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On $L(2,1)$-labellings of distance graphs

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Abstract: The $L(2,1)$-labelling number of distance graphs $G(D)$, denoted by $\lambda(D)$, is studied. It is shown that distance graphs satisfy $\lambda(G) \leqslant \Delta^2$. Moreover, we prove $\lambda(\{1,2,\ldots,k\}) = 2k+2$ and $\lambda(\{1,3,\ldots,2k-1\}) = 2k+2$ for any fixed positive integer k. Suppose $k, a \in \mathbf{N}$ and $k, a \geqslant 2$. If $k \geqslant a$, then $\lambda(\{a, a+1, \ldots, a+k-1\}) = 2(a+k-1)$. Otherwise, $\lambda(\{a, a+1, \ldots, a+k-1\}) \leqslant \min\{2(a+k-1), 6k-2\}$. When D consists of two positive integers, $6 \leqslant \lambda(D) \leqslant 8$. For the special distance sets $D = \{k, k+1\}(\forall k \in \mathbf{N})$, the upper bound of $\lambda(D)$ is improved to 7.

Key words: channel assignment problem; $L(2,1)$-labelling; distance graphs

The channel assignment problem[1] is to assign a channel(nonnegative integer) to each radio transmitter so that interfering transmitters are assigned channels whose separation is not in a set of disallowed separations. Roberts introduced a variation of this problem, where close transmitters must receive different channels and very close transmitters must receive channels at least two apart. To formulate the problem in graphs, the transmitters are represented by the vertices of a graph: two vertices are "very close" if they are adjacent in the graph and "close" if they are of distance two in the graph. Given a graph G with vertex set V and edge set E, for any $u, v \in V$, let $d_G(u,v)$ denote the distance between u and v in G. An $L(2,1)$-labelling of G is a function $f: V \rightarrow \{0, 1, 2, \ldots\}$ such that $|f(u) - f(v)| \geqslant 2$ if $uv \in E(G)$ and $|f(u) - f(v)| \geqslant 1$ if $d_G(u,v) = 2$. Elements of the image of f are called labels. A k-$L(2,1)$-labelling is an $L(2,1)$-labelling such that no label is greater than k. The $L(2,1)$-labelling number of G, denoted by $\lambda(G)$ (or simply λ), is the smallest number k such that G has a k-$L(2,1)$-labelling.

The $L(2,1)$-labelling problem was first researched by Griggs and Yeh[2]. They found exact values of λ for paths, cycles, wheels, etc. For some other kinds of graphs, they presented the bounds of λ. For general graphs G with the maximum degree Δ, they proved that $\lambda(G) \leqslant \Delta^2 + 2\Delta$, which was later improved to $\Delta^2 + \Delta$ by Chang and Kuo[3]. In Ref. [2], Griggs and Yeh proposed the following conjecture:

Conjecture 1[2] For any graph G with the maximum degree $\Delta \geqslant 2$, $\lambda(G) \leqslant \Delta^2$.

This remains an open problem and scholars have studied $\lambda(G)$ for some special graphs[4-7]. To study the conjecture, we focus on the integer distance graphs. Suppose D is a subset of all positive integers. The integer distance graph(or simply distance graph) $G(\mathbf{Z}, D)$ with distance set D is the graph with vertex set $\mathbf{Z}$($\mathbf{Z}$ is the set of all integers) and two vertices u and v are adjacent if and only if $|u-v| \in D$. For simplicity, we denote $G(\mathbf{Z}, D)$ by $G(D)$ and $\lambda(D)$ denotes the labelling number of $G(D)$. For the case where D is a set of positive integers with $g = \gcd(D)$, each component of $G(D)$ is isomorphic to $G(D')$, where $D' = \{d' : gd' \in D\}$. So when we study $\lambda(D)$, we may assume that $\gcd(D) = 1$.

1 Preliminaries

In this section, we summarize some definitions and known results.

Suppose $D = \{a_1, a_2, \ldots, a_k\}$, $0 < a_1 < a_2 < \ldots < a_k$. Let

$$D^2 = \{2a_i \mid 1 \leqslant i \leqslant k\} \cup \{a_j \pm a_i \mid 1 \leqslant i < j \leqslant k \text{ and } i \neq j\} \backslash D$$

So $|D^2| \leqslant k + 2\binom{k}{2} = k^2 = |D|^2$. From the definition we can assert that $\forall u, v \in \mathbf{Z}$, $|u-v| \in D^2$ if and only if

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$d_{G(D)}(u,v)=2$.

An $L(2,1)$-labelling $f: \mathbf{Z}\to\{0,1,2,\dots\}$ is called periodic with period p if $f(i)=f(i+p)$ for all $i\in\mathbf{Z}$. We denote a p-periodic $L(2,1)$-labelling by f_p.

A labelling $f: \mathbf{Z}\to\{0,1,2,\dots\}$ is called D-consistent if it satisfies

① $|f(u)-f(v)|\geqslant 2$, if $|u-v|\in D$;

② $|f(u)-f(v)|\geqslant 1$, if $|u-v|\in D^2$.

The definition above implies that a labelling f is D-consistent if and only if it is an $L(2,1)$-labelling of $G(D)$.

Suppose i is a label, we define the labelling $i^k=\underbrace{i\dots i}_{k}$ of length k. Let $\mathbf{N}$ be the set of all nonnegative integers.

Lemma 1[8] If D is a finite distance set, then

$$2|D|+2\leqslant\lambda(D)\leqslant|D^2|+3|D|$$

If $|D|=1$, by lemma 1 $\lambda(D)=4$. In this case each component of $G(D)$ is actually a path on infinite vertices. Griggs and Yeh[2] proved that $\lambda(P_n)=4$, for $n\geqslant 5$. Thus when $|D|=1$, lemma 1 is equivalent to their result. In the next section, we shall point out the lower bound is sharp for $|D|\geqslant 2$.

From the definitions above, we obtain the following lemmas.

Lemma 2 If f_p is D-consistent with period p where $D=\{d_1,d_2,\dots,d_n\}$, $p>d_i (i=1,2,\dots,n)$, then f_p is also $\{k_1p\pm d_1,k_2p\pm d_2,\dots,k_np\pm d_n\}$-consistent for all $k_i\in\mathbf{N}(i=1,2,\dots,n)$.

Proof Since f_p is $\{d_1,d_2,\dots,d_n\}$-consistent with period p, for all $v\in\mathbf{Z}$ we have

$$|f_p(v)-f_p(v+d_i)|\geqslant 2,\ |f_p(v)-f_p(v+2d_i)|\geqslant 1,\ |f_p(v)-f_p(v+d_i\pm d_j)|\geqslant 1$$

where $1\leqslant i,j\leqslant n$ and $i\neq j$.

It follows that

$$|f_p(v)-f_p(v+k_ip\pm d_i)|=|f_p(v)-f_p(v\pm d_i)|\geqslant 2$$
$$|f_p(v)-f_p[v+2(k_ip\pm d_i)]|=|f_p(v)-f_p(v\pm 2d_i)|\geqslant 1$$
$$|f_p(v)-f_p[v+(k_ip\pm d_i)+(k_jp\pm d_j)]|=|f_p(v)-f_p[v\pm(d_i\pm d_j)]|\geqslant 1$$
$$|f_p(v)-f_p[v+(k_ip\pm d_i)-(k_jp\pm d_j)]|=|f_p(v)-f_p[v\pm(d_i\pm d_j)]|\geqslant 1$$

This yields that f_p is also $\{k_1p\pm d_1,k_2p\pm d_2,\dots,k_np\pm d_n\}$-consistent ($\forall k_i\in\mathbf{N}$, $i=1,2,\dots,n$).

In the proof of our main results, a conclusion is useful.

Lemma 3[9] Let a and b be two positive integers such that the greatest common divisor of a and b is 1. If t is an integer such that $t>ab-a-b$ then the equation $t=na+mb$ has at least one solution with n and m nonnegative integers.

2 *k*-Element Distance Sets($\forall k\in\mathbf{N}$)

In this section we first prove that general distance graphs satisfy conjecture 1.

Theorem 1 Let $G(D)$ be a distance graph with finite distance set D and the maximum degree Δ. Then $\lambda(D)\leqslant\Delta^2$.

Proof Since $G(D)$ is $2|D|$-regular, $\Delta=2|D|\geqslant 2$. It follows from lemma 1 that $\lambda(D)\leqslant|D^2|+3|D|\leqslant|D|^2+3|D|=\dfrac{\Delta^2}{4}+\dfrac{3}{2}\Delta\leqslant\Delta^2$.

We now turn to some special k-element distance sets D. If D is composed of the first k positive integers or odd integers, $\lambda(D)$ can be determined completely by next the two results:

Theorem 2 If $D=\{1,2,\dots,k\}$, then $\lambda(D)=2k+2$.

Proof By lemma 1, $\lambda(D)\geqslant 2|D|+2=2k+2$. It suffices to show $\lambda(D)\leqslant 2k+2$. Consider the periodic labelling $P_{2k+3}=0,2,4,\dots,2k+2,1,3,5,\dots,2k+1$. It is straightforward to check P_{2k+3} is $\{1,2,\dots,k\}$-consistent. So $\lambda(D)\leqslant 2k+2$.

Theorem 3 If $D=\{1,3,5,\dots,2k-1\}$, $k\geqslant 2$, then $\lambda(D)=2k+2$.

Proof $\lambda(D)\geqslant 2k+2$ follows from lemma 1. On the other hand, we define a labelling $P_{2k+3}=2k+2,k,2k+1,k-1,2k,k-2,\dots,k+5,3,k+4,2,k+3,1,k+2,0,k+1$. This makes a $(2k+2)$-$L(2,1)$-labelling of $G(D)$. So $\lambda(D)\leqslant 2k+2$, which completes our proof.

By the previous two theorems, we see that $G(D)$ when $D=\{1,2,\ldots,k\}$ or $\{1,3,5,\ldots,2k-1\}$ are the examples where the lower bound $2|D|+2$ in lemma 1 is attainable.

If D consists of arbitrary k consecutive integers, we have the following theorem.

Theorem 4 If $D=\{a,a+1,\ldots,a+k-1\}$, $a, k\geqslant 2$, then $\lambda(D)\leqslant\min\{2(a+k-1),6k-2\}$.

Proof First define a periodic labelling $P_{2a+2k-1}=0,1,2,\ldots,2(a+k-1)$. It is straightforward to check $P_{2a+2k-1}$ is D-consistent. So $\lambda(D)\leqslant 2(a+k-1)$. Now we prove $\lambda(D)\leqslant 6k-2$. By lemma 1, $\lambda(D)\leqslant|D^2|+3|D|$. Here $D^2=\{1,2,\ldots,k-1,2a,2a+2,\ldots,2a+2k-2,2a+1,2a+3,2a+2k-3\}$, hence $|D^2|\leqslant 3k-2$. It follows that $\lambda(D)\leqslant 3k-2+3k=6k-2$.

In particular, for the case when $k\geqslant a$ we find the exact value of $\lambda(D)$.

Theorem 5 If $D=\{a,a+1,\ldots,a+k-1\}$, $k\geqslant a\geqslant 2$, then $\lambda(D)=2(a+k-1)$.

Proof Recall the definition of the set D^2 at the beginning of this section. Obviously, for all $u,v\in\mathbf{Z}$, $d_G(u,v)\leqslant 2$ if and only if $|u-v|\in D^2\cup D$. Thus if $|u-v|\in D^2\cup D$, then u and v must have distinct labels. Note that $D=\{a,a+1,\ldots,a+k-1\}$ and $k\geqslant a$, so $D^2\cup D=\{1,2,\ldots,a-1,a,a+1,\ldots,a+k-1,a+k,a+k+1,\ldots,2(a+k-1)\}$. For all distinct $u,v\in S=\{1,2,\ldots,2(a+k-1),2(a+k-1)+1\}$, we have $1\leqslant|u-v|\leqslant 2(a+k-1)$, i. e., $|u-v|\in D^2\cup D$. This implies the labels of u and v must be distinct. Since S has $2(a+k-1)+1$ numbers and 0 can be used as a label, $\lambda(D)\geqslant 2(a+k-1)$.

On the other hand, the periodic labelling $P_{2a+2k-1}=0,1,2,\ldots,2(a+k-1)$ is $\{a,a+1,\ldots,a+k-1\}$-consistent. Hence $\lambda(D)\leqslant 2(a+k-1)$. The proof is complete.

3 Two-Element Distance Sets

In this section, we concentrate on the distance sets $D=\{a,b\}$ with $a<b$. From lemma 1, $6\leqslant\lambda(D)\leqslant 10$. The lower bound has been shown to be sharp in section 2. Therefore, we try to improve the upper bound 10.

Theorem 6 If $D=\{a,b\}$ and $a\neq 1$, then $6\leqslant\lambda(D)\leqslant 8$.

Proof The lower bound 6 is immediate from lemma 1. To show the upper bound, we construct a proper 8-$L(2,1)$-labelling for $G(D)$ in all cases.

Case 1 $a<b<2a$. The following periodic 8-$L(2,1)$-labellings are $\{a,b\}$-consistent.

① $\frac{3}{2}a\leqslant b<2a$

$$P_{3a+b}=0^{b-a}1^{2a-b}2^{b-a}3^{2a-b}4^{b-a}5^{2a-b}6^{b-a}7^{2a-b}8^{b-a}$$

② $\frac{4}{3}a\leqslant b<\frac{3}{2}a$

$$P_{3b-a}=0^{b-a}1^{b-a}2^{3a-2b}3^{b-a}4^{b-a}5^{b-a}6^{3a-2b}7^{b-a}8^{b-a}$$

③ $\frac{2n+1}{2n}a\leqslant b<\frac{2n}{2n-1}a$, $n\geqslant 2$, $n\in\mathbf{N}$

$$P_{(2n+1)b-(2n-1)a}=\underbrace{0^{b-a}1^{b-a}\ldots 0^{b-a}1^{2na-(2n-1)b}}_{n\text{ pairs of “01”}}2^{b-a}3^{b-a}4^{b-a}\underbrace{5^{b-a}4^{b-a}\ldots 5^{b-a}4^{2na-(2n-1)b}}_{n-1\text{ pairs of “54”}}6^{b-a}\underbrace{8^{b-a}7^{b-a}\ldots 8^{b-a}7^{b-a}}_{n-1\text{ pairs of “87”}}8^{b-a}$$

④ $\frac{2n+2}{2n+1}a\leqslant b<\frac{2n+1}{2n}a$, $n\geqslant 2$, $n\in\mathbf{N}$

$$P_{(2n+1)b-(2n-1)a}=\underbrace{0^{b-a}1^{b-a}\ldots 0^{b-a}1^{b-a}}_{n\text{ pairs of “01”}}2^{(2n+1)a-2nb}3^{b-a}\underbrace{4^{b-a}5^{b-a}\ldots 4^{b-a}5^{b-a}}_{n\text{ pairs of “45”}}6^{(2n+1)a-2nb}7^{b-a}\underbrace{8^{b-a}7^{b-a}\ldots 8^{b-a}7^{b-a}}_{n-1\text{ pairs of “87”}}8^{b-a}$$

Case 2 $2a<b<3a$. It is straightforward to check $P_{9a}=0^a2^a4^a6^a8^a1^a3^a5^a7^a$ is $\{a,b\}$-consistent.

Case 3 $3a<b<4a$. We construct a proper $L(2,1)$-labelling $P_{9a}=0^a5^a1^a6^a2^a7^a3^a8^a4^a$.

Case 4 $b>4a$. Without loss of generality, we may assume $ma<b<(m+1)a$, $m\geqslant 4$, $m\in\mathbf{N}$. The following 8-$L(2,1)$-labellings are D-consistent.

① $m=3k+1$, $k\geqslant 1$, $k\in\mathbf{N}$

$$P_{3b}=3^a7^a\underbrace{0^a4^a2^a\ldots 0^a4^a2^{b-ma}}_{k\text{ terms of “042”}}6^a1^a\underbrace{3^a7^a5^a\ldots 3^a7^a5^{b-ma}}_{k\text{ terms of “375”}}0^a4^a\underbrace{6^a1^a8^a\ldots 6^a1^a8^{b-ma}}_{k\text{ terms of “618”}}$$

② $m=3k+2$, $k\geqslant 1$, $k\in\mathbf{N}$

$$P_{3b}=3^a7^a2^a\underbrace{0^a4^a2^a\ldots 0^a4^a2^{b-ma}}_{k\text{ terms of “042”}}6^a1^a5^a\underbrace{3^a7^a5^a\ldots 3^a7^a5^{b-ma}}_{k\text{ terms of “375”}}0^a4^a8^a\underbrace{6^a1^a8^a\ldots 6^a1^a8^{b-ma}}_{k\text{ terms of “618”}}$$

③ $m=3k+3$, $k\geqslant 1$, $k\in\mathbf{N}$

$$P_{3b}=3^a5^a7^a2^a\underbrace{0^a4^a2^a\ldots0^a4^a2^{b-ma}}_{k\text{ terms of ``042''}}6^a8^a1^a5^a\underbrace{3^a7^a5^a\ldots3^a7^a5^{b-ma}}_{k\text{ terms of ``375''}}0^a2^a4^a8^a\underbrace{6^a1^a8^a\ldots6^a1^a8^{b-ma}}_{k\text{ terms of ``618''}}$$

From all the cases above, we can conclude $\lambda(D)\leqslant 8$ when $D=\{a,b\}$.

Theorem 7 If $D=\{1,b\}$, then

$$\lambda(D)\leqslant\begin{cases}8 & \text{if } b\equiv 0(\bmod 3)\\ 7 & \text{otherwise}\end{cases}$$

Proof First we consider the case when $b=2,3,4,5,6$.

① $b=2,5$. We set $P_7=0246135$ which is obviously $\{1,b\}$-consistent. Again from lemma 1 this implies $\lambda(D)=6$.

② $b=3,4$. It is easy to see $P_7=0362514$ is $\{1,b\}$-consistent. So $\lambda(D)=6$.

③ $b=6$. $P_8=02461357$ is $\{1,6\}$-consistent, which implies $\lambda(D)\leqslant 7$.

In the following proof we suppose $b>6$.

Case 1 $b=3k+1$, $k\geqslant 2$, $k\in\mathbf{N}$.

We set $P_{6k}=\underbrace{046\ldots046}_{k+1\text{ terms of ``046''}}\underbrace{137\ldots137}_{k\text{ terms of ``137''}}$, which is a proper $L(2,1)$-labelling of $G(\{1,b\})$. This yields $\lambda(D)\leqslant 7$.

Case 2 $b=3k+2$, $k\geqslant 2$, $k\in\mathbf{N}$.

We take a periodic labelling $P_{6(k+1)}=\underbrace{046\ldots046}_{k+1\text{ terms of ``046''}}\underbrace{137\ldots137}_{k+1\text{ terms of ``137''}}$. It is straightforward to verify $P_{6(k+1)}$ is $\{1,b\}$-consistent. Also $\lambda(D)\leqslant 7$.

Case 3 $b=3k$, $k\geqslant 3$, $k\in\mathbf{N}$.

We set $P_{9k}=\underbrace{135\ldots135}_{k\text{ terms of ``135''}}\underbrace{702\ldots702}_{k\text{ terms of ``702''}}\underbrace{468\ldots468}_{k\text{ terms of ``468''}}$, which is obviously $\{1,b\}$-consistent. Thus $\lambda(D)\leqslant 8$.

Combining theorems 6 and 7, we have the following theorem.

Theorem 8 If $|D|=2$, then $\lambda(D)\leqslant 8$.

The remainder of the paper provides the upper bound of λ for $D=\{k,k+1\}$.

Lemma 4 If $D=\{k,k+1\}$, k is an even integer. Then $\lambda(D)\leqslant 7$.

Proof For the case when $k=2$, $P_7=0123456$ is $\{2,3\}$-consistent. This implies $\lambda(D)=6<7$.

Suppose $k>2$. We define a periodic labelling $P_{3k+1}=\underbrace{0101\ldots01}_{\frac{k}{2}\text{ pairs of ``01''}}\;2\;\underbrace{3434\ldots34}_{\frac{k}{2}\text{ pairs of ``34''}}\;5\;\underbrace{6767\ldots67}_{\frac{k}{2}-1\text{ pairs of ``67''}}\;6$, which is $\{k,k+1\}$-consistent.

Theorem 9 If $D=\{k,k+1\}$, $k\geqslant 1$. Then $\lambda(D)\leqslant 7$.

Proof From lemma 4, the theorem holds when k is even. We now construct 7-$L(2,1)$-labellings when k is odd.

We set $P_7=0123456$, $P_8=01234567$. It can be verified that P_7, P_8, P_7P_8, P_8P_7 are all $\{2,3\}$-consistent. Since $\gcd(7,8)=1$, the equation $t=7n+8m$ has a nonnegative solution (n,m) for all $t>7\times 8-7-8=41$ by lemma 3. For such a solution (n,m) we define a periodic 7-$L(2,1)$-labelling of the form $P_t=P_7^nP_8^m=\underbrace{P_7\ldots P_7}_{n}\underbrace{P_8\ldots P_8}_{m}$. It is straightforward to check P_t is $\{2,3\}$-consistent. By lemma 2, P_t is also $\{t-3,t-2\}$-consistent. Setting $k=t-3$, $P_t=P_{k+3}$ is $\{k,k+1\}$-consistent for all $k>41-3=38$.

It suffices to construct a 7-$L(2,1)$-labelling of $G(D)$ when $k\leqslant 38$ and k is odd.

(a) $k=1,5,15,19,29,33$; $P_7=0246135$.

(b) $k=9,11,23,25,37$; $P_7=0123456$.

(c) $k=3$, $P_{50}=76753401024767243501035767342010565732317064540212$.

(d) $k=7$, $P_{22}=7676754510123703434012$.

(e) $k=13,21$; $P_8=01234567$.

(f) $k=17$, $P_8=02461357$.

(g) $k=27$, $P_{30}=P_7{}^2P_8{}^2$, where $P_7=0123456$ and $P_8=01234567$. It is easy to check P_{30} is $\{2,3\}$-consistent. By lemma 2, P_{30} is also $\{30-3,30-2\}$-consistent, i. e., P_{30} is $\{27,28\}$-consistent.

(h) $k=31$, $P_{29}=P_7^3P_8^1$, where P_7, P_8 are the same as in (g). It is $\{31,32\}$-consistent by lemma 2.

(i) $k=35$, $P_{38}=P_7^2P_8^3$, where P_7, P_8 are the same as in (g). We can use the same arguments as above.

From all the cases above, $\lambda(\{k, k+1\}) \leqslant 7$.

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关于距离图的 $L(2,1)$-标号着色

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摘要: 研究了距离图 $G(D)$ 的 $L(2,1)$-标号色数 $\lambda(D)$. 证明了距离图满足 $\lambda(G) \leqslant \Delta^2$. 对于任意给定的正整数 k, 证明了 $\lambda(\{1, 2, \ldots, k\}) = 2k+2$ 和 $\lambda(\{1, 3 \ldots, 2k-1\}) = 2k+2$. 假设 $k, a \in \mathbf{N}$ 且 $k, a \geqslant 2$. 如果 $k \geqslant a$, 则 $\lambda(\{a, a+1, \ldots, a+k-1\}) = 2(a+k-1)$. 否则, $\lambda(\{a, a+1, \ldots, a+k-1\}) \leqslant \min\{2(a+k-1), 6k-2\}$. 若 D 由 2 个正整数构成, 则 $6 \leqslant \lambda(D) \leqslant 8$. 对于特殊的距离集 $D = \{k, k+1\}(\forall k \in \mathbf{N})$, $\lambda(D)$ 的上界改进到了 7.

关键词: 频道分配问题; $L(2,1)$-标号着色; 距离图

中图分类号: O157. 5