{v}_1^{\mathrm{T}}(k) \quad \boldsymbol{v}_2^{\mathrm{T}}(k)]^{\mathrm{T}} \end{aligned}\right\} \tag{6}$$

where in each of $\boldsymbol{r}(k)$, $\boldsymbol{s}(k)$ and $\boldsymbol{v}(k)$, the first and the third part of the respective vectors contain M components and the second part contains the rest $(N-M)$ components. Since $\boldsymbol{s}_0(k)$ corresponds to the CP, $\boldsymbol{s}_0(k) = \boldsymbol{s}_2(k)$.

If we define

$$\left.\begin{aligned} \bar{\boldsymbol{r}}_1(k) &= [\boldsymbol{r}_1^{\mathrm{T}}(k-1) \quad \boldsymbol{r}_2^{\mathrm{T}}(k-1) \quad \boldsymbol{r}_0^{\mathrm{T}}(k)]^{\mathrm{T}} \\ \bar{\boldsymbol{r}}_2(k) &= [\boldsymbol{r}_0^{\mathrm{T}}(k) \quad \boldsymbol{r}_1^{\mathrm{T}}(k) \quad \boldsymbol{r}_2^{\mathrm{T}}(k)]^{\mathrm{T}} \end{aligned}\right\} \tag{7}$$

then

$$\left.\begin{aligned} \bar{\boldsymbol{r}}_1(k) &= \boldsymbol{H} \begin{bmatrix} \boldsymbol{s}_0(k-1) \\ \boldsymbol{s}_1(k-1) \\ \boldsymbol{s}_2(k-1) \\ \boldsymbol{s}_0(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{v}_1(k-1) \\ \boldsymbol{v}_2(k-1) \\ \boldsymbol{v}_0(k) \end{bmatrix} \\ \bar{\boldsymbol{r}}_2(k) &= \boldsymbol{H} \begin{bmatrix} \boldsymbol{s}_2(k-1) \\ \boldsymbol{s}_0(k) \\ \boldsymbol{s}_1(k) \\ \boldsymbol{s}_2(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{v}_0(k) \\ \boldsymbol{v}_1(k) \\ \boldsymbol{v}_2(k) \end{bmatrix} \end{aligned}\right\} \tag{8}$$

where $\boldsymbol{H}$ is an $(N+M) \times (N+2M)$ matrix.

$$\boldsymbol{H} = \begin{bmatrix} h(M) & \ldots & h(0) & & & & \\ & h(M) & \ldots & h(0) & & & \\ & & \ddots & & \ddots & & \\ & & & h(M) & \ldots & h(0) \end{bmatrix} \tag{9}$$

Now, we let

$$\left.\begin{aligned} \tilde{\boldsymbol{r}}(k) &= \bar{\boldsymbol{r}}_1(k) - \bar{\boldsymbol{r}}_2(k) \\ \tilde{\boldsymbol{s}}(k) &= \begin{bmatrix} \boldsymbol{s}_1(k-1) \\ \boldsymbol{s}_2(k-1) \end{bmatrix} - \begin{bmatrix} \boldsymbol{s}_0(k) \\ \boldsymbol{s}_1(k) \end{bmatrix} \\ \tilde{\boldsymbol{v}}(k) &= \begin{bmatrix} \boldsymbol{v}_1(k-1) \\ \boldsymbol{v}_2(k-1) \\ \boldsymbol{v}_0(k) \end{bmatrix} - \begin{bmatrix} \boldsymbol{v}_0(k) \\ \boldsymbol{v}_1(k) \\ \boldsymbol{v}_2(k) \end{bmatrix} \end{aligned}\right\} \tag{10}$$

From Eqs. (8) and (10) and $\boldsymbol{s}_0(k-1) = \boldsymbol{s}_2(k-1)$, $\boldsymbol{s}_0(k) = \boldsymbol{s}_2(k)$, it is easy to verify that

$$\tilde{\boldsymbol{r}}(k) = \boldsymbol{T} \tilde{\boldsymbol{s}}(k) + \tilde{\boldsymbol{v}}(k) \tag{11}$$

with the $(N+M) \times N$ matrix $\boldsymbol{T}$ given by

$$\boldsymbol{T} = \begin{bmatrix} h(0) & & & \\ \vdots & h(0) & & \\ h(M) & \vdots & \ddots & \\ & h(M) & & h(0) \\ & & \ddots & \vdots \\ & & & h(M) \end{bmatrix} \tag{12}$$

In Eq. (11), $\boldsymbol{T}$ is a Toeplitz matrix, and it is a full column rank matrix under $h(0) \neq 0$, so, based on Eq. (11) a subspace algorithm for channel estimation can be developed[8].

The subspace algorithm relies on the second-order statistics of $\tilde{\boldsymbol{r}}(k)$ to estimate the channel. From Eq. (11), the correlation matrix of $\tilde{\boldsymbol{r}}(k)$ is

$$\boldsymbol{R}_{\tilde{r}} = \boldsymbol{T} \boldsymbol{R}_{\tilde{s}} \boldsymbol{T}^{\mathrm{H}} + \boldsymbol{R}_{\tilde{v}} \tag{13}$$

where $\boldsymbol{R}_{\tilde{s}}$ is the correlation matrix of $\tilde{\boldsymbol{s}}(k)$ and $\boldsymbol{R}_{\tilde{v}}$ is the correlation matrix of $\tilde{\boldsymbol{v}}(k)$. It should be noted that, although $\boldsymbol{v}(k)$ is white noise, the noise term $\tilde{\boldsymbol{v}}(k)$ in Eq. (11) is colored noise. The correlation matrix $\boldsymbol{R}_{\tilde{v}}$ of $\tilde{\boldsymbol{v}}(k)$ is an $(N+M) \times (N+M)$ matrix:

$$\boldsymbol{R}_{\tilde{v}} = \sigma_{\mathrm{v}}^2 \begin{bmatrix} 2\boldsymbol{I}_{M \times M} & \boldsymbol{0}_{M \times (N-M)} & -\boldsymbol{I}_{M \times M} \\ \boldsymbol{0}_{(N-M) \times M} & 2\boldsymbol{I}_{(N-M) \times (N-M)} & \boldsymbol{0}_{(N-M) \times M} \\ -\boldsymbol{I}_{M \times M} & \boldsymbol{0}_{M \times (N-M)} & 2\boldsymbol{I}_{M \times M} \end{bmatrix} \tag{14}$$

where σ_{v}^2 is the variance of the additive white noise $\boldsymbol{v}(k)$, $\boldsymbol{I}$ is an identity matrix, and $\boldsymbol{0}$ is a zero matrix.

So it is necessary to whiten it by multiplying $\tilde{\boldsymbol{r}}(k)$ by a whitening matrix, which is the inverse of the Hermitian square root of $\boldsymbol{R}_{\tilde{v}}$, denoted $\boldsymbol{W}_{\tilde{v}} = \boldsymbol{R}_{\tilde{v}}^{-\frac{1}{2}}$ (see Ref. [8]). The correlation matrix of the whitened signals is

$$\underline{\boldsymbol{R}}_{\tilde{r}} = \boldsymbol{W}_{\tilde{v}} \boldsymbol{R}_{\tilde{r}} \boldsymbol{W}_{\tilde{v}}^{\mathrm{H}} = \boldsymbol{W}_{\tilde{v}} \boldsymbol{T} \boldsymbol{R}_{\tilde{s}} \boldsymbol{T}^{\mathrm{H}} \boldsymbol{W}_{\tilde{v}}^{\mathrm{H}} + \boldsymbol{I}_{(M+N)\times(M+N)} \quad (15)$$

Since the input data symbols are i. i. d., $\boldsymbol{R}_{\tilde{s}}$ has full rank N. Notice that channel matrix $\boldsymbol{T}$ is a full column rank matrix under $h(0) \neq 0$, and $\boldsymbol{W}_{\tilde{v}}$ is a full rank matrix, so the signal subspace of $\underline{\boldsymbol{R}}_{\tilde{r}}$ has a dimension N, and can be spanned by the columns of $\boldsymbol{W}_{\tilde{v}}\boldsymbol{T}$, and the noise subspace of $\underline{\boldsymbol{R}}_{\tilde{r}}$ has a dimension M, and is spanned by the basis $\boldsymbol{g}_0, \boldsymbol{g}_1, \ldots, \boldsymbol{g}_{M-1}$, which are the M eigenvectors associated with the M smallest eigenvalues of $\underline{\boldsymbol{R}}_{\tilde{r}}$. Using the orthogonality between the signal subspace and the noise subspace, we have

$$\underline{\boldsymbol{G}}^{\mathrm{H}} \boldsymbol{W}_{\tilde{v}} \boldsymbol{T} = \boldsymbol{0} \quad (16)$$

where $\underline{\boldsymbol{G}} = [\boldsymbol{g}_0, \boldsymbol{g}_1, \ldots, \boldsymbol{g}_{M-1}]$. By theorem 1 in Ref. [6], due to the Toeplitz structure of $\boldsymbol{T}$, $\boldsymbol{h}$ can be uniquely identified (within a scaling ambiguity) from Eq. (16).

In practical computation, $\boldsymbol{R}_{\tilde{r}}$ is not exactly known in the receiver, and it is estimated by an averaging in time over $\tilde{\boldsymbol{r}}(k)$ that is

$$\hat{\boldsymbol{R}}_{\tilde{r}} = \frac{1}{K} \sum_{k=0}^{K-1} \tilde{\boldsymbol{r}}(k) \tilde{\boldsymbol{r}}^{\mathrm{H}}(k) \quad (17)$$

Hence Eq. (16) has to be solved in the least square sense to estimate $\boldsymbol{h}$, which is

$$\hat{\boldsymbol{h}} = \arg \min_{\|\boldsymbol{h}\|=1} \|\underline{\boldsymbol{G}}^{\mathrm{H}} \boldsymbol{W}_{\tilde{v}} \boldsymbol{T}\|_2^2 \quad (18)$$

where $\hat{\boldsymbol{h}}$ is the estimate of $\boldsymbol{h}$. The method to solve Eq. (18) for $\hat{\boldsymbol{h}}$ can be seen in Ref. [8].

The subspace algorithm developed in this section is based on Eq. (11) and its channel matrix $\boldsymbol{T}$ has a simple form. Compared with the subspace algorithm in Ref. [5], the simple form of $\boldsymbol{T}$ greatly eases the derivation of the subspace algorithm and practical computation for $\hat{\boldsymbol{h}}$. Since, under channel order overdetermination and no matter where the channel zero locations are, $\boldsymbol{T}$ in Eq. (11) is always full column rank under $h(0) \neq 0$ and the subspace algorithm developed in this paper is robust to channel order overdetermination and channel zero locations. However, in the conventional subspace algorithm in Ref. [8], the channel order must be estimated exactly, which is impractical under the additive noise. Although the subspace algorithm in Ref. [5] is also robust to channel order overdetermination, and the algorithm is not sensitive to the channel zero locations in practical simulation, these performances cannot be proved clearly because of the complexity of derivation in Ref. [5]. Moreover the algorithm in Ref. [5] is derived only under $N \geqslant 2M$. The proposed algorithm presents no limitations on the size of OFDM symbols.

3 Simulations

We use Monte Carlo simulations to examine the performances of the proposed algorithm. We use the root-mean-square error (RMSE), which is defined as $\frac{1}{\|\boldsymbol{h}\|}\sqrt{\frac{1}{I(L+1)}\sum_{i=1}^{I}\|\hat{\boldsymbol{h}}^{(i)} - \boldsymbol{h}\|^2}$, and the channel average bias as $\frac{1}{I(L+1)\|\boldsymbol{h}\|}\sum_{l=0}^{L}\left|\sum_{i=1}^{I}[\hat{h}^{(i)}(l) - h(l)]\right|$ to measure the error performance of channel estimation. In RMSE and Bias, I denotes the number of Monte Carlo trials, and $\hat{\boldsymbol{h}}^{(i)}$ is the estimate of the channel in the i-th trial. In the simulations, transmitted data symbols are 4-QAM i. i. d, the simulation channel is $\boldsymbol{h} = \{1, -3.2, -0.1296, 0.4147, -0.792\}$, and Monte Carlo trials $I = 120$. For comparison, the results by the algorithm in Refs. [4, 5], are also displayed, which are denoted as HG and MCD respectively. Fig. 1 illustrates the channel error versus the SNR. In the simulation, $M = 4$, $N = 20$ and the number of the received OFDM symbols used to estimate $\boldsymbol{R}_{\tilde{r}}$ in Eq. (17) is $K = 120$. Fig. 2 illustrates the channel error versus the received OFDM symbols K under SNR = 15 dB and $M = 4$, $N = 20$. Fig. 3 tests the performance of the algorithm under channel overdetermination. From Fig. 1 and Fig. 2, the proposed algorithm offers satisfactory performance as an MCD algorithm, and its estimation errors are much smaller than the HG algorithm. From Fig. 3, the proposed algorithm is robust to the channel overdetermination.

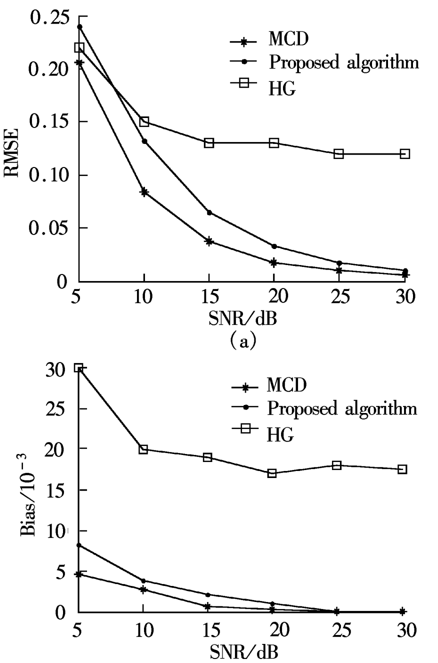

Fig. 1 Channel error versus SNR

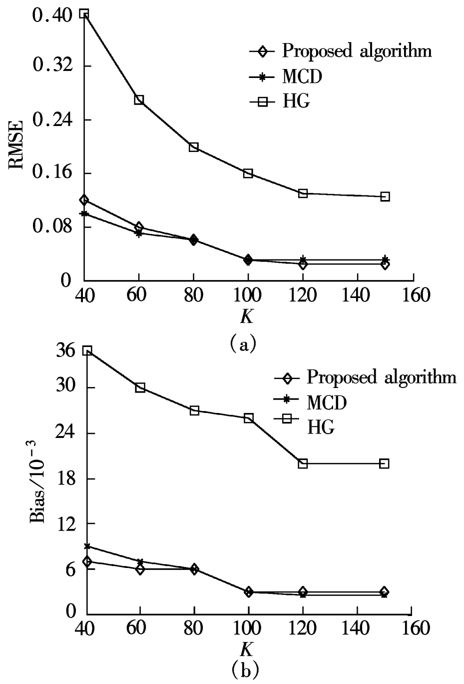

Fig. 2 Channel error versus number K

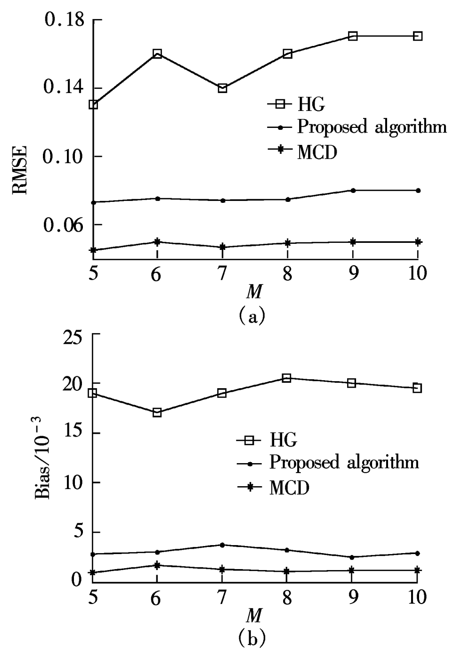

Fig. 3 Channel error under channel overdetermination

4 Conclusion

This paper proposes a new subspace algorithm for blind channel estimation in OFDM systems. Since the algorithm makes use of the inherent redundancy introduced by the CP for channel estimation, it does not need additional changes to the structure of OFDM systems and can be used for traditional OFDM systems directly. Moreover the proposed algorithm has high accuracy for channel estimation, and is not sensitive to channel order overdetermination and the locations of channel zeros. The simulation results show that the algorithm offers satisfactory performance.

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一种新的 OFDM 系统信道盲估计的子空间算法

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摘要: 为提高 OFDM 系统的传输效率, 提出了一种利用接收信号的二阶统计特性实现信道盲估计的子空间算法. 首先将 OFDM 信号分成 3 部分, 利用循环前缀(CP) 引入的信息冗余, 对 OFDM 系统的传输方程作矩阵变换, 得到一个信道矩阵为 Toeplitz 矩阵的新方程. 基于此方程推导出信道估计的子空间算法. Toeplitz 矩阵结构使算法的推导和实际的计算大为简化. 该算法不需要改变 OFDM 系统结构, 不受信道零点位置的限制, 在信道过估计的情况下也适用. 实验结果验证了算法的有效性.

关键词: 正交频分复用; 信道盲估计; 子空间算法

中图分类号: TN911.5