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Reliability analysis for anchorage of reinforced concrete beams with longitudinal cracks due to corrosion at anchorage zone

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Abstract: An anchorage reliability analysis approach for simply supported reinforced concrete beams under corrosion attack in the anchorage zone is developed. The first-order second-moment method is employed to analyze the effects of various factors on the anchorage reliability. These factors include both the length and width of cover cracking due to reinforcement corrosion, the cover thickness, the anchorage length, and the stirrup ratio. The results show that the effect of corrosion-induced crack length on the reliability index for anchorage, β_0, is negligible when the crack on the concrete surface is just appearing, but with the crack widening, the β_0 value is reduced significantly; the considerable changes in β_0 result from a variation in cover depth and anchorage length; the effect of changes in the diameter or space of stirrups on the anchorage resistance is very limited, and the variation in β_0 is also very low.

Key words: reinforcement corrosion; bond behavior; anchorage; reinforced concrete structure; corrosion-induced crack

The structural performances of reinforced concrete beams, such as the load bearing capacity, the deflection at the midspan, and the ductility, are based on sufficient bond stress between concrete and reinforcement and on the adequate end anchorage of bars at supports. However, steel corrosion in concrete is induced by environmental attacks, e. g., chloride contamination and carbonation of cover. Furthermore, cover cracking along the longitudinal bars is caused by increasing corrosion level. Thereby, reduction in serviceability and load carrying capacity is produced by insufficient bond stress[1,2]. In addition, the higher corrosion level which causes the cover cracking in anchorage zones at beam ends can lead to failure in anchorage for simply supported beams[2,3]. It is explained that, the corrosion products change the physical characteristics of bar/concrete interface, i. e. the friction between the bar and the concrete is reduced by a weak layer of oxides at the interface between the two, and the formation of cracks caused by corrosion reduces its confinement in the concrete. Thus, the anchorage resistance is significantly reduced.

1 Limit State for Anchorage

Two failure states for anchorage may be caused by the tensile force of steel bars anchored in the concrete[4], namely: ① Failure state for anchorage strength, i. e. the bond stress exceeds the ultimate bond strength; ② Failure state for anchorage rigidity, i. e. the local slip or its increase rate is excessive.

For certain reinforcement and concrete, the yield force of a bar is constant, whereas the anchorage resistance caused by the bond stress between the concrete and the bar increases with increasing anchorage length, l_a, as shown in Fig. 1[5]. This figure shows that, when the anchorage length is equal to a certain critical value of l_a^{cr}, both the failure in anchorage and the yield state of reinforcement will occur at the same time. Accordingly, the so-called limit state for anchorage is that the ultimate bond stress between rebar and concrete reaches the permissible value when the tension stress of anchored bar in concrete is the maximum.

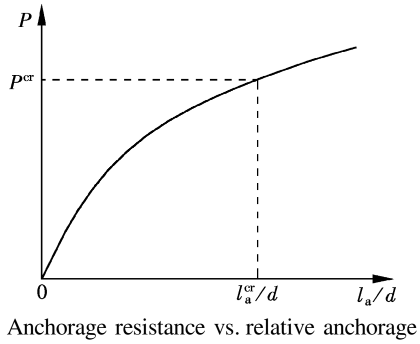

Fig. 1 Anchorage resistance vs. relative anchorage length

2 Limit State Equation of Anchorage

To simplify the analytic process, the assumptions

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of uniform corrosion for the reinforcement bar in the anchorage zone and of ignoring variation in both diameter and yield strength of the bar due to the corrosion are taken.

As shown in Fig. 2, if a longitudinal crack at the anchorage zone occurs due to steel corrosion, then the anchorage resistance of the bar can divide into two parts, i. e. the component of that in the non-cracking region and that in the cracking region. Let τ_{u1} and τ_{u2} represent the ultimate bond strength for the non-cracking and cracking region, respectively. Moreover, the τ_{u2} value may be expressed as a product of the τ_{u1} value and a reduction coefficient of the ultimate bond strength[6,7]. Thus, the components of anchorage resistance can be expressed as

$$P_1 = \pi d(1-\alpha) l_a \tau_{u1} \tag{1}$$

$$P_2 = \pi d\alpha l_a \tau_{u2} = \pi d\alpha l_a \eta_b \tau_{u1} \tag{2}$$

where P_1 and P_2 are the components of anchorage resistance in non-cracking and cracking regions, respectively; d is the bar diameter; l_a is the anchorage length; α is the ratio of longitudinal crack length to anchorage length; η_b is the reduction coefficient of bond strength.

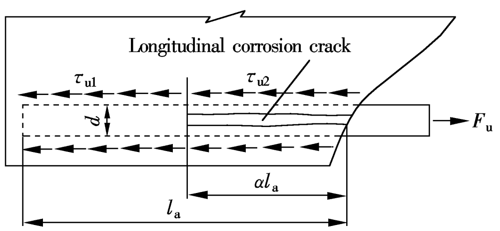

Fig. 2 Scheme of longitudinal cover cracking due to corrosion in anchorage zone at supports for simple concrete beam

From Eqs. (1) and (2) the total anchorage resistance of the bar can be expressed as

$$P = P_1 + P_2 = [1-\alpha(1-\eta_b)]\pi d l_a \tau_{u1} \tag{3}$$

In terms of the equilibrium condition of the limit state for anchorage, the ultimate tension applied to the steel bar, F_u, is equal to the anchorage resistance P. However, the F_u value is also expressed as[4]

$$F_u = \eta \frac{\pi d^2}{4} f_s \tag{4}$$

where f_s is the tensile strength of a steel bar in MPa; η is the abundance ratio of reinforcement stress. For both the end anchorage at supports and the anchorage at midspan for simple beam the η value is equal to 0.617, or else to 1.0.

From Eqs. (3) and (4),

$$\frac{4[1-\alpha(1-\eta_b)] l_a \tau_{u1}}{d} - \eta f_s = 0 \tag{5}$$

It is obvious that Eq. (5) is the limit state equation for anchorage for the simple beam with a corrosion-induced cover cracking in the anchorage zone at supports. If it is assumed that

$$R = \frac{4[1-\alpha(1-\eta_b)] l_a \tau_{u1}}{d} \tag{6}$$

$$S = \eta f_s \tag{7}$$

then Eq. (5) becomes the general limit state equation as follows:

$$Z = R - S = 0 \tag{8}$$

where Z is the performance function for structural members, R is the generalized resistance, and S is the generalized load effect.

3 Reliability Analysis for Anchorage

3.1 Statistic parameters of random variables in limit state equation

For sound, simply supported beams, the ultimate bond stress in the anchorage zone at supports can be calculated by[8]

$$\tau_{u1} = \left(0.82 + 0.9\frac{d}{l_a}\right)\left(1.6 + 0.7\frac{c}{d} + 20\rho_{sv}\right) f_t \tag{9}$$

where c is the cover depth in mm, ρ_{sv} is the stirrup ratio, and f_t is the tensile strength of concrete in MPa.

Therefore, from Eqs. (6) and (9), the generalized resistance, R, can be calculated by

$$\begin{aligned} R &= \Omega_P R_P = \Omega_P \frac{4[1-\alpha(1-\eta_b)] l_a}{d} \cdot \\ &\quad \left(0.82 + 0.9\frac{d}{l_a}\right)\left(1.6 + 0.7\frac{c}{d} + 20\rho_{sv}\right) f_t = \\ &\quad 4[1-\alpha(1-\eta_b)]\Omega_P \left(0.9 + 0.82\frac{l_a}{d}\right) \cdot \\ &\quad \left[1.6 + 0.7\frac{c}{d} + 15.7\frac{(d_{sv}/d)^2}{(c/d)(s_{sv}/d)}\right] f_t \end{aligned} \tag{10}$$

Furthermore, the performance function for anchorage can be expressed as

$$\begin{aligned} Z &= 4[1-\alpha(1-\eta_b)]\Omega_P \left(0.9 + 0.82\frac{l_a}{d}\right) \cdot \\ &\quad \left[1.6 + 0.7\frac{c}{d} + 15.7\frac{(d_{sv}/d)^2}{(c/d)(s_{sv}/d)}\right] f_t - \eta f_s \end{aligned} \tag{11}$$

where Ω_P is the random variable for uncertainty associated with resistance modeling, R_P is the calculated resistance by Eq. (6), d_{sv} is the stirrup diameter, and s_{sv} is the spacing between stirrups.

Hypothesis testing indicates that, for the random variables in Eq. (11), i. e. Ω_P, f_t, l_a, c, d, d_{sv}, s_{sv}, η, and f_s, the distributional assumption does not require a rejection of the normal distribution[4,9]. Thereby, the statistical parameters of these stochastic variables are given in Tab. 1 to Tab. 3[4,9].

Tab. 1 Statistical parameters of geometric variables

Variable	Ω_P	η	Spacing of stirrups① s_{sv}/s_{svk}	Cover thickness c/c_k	Bar diameter② d/d_k, d_{sv}/d_{svk}	Anchorage length l_a/l_{ak}	
						Deformed bar	Plain bar
Mean value	1.065	0.617	1.00	1.017 8	1.00	1.025	1.007
Coefficient of variation	0.229	0.076	0.06	0.0496	0.024 7	0.077	0.128

Note: ① The symbols with and without the subscript "k" represent the design value and the measured value, respectively.
② The statistical parameters of bar diameter are calculated from cross section area of bar.

Tab. 2 Statistical parameters for f_t value in members

Grade of concrete	Mean value/MPa	Coefficient of variation
C15	1.69	0.239
C20	2.05	0.227
C30	2.51	0.189
C40	2.93	0.151

Tab. 3 Statistical parameters for f_s value in members

Steel grade	Mean value/MPa	Coefficient of variation
Ⅰ	259.71	0.121 1
Ⅱ	368.87	0.071 9
Ⅲ	413.16	0.064 5

3.2 Target reliability index for anchorage

The limit state for anchorage is deemed to occur if both the bar stress in the anchorage zone at supports and the bond stress reach the acceptable maximum values together. Thereby, the occurrence probability of the limit state for anchorage, p_f, can be expressed as

$$p_f = P(\sigma_s = \eta f_s, \tau = \tau_u) = P(\sigma_s = \eta f_s) P(\tau = \tau_u \mid \sigma_s = \eta f_s) \tag{12}$$

In view of the importance of adequate anchorage for concrete structural members, the acceptable level of reliability for anchorage is limit states should be greater than that for other limit states(e.g. serviceability or ultimate limit states)[4].

According to Ref. [9], for bridge elements with secondary safety classes, the target reliability index for ductile fracture is equal to 4.2 and for brittle failure 4.7. In order to ensure the reliability index for anchorage is adequately greater than that for other limit states, it is suggested that the total reliability index for anchorage, β, and the corresponding failure probability, p_f, have been given by

$$\beta = 4.95, \quad p_f = 3.71 \times 10^{-7} \tag{13}$$

For a steel bar anchored in concrete at supports, the occurrence probability that the bar stress will reach the maximum acceptable value (i.e. $\sigma_s = \eta f_s$), and the corresponding reliability index can be given according to Ref. [9] as follows:

$$\beta_1 = 4.2, \quad p_{f1} = 1.34 \times 10^{-5} \tag{14}$$

Under the condition of the event "the bar stress in the anchorage zone at supports reached the acceptable value (i.e. $\sigma_s = \eta f_s$)", the occurrence probability of the event "the bond stress also reached the acceptable value (i.e. $\tau = \tau_u$)" is

$$p_{f0} = P(\tau = \tau_u \mid \sigma_s = \eta f_s) = \frac{p_f}{p_{f1}} = 2.77 \times 10^{-2} \tag{15}$$

Further, the corresponding reliability index is $\beta_0 = 1.92$.

Thus, from the analysis mentioned above it can be seen that, for concrete bridge members with secondary safety classes, in order to ensure that the total reliability index for anchorage is not less than 4.95 under the condition that the reliability index for bearing capacity of normal section has value 4.2, the target reliability index, β_0^*, which is as high as 1.92 determined by Eq. (11), should be considered.

3.3 Reliability analysis for anchorage

The basic variables in Eq. (11), such as Ω_P, f_t, l_a, c, d, d_{sv}, s_{sv}, η, and f_s, can be expressed as parameters X_i ($i = 1, 2, \ldots, 9$). In addition, these variables are assumed to be statistically independent and have a normal distribution. In terms of design point approach, the equation set is given as follows:

$$\left.\begin{aligned} x_i^* &= \mu_{X_i} + \beta_0 \sigma_{X_i} \cos\theta_{X_i} \qquad i = 1, 2, \ldots, 9 \\ \cos\theta_{X_i} &= \frac{-\left.\dfrac{\partial g}{\partial X_i}\right|_{P^*} \sigma_{X_i}}{\left[\sum_{i=1}^{9}\left(\left.\dfrac{\partial g}{\partial X_i}\right|_{P^*} \sigma_{X_i}\right)^2\right]^{\frac{1}{2}}} \\ g(x_1^*, x_2^*, \ldots, x_9^*) &= 0 \end{aligned}\right\} \tag{16}$$

where μ_{X_i} and σ_{X_i} are the mean value and the standard deviation for basic variables, respectively; x_i^* is the original coordinate value of the design point P^*; $g(\cdot)$ is the performance function which is expressed as Eq. (11); $\left.\dfrac{\partial g}{\partial X_i}\right|_{P^*}$ is the value of the first-order partial derivative of function $g(\cdot)$ with respect to X_i at design point P^*.

The statistical parameters for the variables in Eq. (11) have been given in Tab. 1 to Tab. 3. Further, the β_0 value can be calculated by Eq. (16) with statistical parameters listed in Tab. 1 to Tab. 3.

4 Results and Discussion

From Eq. (10) it can be seen that, the generalized resistance can change as the length and width of longitudinal corrosion crack, the anchorage length, the cover thickness, and the stirrup ratio vary. Thereby, the β_0 value will also be influenced by these variables. For illustrative purpose, a reinforced concrete beam with a concrete of grade C30 and a steel bar of grade Ⅱ is considered.

4.1 Effect of both length and width of longitudinal crack

Once the cracking of cover concrete due to steel corrosion along the longitudinal reinforcement has occurred, the reduction coefficient of ultimate bond stress for structural members can be expressed as[7]

$$\eta_b = 0.9495 e^{-1.093w} \tag{17}$$

where w is the width of longitudinal crack in mm.

For anchorage reliability analysis, the adverse conditions are considered. Namely, it is assumed that $c/d = 1$, $l_a/d = 30$, $d_{sv}/d = 0.25$, $s_{sv}/d = 15$.

The variation in β_0 with both length and width of longitudinal crack is shown in Fig. 3. When the cover concrete cracks just due to corrosion ($w = 0.05$ mm), the reduction of β_0 with the increasing crack length is negligible. Thereafter, the β_0 value is significantly reduced by the increase in length or width of the corrosion crack. For example, if the crack width increases from 0.3 to 0.6, 0.8, and 1.0 mm, then β_0 reduces by 11%, 18%, and 23% for $\alpha = 0.5$, and by 20%, 32%, and 44% for $\alpha = 0.7$, respectively.

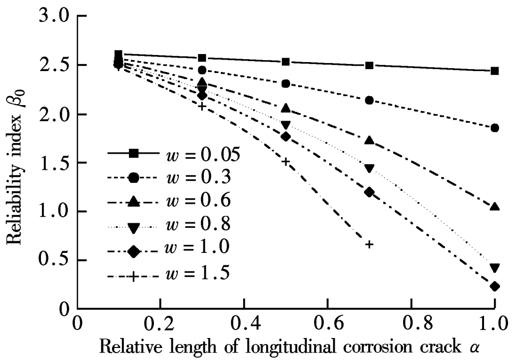

Fig. 3 Effect of both length and width of corrosion cracking on the β_0 value

Moreover, the effect of crack width on the β_0 value is insignificant if the length of corrosion crack is shorter (e. g. $\alpha \leqslant 0.3$). In addition, the reduction in the β_0 value is also comparatively lower for $w \leqslant 0.3$ mm. However, the β_0 value is already below 1.92 if $w \geqslant 0.8$ mm and $\alpha > 0.5$.

4.2 Effect of cover depth

The test results show that the loss in bond strength of specimens is increasingly reduced as a relative cover thickness (i. e. cover-to-bar diameter ratio, c/d) is increased[10]. This implies that the bond strength, especially the residual bond strength, is significantly influenced by the cover depth. Further, the β_0 value will increase with increasing cover depth. Fig. 4 shows the changes in the β_0 value when $w = 1.0$ mm and c/d ratio has a value in the range 1.0 to 3.0. It can be seen that, the increment of 0.5 for c/d ratio leads to increase in ultimate bond stresses about 10% to 14%, and the β_0 value correspondingly increases by 0.2 to 0.3 approximately.

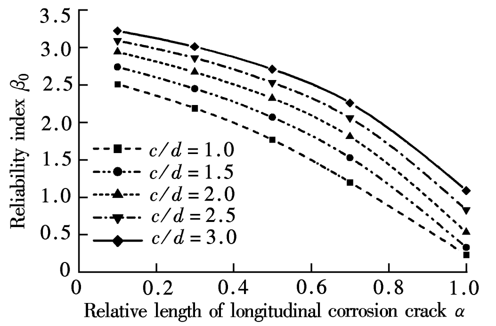

Fig. 4 Effect of cover depth on the β_0 value

4.3 Effect of anchorage length

From Fig. 1 it can be seen that the anchorage resistance increases by increasing relative anchorage length (l_a/d). Therefore, the β_0 value also increases with the value of l_a/d ratio. It is assumed that $w = 1.0$ mm, the l_a/d ratio ranges from 25 to 40, and other conditions are unchanging. A variation of β_0 is shown in Fig. 5. It is indicated that the β_0 value is significantly influenced by the l_a/d ratio. For a smaller l_a/d ratio, β_0 may be less than the target reliability index of 1.92 even though the corrosion crack length is still shorter (e. g. $\alpha \leqslant 0.3$). Hence, a severe restraint of development of both length and width of the corrosion cracking in the anchorage zone, beyond doubt, is required for a structural member with a short anchorage length.

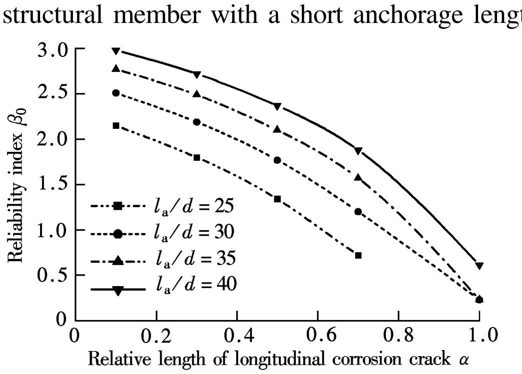

Fig. 5 Effect of anchorage length on the β_0 value

4.4 Effect of stirrup ratio

The stirrups can provide an effective confinement to prevent excessive cracking in concrete elements. For a reinforced concrete member with stirrups, bond failure results from the crushing of concrete keys adjacent to the rib lugs without splitting of cover. Experimental results show that bond strength for specimens with hoop reinforcement is linearly increased by the stirrup ratio[6]. Apparently, the stirrup ratio will increase with the reduction of stirrup spacing or the increase in the diameter of the stirrups. Fig. 6 and Fig. 7 indicate the variation of β_0 by changing the stirrup diameter (d_{sv}/d) and the stirrup spacing (s_{sv}/d). It can be seen that, the increment of β_0 resulting from either increasing stirrup diameter or reducing stirrup space is very small, as the influence of changes in value of d_{sv}/d ratio or s_{sv}/d ratio on the anchorage resistance is very limited. For instance, the resistance increased by 1.5% as the value of d_{sv}/d was enhanced from 0.25 to 0.3, and by 4.5% as the value of d_{sv}/d increased from 0.25 to 0.4. In addition, the increment of 1.5% for resistance results from the reduction in s_{sv}/d value from 15 to 10. Furthermore, the resistance was increased by 2.5% as the value of s_{sv}/d was reduced from 15 to 8. An explanation for this is that the ultimate bond strength of members with stirrups depends on the resistance of crushing of concrete keys. Although the presence of links enhances the split resistance for concrete cover, its effect on the resistance of crushing of concrete keys is insignificant.

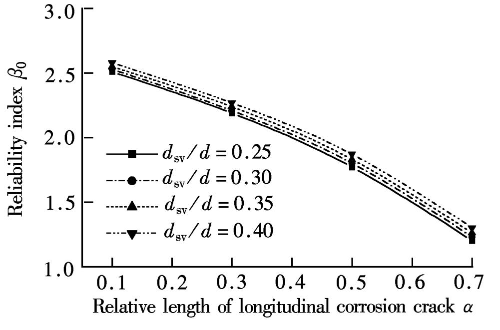

Fig. 6 Effect of stirrups diameter on the β_0 value

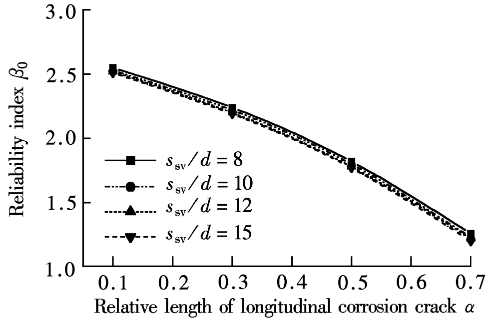

Fig. 7 Effect of stirrups spacing on the β_0 value

5 Conclusion

Corrosion-induced cover cracking in the anchorage zone at supports can result in loss of the effective anchorage length; furthermore, the failure probability in anchorage for members is also enhanced. In order to ensure the structural safety, the total reliability index for anchorage should be greater than that for ultimate or serviceability limit states.

The analysis results show that the effect of corrosion crack length on the reliability index for anchorage, β_0, is negligible when the crack on the concrete surface is just appearing, but with the crack widening, the β_0 value is reduced significantly; the considerable changes in β_0 result from a variation in cover depth and anchorage length; the effect of changes in diameter or space of stirrups on the anchorage resistance is very limited, and the variation in β_0 is also very low.

References

[1] Cabrera J G. Deterioration of concrete due to reinforcement steel corrosion [J]. *Cement & Concrete Composites*, 1996, **18**(1): 47 - 59.

[2] Yuan Yingshu, Jia Fuping, Cai Yue. The structural behavior deterioration model for corroded reinforced concrete beams [J]. *China Civil Engineering Journal*, 2001, **34**(3): 47 - 52. (in Chinese)

[3] Fan Yingfang, Zhou Jing, Huang Zhenguo. Experimental research on strength of R. C. member corroded by chloride [J]. *Industrial Construction*, 2001, **31**(5): 3 - 5. (in Chinese)

[4] Shao Zhuomin, Shen Wendu, Xu Youlin. The reliability and design for anchorage of reinforcing bars [J]. *Journal of Building Structures*, 1987, **8**(4): 36 - 49. (in Chinese)

[5] Xu Youlin, Shen Wendu, Wang Hong. An experimental study of bond-anchorage properties of bars in concrete [J]. *Journal of Building Structures*, 1994, **15**(3): 26 - 37. (in Chinese)

[6] Wang Linke, Tao Feng, Wang Qinglin, et al. Experimental study on bond and anchorage of corroded reinforcement in concrete [J]. *Industrial Construction*, 1996, **26**(4): 14 - 16. (in Chinese)

[7] Zhang Weiping, Zhang Yu. Test study on deterioration model of bond behavior between corroded reinforcement and concrete [J]. *Building Structure*, 2002, **32**(1): 31 - 33. (in Chinese)

[8] Shao Zhuomin, Xu Youlin. The study and design proposition for bond-anchorage of reinforced concrete [J]. *Building Structure*, 1986, **16**(4): 2 - 12. (in Chinese)

[9] China Highway Planning and Design Institute. GB/T 50283—1999 Unified standard for reliability design of highway engineering structures [S]. Beijing: China Planning Press, 1999. (in Chinese)

[10] Al-Sulaimani G J, Kaleemullah M, Basunbul L A, et al. In-

fluence of corrosion and cracking on bond behavior and strength of reinforced concrete members [J]. *ACI Structural Journal*, 1990, **87**(2): 220 - 231.

钢筋混凝土简支梁锚固区锈胀开裂后锚固可靠度分析

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摘要:为了研究锚固区混凝土锈胀开裂后构件的锚固可靠度的变化情况,采用一次二阶矩方法分析了锈胀裂缝的长度及宽度、保护层厚度、钢筋锚固长度以及配箍率等参数对混凝土简支梁锚固可靠度的影响. 分析结果表明:当保护层刚开裂时,锈胀裂缝长度对锚固可靠指标 β_0 的影响很小,此后,随着裂缝宽度的增加,β_0 有了明显的降低;保护层厚度和锚固长度的变化对 β_0 有相当大的影响;改变箍筋的直径或间距对锚固抗力的影响很小,因此,β_0 变化的幅度也很小.

关键词:钢筋锈蚀;粘结性能;锚固;钢筋混凝土结构;锈胀裂缝

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