Journal of Southeast University (English Edition) Vol. 21, No. 3, pp. 348 – 352 Sept. 2005 ISSN 1003—7985

Modeling of rock failure based on physical cellular automata

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Abstract: To analyze the effects of heterogeneous material characteristics on rock failure, a micro-heterogeneous physical cellular automata (Mh-PCA) model is introduced according to the cellular automata theory from a general power view. In this model, the neighbor is the Moore pattern and the Weibull distribution is adopted to simulate the rock heterogeneousness. Using this model, the evolvements and acoustic emission of rock failure are simulated for four materials of different degree of homogeneousness ($m = 1, 5, 10, 15$). The results show that the heterogeneous characteristic has a great effect on the rock failure, the more the homogeneousness, the fewer the crack branches and the more concentrated acoustic emissions. The physical cellular automata theory gives a new idea for studying rock failure.

Key words: rock failure; acoustic emission; heterogeneous characteristic; physical cellular automata; Weibull distribution

The mechanism of rock failure is recognized as one of the most important aspects of rock behavior. However, the solution to this problem is still not conclusive due to the complexity nature of the rock micro-structure. Recently, more researchers have identified that nonlinearity is the primary character of rock failure[1], and that the micro-heterogeneous of rock structure is one of the main difficulties[2].

Cellular automata (CA) was first introduced by Von Neumann[3], the world famous mathematician, in the 1950s, being applied to develop models to simulate the evolvement of the self-organization phenomenon in some discrete systems resulting from the nonlinear interaction between cells. Both time and space in CA are discrete. With the fast development of computers, the CA method has been used in many fields. In the study of rock medium related earthquake problems, some researchers have constructed CA models[4 – 8] to study the characteristics of earthquakes. However, these models are mostly based on the spring-block theory, which cannot take into account some special structure characteristics of rock medium, such as homogeneousness, anisotropy etc. Recently, Tan Yunliang and Zhou Hui et al.[9 – 11] have proposed a new model known as the physical cellular automata (PCA) model. Heterogeneousness and anisotropy can be considered in their models and the chaos during the evolvement of rock failure can be studied. But in the PCA model, the neighbor cells of the four-cell neighbor model are few, and the homogeneousness of rock medium described in the PCA model is too rough. It means that the study of CA in the rock field has just started. In this paper, the authors attempt to model and analyze the evolvement of rock failure by using an improved PCA model, the micro-heterogeneous physical cellular automata (Mh-PCA) model.

1 Mh-PCA Model

The Mh-PCA model is a two-dimensional model, where the cell form is a foursquare lattice, and the neighbor model is an eight-cell neighbor as in the Moore model. The mesh is given in Fig. 1.

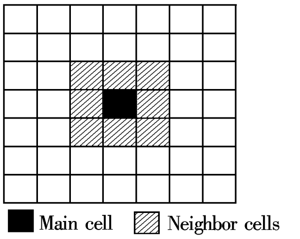

Fig. 1 Moore neighbor model

In the Mh-PCA model, the object being studied will be dispersed into $N \times N$ cells labeled as (x_i, y_j) $(i, j = 1, 2, \ldots, N)$ on 2-D space. Each cell has two statuses, intact or destructive, which are marked with 1 or 0 in the simulation program. The variable $E(x_i, y_j, t)$ is the generalized energy of cell (x_i, y_j) at step t. The generalized energy is equal to the cell's elastic strain energy in the system. Therefore, the initial energy $E(x_i, y_j, 0)$ is equal to the cell's initial stress and the ultimate energy $E_{\max}(x_i, y_j, t)$ will replace the utmost intensity of each cell according to a certain probability distribution. Finally, we use $N(t)$ to account for the

Received 2005-01-14.

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number of destructed cells at each step. Because we use the energy to replace the stress, in each step, the generalized energy input into cells in a certain way can take the place of the increased stress. To dispose the destructive cells, firstly, the energy $E(x_i, y_j, t)$ in each cell at step t will be computed and compared with $E_{max}(x_i, y_j, t)$ as given in Eq. (1).

$$E(x_i, y_j, t) \geqslant E_{max}(x_i, y_j, t) \tag{1}$$

If Eq. (1) is satisfied, the cell begins to fail and release its energy. The energy of the destruct cell will be set to zero, that is $E(x_i, y_j, t) = 0$. The energy released from the cell will then be transferred to its neighboring cells, and the energy in each cell will be redistributed accordingly. If the neighboring cells' energy also meets Eq. (1), it can be dealt with in the same way as above. The repetition will continue until the energy of each cell is less than its maximal energy. If you put energy into the system continually, a new computation will take place.

2 Descriptions of Material Characters

In the Mh-PCA model, two main factors, homogeneousness and anisotropy that have significant influence on the fracture pattern, are taken into account.

2.1 Homogeneousness of material

The homogeneousness of rock material is simply explained by the fact that at different cell locations, the mechanical parameters, such as elastic modulus, Poisson's ratio, ultimate intensity and so on, also differ. In this model, we mainly consider the difference in ultimate intensity, namely, the difference in $E_{max}(x_i, y_j, t)$. As suggested by Tang[2], the ultimate intensity of each cell follows the Weibull distribution given as

$$\varphi(u) = \frac{m}{u_0}\left(\frac{u}{u_0}\right)^{m-1} e^{-\left(\frac{u}{u_0}\right)^m} \tag{2}$$

where u is the intensity of material, $\varphi(u)$ is the probabilistic density of the cell's intensity, u_0 is the mean of intensity, and m is the degree of homogeneousness. In Eq. (2), the larger the m, the higher the degree of homogeneousness is. The curves with different m are given in Fig. 2.

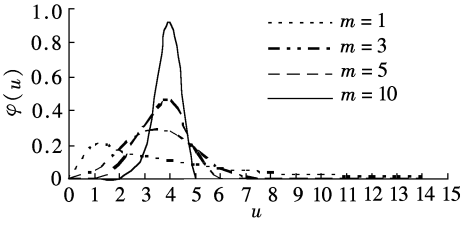

Fig. 2 Curves of Weibull distribution

In the Mh-PCA model, the simulation of homogeneousness of rock material is carried out by evaluating the ultimate intensity $E_{max}(x_i, y_j, t)$ for each cell in terms of the Weibull distribution after the lattices are set. Fig. 3 shows the simulation result, with m equal to 1 and 3 respectively. From Fig. 3, we can see that the smaller the m, the rougher the surface is. Fig. 4 gives the statistic of cellular strength.

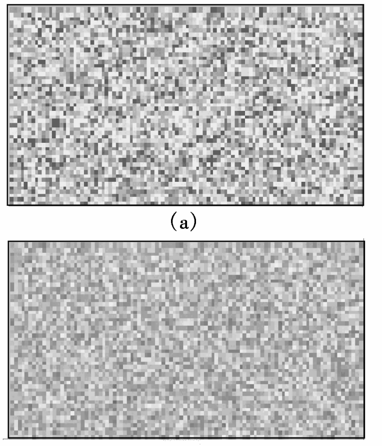

Fig. 3 Simulation of material with different m. (a) $m = 1$; (b) $m = 3$

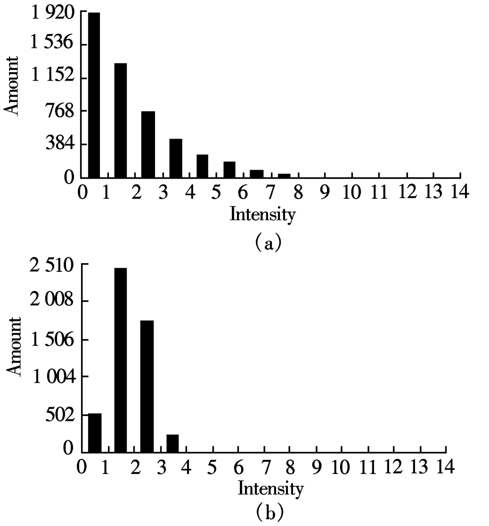

Fig. 4 Statistic of cellular intensity (the number of cell 100×50). (a) $m = 1$; (b) $m = 3$

2.2 Anisotropy description

Compared with other materials such as metal etc., the rock material is obviously anisotropic. According to the Mh-PCA theory, when one cell fails, its energy will be transferred to its eight neighbors. The energy that each neighbor cell takes from the failed cell follows Eq. (3).

$$E(j, t+1) = E(j, t) + w(j, i) \cdot \Delta E(i, t) \tag{3}$$

where $E(j, t+1)$ is the energy that the neighbor cell j can take in at step $t+1$, $E(j, t)$ is the energy that the neighbor cell j can take in at step t, $\Delta E(i, t)$ is the energy that the failed cell i releases at step t, and $w(j, i)$ is the contact coefficient between the destroyed cell i

and the neighbor cells $j(j=1, 2, \ldots, 8)$, $\sum_{j=1}^{8} w(j, i) = 1$. If the value of $w(j, i)$ $(j = 1, 2, \ldots, 8)$ is not changed, the material is considered to be isotropic; otherwise, the material is anisotropic. As a result, the use of $w(j, i)$ allows a more universal model.

3 Loading Rules

The loading rules in this model follow Eq. (4).

$$E(x_i, y_j, t) = E(x_i, y_j, 0) + \varepsilon(x_i, y_j) t \qquad (4)$$

where $E(x_i, y_j, 0)$ is the initial energy of cell(x_i, y_j), $\varepsilon(x_i, y_j)$ is the speed of energy accumulation, and t is the step of the system evolvement. Different load patterns can be obtained by choosing different forms of $\varepsilon(x_i, y_j)$.

4 Simulation of Rock Failure

4.1 Simulation conditions

In the simulation, the number of cells is 50×50, the value of u_0 in Eq. (2) is 4, and m is 1, 5, 10 and 15. In addition, $\varepsilon(x_i, y_j)$ is set to the constant 0.05. It is assumed that the initial press of cells follows even distribution in the [0, 1] zone, and the energy from the failed cell is transferred to its neighboring cells equiprobably.

4.2 Results and analyses

According to the rules and the simulation conditions given above, the simulation results are given in Fig. 5. The results shown in Fig. 5 have indicated the following: ① In the case of a high degree of homoge-

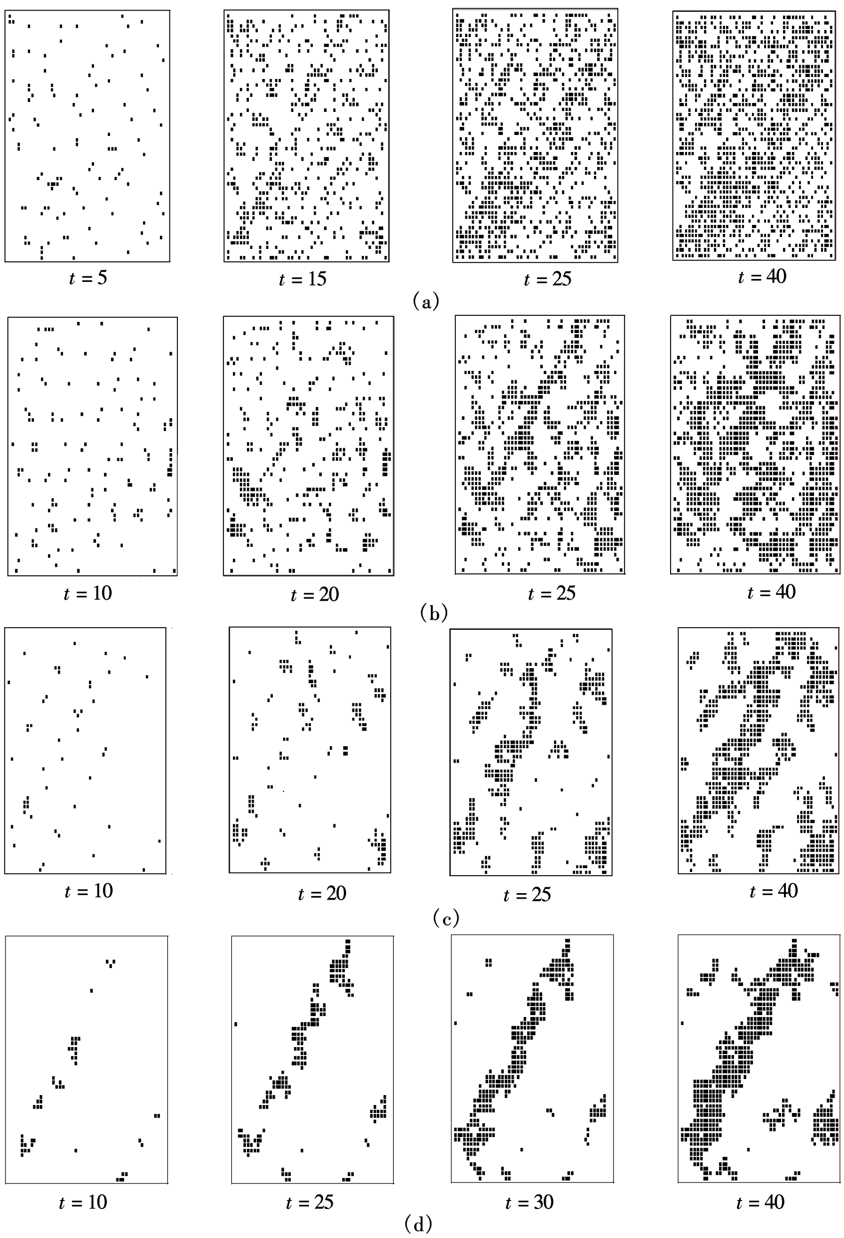

Fig. 5 Fracture simulations of different m. (a) $m=1$; (b) $m=5$; (c) $m=10$; (d) $m=15$

neousness, the failed cells are few before the main fracture forms, and the branches of the main fracture are also few, the distribution of which is more concentrated; however in the case of a lower degree of homogeneousness, the number of the failed cells is greater before the main fracture forms, and their distribution is more random and not concentrated. ② The progress of the rock fracture clearly shows that there are a few failed cells at the start, and then some microcracks converge. ③ The macrocrack comes into being. These results are consistent with the discovery in Ref. [12].

4.3 Simulation of acoustic emission

In the Mh-PCA model, if the cell is in failure, it will release its energy, and we call this an acoustic emission (AE). Based on this theory, the character of the AE character is also modeled during the rock failure and the results are shown in Fig. 6.

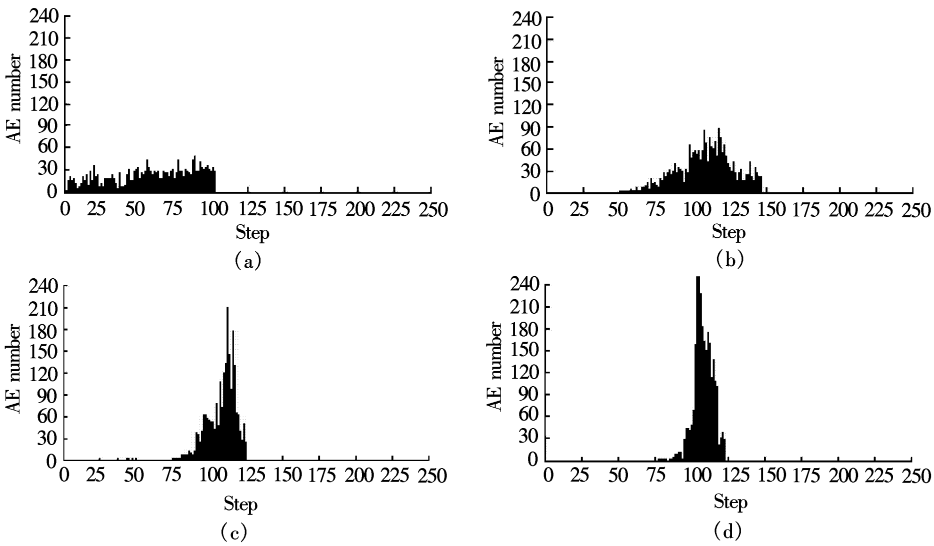

Fig. 6 AE simulations. (a) $m=1$ (b) $m=5$; (c) $m=10$; (d) $m=15$

From Fig. 6 it can also be found that a larger m will result in more concentrated acoustic emission. The reason is that the mode of cell failure is concentrated. These results are consistent with the failure patterns.

5 Conclusion

In this paper, a micro-heterogeneous physical cellular automata model is illustrated based on energy theory. Rock failure and acoustic emissions for materials of various degree of homogeneousness have been simulated. Although the study is preliminary, the results have validated the known theory that the homogeneousness has significant influence on the rock fracture.

References

[1] Zheng Y R, Liu X H. The problems of neoteric nonlinear science and rock mechanics [J]. *Chinese Journal of Geotechnical Engineering*, 1996, **18**(1): 98 − 100. (in Chinese)

[2] Tang Chun'an. The effect of micro heterogeneous during the rock failure[J]. *Chinese Journal of Geotechnical Engineering*, 2000, **22**(6): 706 − 710. (in Chinese)

[3] Von Neumman. *Theory of self-reproducting automa* [M]. Illinois: University of Illinois Press, 1996. 157.

[4] Nakanishi Hiizu. Cellular-automata of earthquakes with deterministic dynamics [J]. *Physical Review A*, 1990, **41** (12): 7086 − 7089.

[5] Liu Jie. Simplified dynamic model based on the characteristics of continental seismicity—cell automata model[J]. *Earthquake*, 1999, **19**(3): 230 − 238. (in Chinese)

[6] Liu Guiping. A cellular automaton model of seismicity [J]. *Northwestern Seismological Journal*, 1995, **17**(2): 20 − 25. (in Chinese)

[7] Li Youming. Application of cellular automata approach to the study of seismic wave propagation [J]. *Acta Phys Sin*, 1995, **38**(5): 651 − 661. (in Chinese)

[8] Hu J X. A study of the cellular automata modeling for P wave propagation in complex media[J]. *Acta Phys Sin*, 1997, **40**(1): 120 − 126. (in Chinese)

[9] Tan Yunliang, Zhou Hui. Physical cellular automaton theory for simulating the failure progress of micro-heterogeneous material [J]. *Acta Phys Sin*, 2001, **50**(4): 704 − 709. (in Chinese)

[10] Zhou Hui, Wang Yongjia, Tan Yunliang, et al. Study on physical cellular automata model of rock mass failure (Ⅰ)—basic model [J]. *Chinese Journal of Rock Mechanics and Engineering*, 2002, **21**(4): 475 − 478. (in Chinese)

[11] Zhou Hui, Tan Yunliang, Feng Xiating, et al. Study on physical cellular automata model of rock mass failure (Ⅱ)—examples[J]. *Chinese Journal of Rock Mechanics and Engineering*, 2002, **21**(6): 782 − 786. (in Chinese)

[12] Sun J Z, Zhou J. The factors affect the acoustic emission of rock [J]. *Lithosphere Deformation and Earthquake*, 1997, **17**(2): 2 − 5. (in Chinese)

基于物理元胞自动机的岩石破坏模拟

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摘要:为了分析岩石材料的非均质性对其破坏演化的影响,根据元胞自动机理论,从能量的角度建立了一种能够从细观层次上对岩石破坏演化进行模拟的物理元胞自动机模型(Mh-PCA 模型),模型引用的 Weibull 随机分布函数对材料的非均质性进行描述. 运用该模型,对 m 分别为 1,5,10,15 四种不同均质度材料的破坏模式及其破坏过程中的声发射现象进行了模拟分析. 结果表明:材料的非均质性对其破坏有重要的影响,均质度越高,破坏过程中的分支裂纹越少,声发射也越集中. 物理元胞自动机理论为岩石的破坏研究提供了一种新的研究思路.

关键词:岩石破坏;声发射;非均质性;物理元胞自动机;Weibull 随机分布

中图分类号:TU452