Journal of Southeast University (English Edition) Vol. 21, No. 4, pp. 379 – 384 Dec. 2005 ISSN 1003—7985

Lower bound on BER performance for maximal ratio combining with weighting errors

Sheng Bin You Xiaohu

(National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China)

Abstract: The theoretical lower bounds on mean squared channel estimation errors for typical fading channels are presented by the infinite-length and non-causal Wiener filter and the exact closed-form expressions of the lower bounds for different channel Doppler spectra are derived. Based on the obtained lower bounds on mean squared channel estimation errors, the limits on bit error rate (BER) for maximal ratio combining (MRC) with Gaussian distributed weighting errors on independent and identically distributed (i. i. d) fading channels are presented. Numerical results show that the BER performances of ideal MRC are the lower bounds on the BER performances of non-ideal MRC and deteriorate as the maximum Doppler frequency increases or the SNR of channel estimate decreases.

Key words: lower bound; bit error rate; minimum mean-square error; channel estimation; maximal ratio combining

Of all diversity linear-combining schemes, maximal ratio combining is a theoretically optimal combiner since it provides the highest average output signal-to-noise ratio (SNR) and the lowest probability of deep fades. Often previous works assume that the weighting of the diversity branches is perfectly done by using perfect knowledge of the branch signal-to-noise ratios. Few results regarding the effects of imperfect weighting on the performance of maximal ratio combining (MRC) are available.

In Ref. [1], Gans models the channel estimation errors in the practical MRC as being complex Gaussian and the probability density function (PDF) of the output SNR is found based on this assumption. The effects of weighting errors on bit error rates (BERs) are not investigated. In Refs. [2 – 4], BERs are given for an arbitrary modulation format used MRC when an ad hoc estimator based on pilot signal is used to find the channel weights. However, the lower bounds on the mean squared channel estimation errors are still to be solved.

In this paper, we present the lower bounds on the mean squared channel estimation errors by using the infinite-length and non-causal Wiener filter and derive the exact closed-form expressions of the lower bounds for the typical Doppler power spectra introduced by the mobile channels. The lower bounds on BER for maximal ratio combining with Gaussian distributed weighting errors on independent and identically distributed (i. i. d) fading channels are then obtained according to the analytical expressions that have been derived.

1 System Model

Consider the operation of predetection L-branch MRC on independent and identically distributed fading channels. The received signals are assumed to be corrupted by independent additive white Gaussian noises (AWGNs) with power spectral density $N_0/2$. Thus, the combiner input from the i-th channel, $i=1, 2, \ldots, L$ is

$$z_i(n) = \alpha_i(n) u_i(n) + w_i(n) \tag{1}$$

where $\alpha_i(n)$ is the complex Gaussian channel gain multiplying the transmitted information signal $u_i(n)$, $w_i(n)$ is the noise, and n is the discrete time index. The ideal MRC output is given by[5, 6]

$$z_0 = \sum_{i=1}^{L} \frac{\alpha_i^*(n)}{N_i} z_i(n) \tag{2}$$

where $\alpha_i^*(n)$ represents the complex conjugate of $\alpha_i(n)$, and N_i is the mean square noise power on the i-th channel.

In a practical combiner, the combiner weights $\alpha_i^*(n)/N_i$ cannot be determined perfectly. In this paper, it is assumed that a pilot channel is transmitted with the data signal for combining purposes and a complex Gaussian error will result in the weighting factors $\hat{\alpha}_i^*(n)$, which are estimates of $\alpha_i^*(n)$ derived from the pilot signal.

Received 2005-03-01.

Foundation item: The National Natural Science Foundation of China (No. 60496311).

Biographies: Sheng Bin (1974—), male, doctor, sbdtt@seu.edu.cn; You Xiaohu (1962—), male, doctor, professor, xhyu@seu.edu.cn.

2 Lower Bound on Mean Squared Channel Estimation Error

2.1 Minimum mean-square error (MMSE) of channel estimate

The received channel gain information $X(n)$ of each path can be written as

$$X(n) = \alpha(n) + v(n) \tag{3}$$

where $\alpha(n)$ is an arbitrary one-path channel parameter to be considered later, which is hereby modeled as a wide-sense stationary discrete-time complex Gaussian random process, and $v(n)$ is the zero-mean additive complex white Gaussian noise.

An estimate of $\alpha(n)$ based on Eq. (3) can be obtained by passing $X(n)$ through an infinite-length, non-causal filter, say $\overset{\infty}{h}(k)$, resulting in

$$\hat{\alpha}(n) = \sum_{k=-\infty}^{\infty} X(n-k)\overset{\infty}{h}(k)^{*} \tag{4}$$

The optimal filter minimizing $E[\,|\alpha(n) - \hat{\alpha}(n)|^2]$ is known as the ideal Wiener filter, and its frequency response obeys

$$\overset{\infty}{H}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})}{S_{XX}(\mathrm{e}^{\mathrm{j}\omega})} = \frac{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})}{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega}) + S_{vv}(\mathrm{e}^{\mathrm{j}\omega})} \tag{5}$$

where $S_{XX}(\mathrm{e}^{\mathrm{j}\omega})$, $S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})$ and $S_{vv}(\mathrm{e}^{\mathrm{j}\omega})$ are the power spectra of the process $X(n)$, $\alpha(n)$ and $v(n)$, respectively; and $\{\alpha(n)\}$ and $\{v(n)\}$ are assumed to be uncorrelated. Furthermore, the channel estimate $\hat{\alpha}(n)$ and the estimation error $e(n)$ are stationary processes with their power spectra shown as (see appendix A)

$$S_{\hat{\alpha}}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{S_{\alpha\alpha}^2(\mathrm{e}^{\mathrm{j}\omega})}{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega}) + S_{vv}(\mathrm{e}^{\mathrm{j}\omega})} \tag{6}$$

and

$$S_{e}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})S_{vv}(\mathrm{e}^{\mathrm{j}\omega})}{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega}) + S_{vv}(\mathrm{e}^{\mathrm{j}\omega})} \tag{7}$$

From the above, the minimum mean-square error of the channel estimate is obtained as

$$E[\,|e(n)|^2] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})S_{vv}(\mathrm{e}^{\mathrm{j}\omega})}{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega}) + S_{vv}(\mathrm{e}^{\mathrm{j}\omega})}\mathrm{d}\omega \tag{8}$$

The ideal Wiener filter provides a lower bound on the attainable error. This can be verified as follows: when we consider a length-N filter, say $h^N(k)$, which is commonly used for practical applications, the cost function can be expressed as

$$J_{\min}^{N} = \sigma^2 - \boldsymbol{P}_N^{\mathrm{H}}\boldsymbol{R}_N^{-1}\boldsymbol{P}_N \tag{9}$$

where $\boldsymbol{R}_N = E[\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{H}}(n)]$, $\boldsymbol{P}_N = E[\boldsymbol{X}(n)\alpha^{*}(n)]$ and σ^2 is the channel average power. $\boldsymbol{X}(n) = \{X(n-N), \ldots, X(n+N)\}^{\mathrm{T}}$ for odd N, and $\boldsymbol{X}(n) = \{X(n-N+1), \ldots, X(n+N)\}^{\mathrm{T}}$ for even N.

When the filter tap becomes $N+1$, the decrement of the cost function can be given by

$$J_{\min}^{N} - J_{\min}^{N+1} = \boldsymbol{P}_{N+1}^{\mathrm{H}}\boldsymbol{R}_{N+1}^{-1}\boldsymbol{P}_{N+1} - \boldsymbol{P}_N^{\mathrm{H}}\boldsymbol{R}_N^{-1}\boldsymbol{P}_N \tag{10}$$

After some mathematical manipulations, Eq. (10) can be rewritten as (see appendix B)

$$\left.\begin{aligned} J_{\min}^{N} - J_{\min}^{N+1} &= \frac{(r_{(N+1)/2} - \boldsymbol{w}_{\mathrm{opt}}^{\mathrm{H}}\tilde{\boldsymbol{P}})^2}{\sigma^2 - \tilde{\boldsymbol{P}}^{\mathrm{T}}\boldsymbol{R}_N^{-1}\tilde{\boldsymbol{P}}} \qquad \text{for odd } N \\ J_{\min}^{N} - J_{\min}^{N+1} &= \frac{(r_{N/2} - \boldsymbol{w}_{\mathrm{opt}}^{\mathrm{H}}\tilde{\boldsymbol{P}})^2}{\sigma^2 - \tilde{\boldsymbol{P}}^{\mathrm{T}}\boldsymbol{R}_N^{-1}\tilde{\boldsymbol{P}}} \qquad \text{for even } N \end{aligned}\right\} \tag{11}$$

where $\tilde{\boldsymbol{P}} = \{r_N, r_{N-1}, \ldots, r_1\}^{\mathrm{T}}$, $\boldsymbol{w}_{\mathrm{opt}} = \boldsymbol{R}_N^{-1}\boldsymbol{P}_N^{\mathrm{H}}$, and $r_i = E[X(n)X^{*}(n-i)]$ denotes the auto-correlation function of the received channel gain information. According to the Wiener filter theorem[7], the decrement of the cost function must be larger than or equal to 0. This result clearly indicates that the MMSE for a finite-length filter takes $J_{\min}^{\infty}$ as its lower limit.

2.2 Closed-form expressions for different Doppler spectra

In this section we will derive the exact closed-form expressions of the lower bounds for different channel Doppler spectra.

When the mobile channel has a flat Doppler power spectrum with cutoff frequency at $\pm f_{\mathrm{d}}$, where f_{d} denotes the maximum Doppler frequency, the frequency response of the ideal Wiener filter has the following expression:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \frac{\sigma^2}{T_{\mathrm{S}}(\sigma^2 + 2f_{\mathrm{d}}\sigma_{\mathrm{n}}^2)} \tag{12}$$

where σ_n^2 is the power spectrum density of noise, σ^2 is the average power of the fading channel, and T_S represents the sampling period of channel parameter. The lower bound on the normalized mean squared channel estimation error can be derived as

$$\frac{J_{\min}(h(k))}{\sigma^2} = \frac{1}{2\pi}\int_{-\omega_d}^{\omega_d} \frac{\sigma_n^2}{T_S(\sigma^2 + 2f_d\sigma_n^2)}d\omega = \frac{2f_dT_S}{\zeta_{SNR} + 2f_dT_S} \tag{13}$$

where signal-to-noise ratio $\zeta_{SNR} = \sigma^2 T_S/\sigma_n^2$.

When the mobile channel has the classical U-shaped Doppler power spectrum[8] characteristic of isotropic scattering with cutoff frequency at $\pm f_d$, the frequency response of the ideal Wiener filter has the following expression:

$$H(e^{j\omega}) = \frac{2\sigma^2}{2\sigma^2 T_S + \sigma_n^2\sqrt{\omega_d^2 - \omega^2}} \tag{14}$$

where the digital frequency $\omega_d = 2\pi f_d T_S$. The lower bound on normalized mean squared channel estimation error can be derived as

$$\frac{J_{\min}(h(k))}{\sigma^2} = \frac{1}{2\pi}\int_{-\omega_d}^{\omega_d} \frac{2\sigma_n^2}{2\sigma^2 T_S + \sigma_n^2\sqrt{\omega_d^2 - \omega^2}}d\omega = \frac{\pi\sqrt{4\zeta_{SNR}^2 - \omega_d^2} - 8\zeta_{SNR}\arctan\left(\frac{2\zeta_{SNR} - \omega_d}{\sqrt{4\zeta_{SNR}^2 - \omega_d^2}}\right)}{\pi\sqrt{4\zeta_{SNR}^2 - \omega_d^2}} \tag{15}$$

In some situations, it is appropriate to model the propagation environment as consisting of a strong specular component plus a scatter component. In this case, the frequency response might have the form as

$$H(e^{j\omega}) = \frac{2\sigma^2 T_S^2 + K\sigma^2\sqrt{\omega_d^2 - \omega^2}\,\delta(\omega - \omega_d\cos\theta_0)}{2\sigma^2 T_S^3 + T_S K\sigma^2\sqrt{\omega_d^2 - \omega^2}\,\delta(\omega - \omega_d\cos\theta_0) + T_S^2(K+1)\sigma_n^2\sqrt{\omega_d^2 - \omega^2}} \tag{16}$$

where θ_0 is the angle of the arrival of the specular component, and K is the ratio of the received specular to the scattered power. The lower bound can be expressed approximately as

$$E[e^2(n)] = \frac{1}{2\pi}\int_{-\omega_d}^{\omega_d} H(e^{j\omega})\sigma_n^2\,d\omega = \lim_{a\to\omega_d\cos\theta_0-}\frac{1}{2\pi}\int_{-\omega_d}^{a}\frac{2\sigma^2\sigma_n^2}{2\sigma^2T_S + \sigma_n^2(K+1)\sqrt{\omega_d^2-\omega^2}}d\omega +$$

$$\lim_{b\to\omega_d\cos\theta_0+}\frac{1}{2\pi}\int_{b}^{\omega_d}\frac{2\sigma^2\sigma_n^2}{2\sigma^2T_S + \sigma_n^2(K+1)\sqrt{\omega_d^2-\omega^2}}d\omega + \lim_{\substack{c\to\omega_d\cos\theta_0-\\ d\to\omega_d\cos\theta_0+}}\frac{1}{2\pi}\int_{c}^{d}\frac{K\sigma^2\sigma_n^2\sqrt{\omega_d^2-\omega^2}\,\delta(\omega-\omega_d\cos\theta_0)}{K\sigma^2T_S\sqrt{\omega_d^2-\omega^2}\,\delta(\omega-\omega_d\cos\theta_0)}d\omega \approx$$

$$\frac{\pi\sqrt{4\zeta_{SNR}^2 - (K+1)^2\omega_d^2} - 8\zeta_{SNR}\arctan\left(\frac{2\zeta_{SNR} - (K+1)\omega_d}{\sqrt{4\zeta_{SNR}^2 - (K+1)^2\omega_d^2}}\right)}{\pi(K+1)\sqrt{4\zeta_{SNR}^2 - (K+1)^2\omega_d^2}} \tag{17}$$

3 Lower Bound on BER of Diversity System

The PDF of the output SNR γ of a maximal ratio (MR) combiner with Gaussian distributed weighting errors has been derived by Gans[1], and is given by

$$p(\gamma) = \frac{(1-\rho^2)^{L-1}\exp(-\gamma/\Gamma)}{\Gamma}\sum_{n=0}^{L-1}\binom{L-1}{n}\left[\frac{\rho^2\gamma}{(1-\rho^2)\Gamma}\right]^n\frac{1}{n!} \tag{18}$$

where

$$\Gamma = E[\xi] \tag{19}$$

the instantaneous SNR per branch is given by ξ, and ρ is the correlation between the actual complex channel gains $\alpha(k)$ and their estimates $\hat{\alpha}(k)$. The squared correlation is defined as[2]

$$\rho^2 = \left|\frac{E[\hat{\alpha}^*(k)\alpha(k)]}{E[\hat{\alpha}^*(k)\hat{\alpha}(k)]}\right|^2 \tag{20}$$

After some further manipulations, Eq. (20) can be rewritten as

$$\rho = \left|\frac{\sigma^2}{\sigma^2 + \sigma_e^2}\right| = \frac{1}{1 + \sigma_e^2/\sigma^2} = \frac{1}{1 + \gamma_\alpha} \tag{21}$$

where γ_α is normalized mean squared channel estimation error. When the lower bounds on γ_α are applied to Eq. (21), the maximal squared correlations can be obtained.

Using the form of the PDF of the output SNR in Eq. (18), the BER for a particular modulation scheme can be expressed as[2]

$$P_e = \sum_{s=1}^{L} W(s) \int_0^{\infty} \phi(\gamma) f_\gamma(\gamma, s)\, d\gamma \tag{22}$$

where $\phi(\gamma)$ is the BER for different modulation schemes dependent on γ, and $W(s)$ is the weighting coefficient, which is not a function of γ, and it can be given by

$$W(s) = \binom{L-1}{s-1}(1-\rho^2)^{L-s}\rho^{2(s-1)} \tag{23}$$

$f_\gamma(\gamma, s)$ is the PDF of the output SNR of an ideal s-branch MR combiner and can be written as

$$f_\gamma(\gamma, s) = \frac{e^{-\gamma/\Gamma}\gamma^{s-1}}{(s-1)!\,\Gamma^s} \tag{24}$$

4 Numerical Results

In this section we will indicate the effect of Gaussian errors in the weighting factors on the BER performance for binary phase shift keying (BPSK)[9] of a six-branch combiner.

Fig. 1(a), Fig. 2(a) and Fig. 3(a) illustrate the lower bounds on BER surfaces under different fading power spectra as a function of the SNR of branch and the SNR of channel estimation, where f_d takes the value 185 Hz, T_S and K are assumed to be 66.66 μs and 10, respectively.

The numerical results of the lower bounds on the BER performance as a function of the maximum Doppler frequency and the SNR of branch are shown in Fig. 1(b), Fig. 2(b) and Fig. 3(b) for different fading power spectra, where the SNR of channel estimation takes the value 5 dB, T_S is assumed to be 66.66 μs and $K = 10$.

In all these cases, the lower surfaces denote the BER performance of ideal six-branch MR combiners. It can be noted from all the plots, the BER performances of ideal MR combiners are the lower bounds on the BER performances of non-ideal MR combiners which deteriorate as the maximum Doppler frequency increases or the SNR of channel estimation decreases. It also can be noted that the BER performance in the fading channel with U-shaped power spectra outperforms the performance for flat power spectra and when there exists a specular component, better BER performance can be obtained.

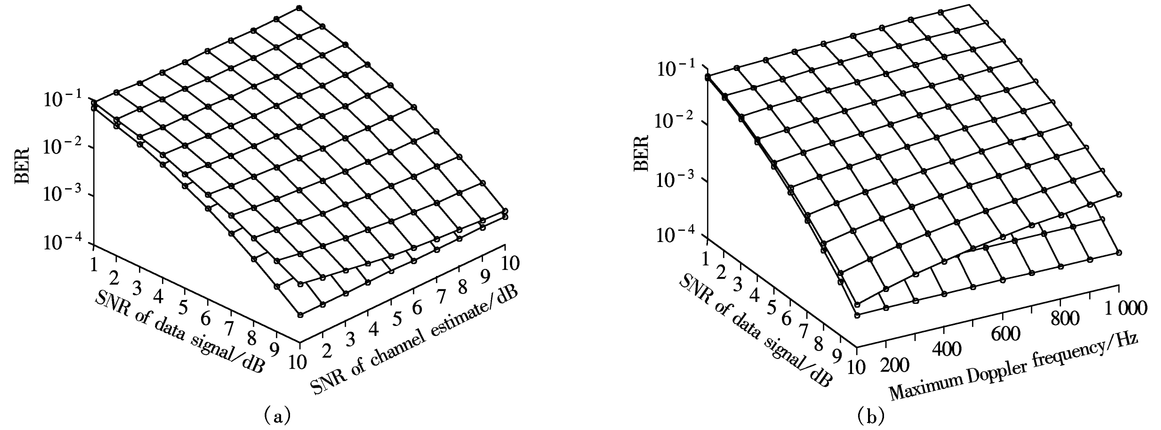

Fig. 1 BER performance for flat power spectrum. (a) Effect of channel estimate SNR; (b) Effect of maximum Doppler frequency

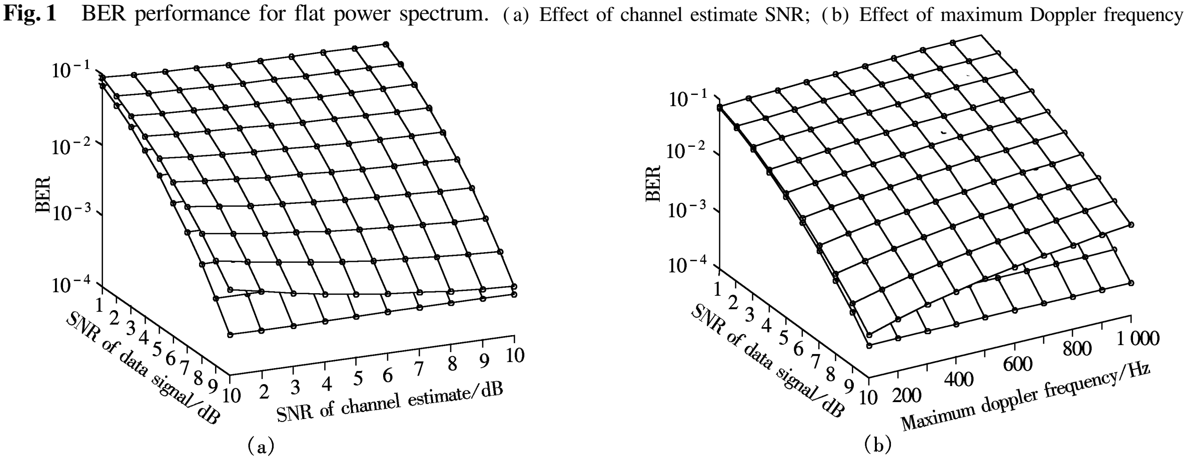

Fig. 2 BER performance for U-shaped power spectrum. (a) Effect of channel estimate SNR; (b) Effect of maximum Doppler frequency

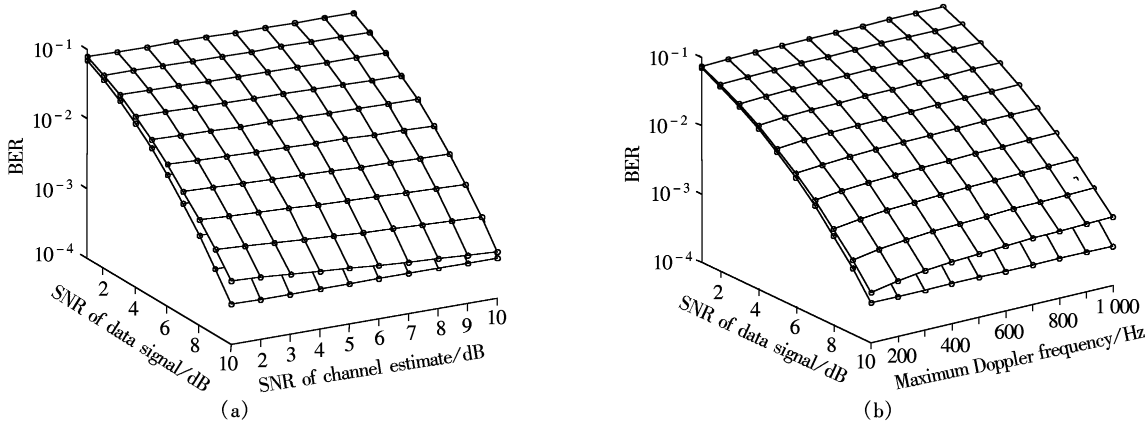

Fig.3 BER performance for U-shaped plus specular component. (a) Effect of channel estimate SNR; (b) Effect of maximum Doppler frequency

5 Conclusion

In this paper, the exact closed-form expressions of the lower bounds on channel estimate for different channel Doppler spectra have been derived. The lower bound on BER for maximal ratio combining with Gaussian distributed weighting errors on independent and identically distributed Rayleigh fading channels has also been obtained. These expressions have presented criteria for us to access the system performance and compare the gain in BER performance for any of the analyzed schemes in the presence of a realistic environment.

Appendix A

In this appendix we derive the power spectra of the channel estimate and the estimation error.

The cost function is defined as mean-square error and can be expressed as

$$J_{\min}(h(k)) = E[\,|\alpha(n) - \hat{\alpha}(n)|^2] \tag{A1}$$

Using Eq. (4) we can write the cost function in Eq. (25) as

$$\begin{aligned} J_{\min}(h(k)) &= R_{\alpha\alpha}(0) - 2\mathrm{Re}\Big\{\sum_{k=-\infty}^{\infty} h(k) R_{\alpha X}(k)\Big\} + \sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h^*(k) h(n) R_{XX}(k-n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})\mathrm{d}\omega - \\ &\frac{2}{2\pi}\mathrm{Re}\Big\{\sum_{k=-\infty}^{\infty} h(k)\int_{-\pi}^{\pi} S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}\omega k}\mathrm{d}\omega\Big\} + \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h^*(k)h(n)\int_{-\pi}^{\pi} S_{XX}(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}\omega(k-n)}\mathrm{d}\omega = \\ &\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})\mathrm{d}\omega - \frac{2}{2\pi}\mathrm{Re}\Big\{\int_{-\pi}^{\pi} H^*(\mathrm{e}^{\mathrm{j}\omega}) S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})\mathrm{d}\omega\Big\} + \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(\mathrm{e}^{\mathrm{j}\omega})|^2 S_{XX}(\mathrm{e}^{\mathrm{j}\omega})\mathrm{d}\omega \end{aligned} \tag{A2}$$

For the typical Doppler power spectra which are normally real symmetry function, Eq. (A2) can be simplified into

$$\begin{aligned} J_{\min}(h(k)) &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\Big\{\Big[1 - \frac{2S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})}{S_{XX}(\mathrm{e}^{\mathrm{j}\omega})}\Big] S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega}) + \Big[\frac{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})}{S_{XX}(\mathrm{e}^{\mathrm{j}\omega})}\Big]^2 S_{XX}(\mathrm{e}^{\mathrm{j}\omega})\Big\}\mathrm{d}\omega = \\ &\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})S_{XX}(\mathrm{e}^{\mathrm{j}\omega}) - [S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})]^2}{S_{XX}(\mathrm{e}^{\mathrm{j}\omega})}\mathrm{d}\omega \end{aligned} \tag{A3}$$

Substituting Eq. (5) into Eq. (A3) yields

$$J_{\min}(h(k)) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})S_{vv}(\mathrm{e}^{\mathrm{j}\omega})}{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega}) + S_{vv}(\mathrm{e}^{\mathrm{j}\omega})}\mathrm{d}\omega \tag{A4}$$

Furthermore, the average power of the channel estimate can be obtained as

$$E[\,|\hat{\alpha}(n)|^2] = \sigma^2 - J_{\min}(h(k)) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{S^2_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega})}{S_{\alpha\alpha}(\mathrm{e}^{\mathrm{j}\omega}) + S_{vv}(\mathrm{e}^{\mathrm{j}\omega})}\mathrm{d}\omega \tag{A5}$$

Appendix B

In this appendix, we describe the important intermediate steps involved in the transformation of Eq. (10) into Eq. (11).

When the filter tap N is odd, the first term on the right-hand side of Eq. (10) can be restated as

$$\boldsymbol{P}^{\mathrm{H}}_{N+1}\boldsymbol{R}^{-1}_{N+1}\boldsymbol{P}_{N+1} = \begin{bmatrix}\boldsymbol{P}_N \\ r_{(N+1)/2}\end{bmatrix}^{\mathrm{H}}\begin{bmatrix}\boldsymbol{R}_N & \tilde{\boldsymbol{P}} \\ \tilde{\boldsymbol{P}}^{\mathrm{T}} & r_0\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{P}_N \\ r_{(N+1)/2}\end{bmatrix} \tag{B1}$$

By using the lemma of matrix inversion, the middle term on the right-hand side of Eq. (B1) can be expressed as

$$\begin{bmatrix} \boldsymbol{R}_N & \tilde{\boldsymbol{P}} \\ \tilde{\boldsymbol{P}}^{\mathrm{T}} & r_0 \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \boldsymbol{C}_{21} & C_{22} \end{bmatrix} \tag{B2}$$

where

$$C_{22} = (r_0 - \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}})^{-1}, \quad \boldsymbol{C}_{12} = -\boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}} C_{22}, \quad \boldsymbol{C}_{21} = -C_{22} \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1}, \quad \boldsymbol{C}_{11} = \boldsymbol{R}_N^{-1} - \boldsymbol{C}_{12} \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \tag{B3}$$

And, C_{22} can be manipulated into

$$C_{22} = (r_0 - \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}})^{-1} = \frac{1}{r_0} + \frac{1}{r_0} \tilde{\boldsymbol{P}}^{\mathrm{T}} \left(\boldsymbol{R}_N - \tilde{\boldsymbol{P}} \frac{1}{r_0} \tilde{\boldsymbol{P}}^{\mathrm{T}} \right)^{-1} \tilde{\boldsymbol{P}} \frac{1}{r_0} =$$
$$\frac{1}{r_0^2} \tilde{\boldsymbol{P}}^{\mathrm{T}} (\boldsymbol{R}_N^{-1} + \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}} (r_0 - \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}})^{-1} \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1}) \tilde{\boldsymbol{P}} = \frac{1}{r_0 - \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}}} \tag{B4}$$

Inserting Eq. (B3) into Eq. (20) gives

$$\begin{bmatrix} \boldsymbol{P}_N \\ r_{(N+1)/2} \end{bmatrix}^{\mathrm{H}} \begin{bmatrix} \boldsymbol{R}_N & \tilde{\boldsymbol{P}} \\ \tilde{\boldsymbol{P}}^{\mathrm{T}} & r_0 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{P}_N \\ r_{(N+1)/2} \end{bmatrix} = \boldsymbol{P}_N^{\mathrm{H}} \boldsymbol{R}_N^{-1} \boldsymbol{P}_N + \boldsymbol{P}_N^{\mathrm{H}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}} C_{22} \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \boldsymbol{P}_N -$$
$$r_{(N+1)/2} C_{22} \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \boldsymbol{P}_N - r_{(N+1)/2} C_{22} \boldsymbol{P}_N^{\mathrm{H}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}} + r_{(N+1)/2}^2 C_{22} \tag{B5}$$

Combining Eq. (9) and Eq. (B1), we finally obtain

$$J_{\min}^{N} - J_{\min}^{N+1} = \boldsymbol{P}_N^{\mathrm{H}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}} C_{22} \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \boldsymbol{P}_N - r_{(N+1)/2} C_{22} \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \boldsymbol{P}_N - r_{(N+1)/2} C_{22} \boldsymbol{P}_N^{\mathrm{H}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}} + r_{(N+1)/2}^2 C_{22} =$$
$$C_{22} (r_{(N+1)/2} - \boldsymbol{P}_N^{\mathrm{H}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}})(r_{(N+1)/2} - \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \boldsymbol{P}_N) = \frac{(r_{(N+1)/2} - \boldsymbol{w}_{\mathrm{opt}}^{\mathrm{H}} \tilde{\boldsymbol{P}})^2}{r_0 - \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}}} \tag{B6}$$

The transformation of Eq. (10) into Eq. (11) is also the same for even N and the final result can be derived as

$$J_{\min}^{N} - J_{\min}^{N+1} = \frac{(r_{N/2} - \boldsymbol{w}_{\mathrm{opt}}^{\mathrm{H}} \tilde{\boldsymbol{P}})^2}{r_0 - \tilde{\boldsymbol{P}}^{\mathrm{T}} \boldsymbol{R}_N^{-1} \tilde{\boldsymbol{P}}} \tag{B7}$$

References

[1] Gans M J. The effect of Gaussian error in maximal ratio combiners [J]. *IEEE Trans Commun Tech*, 1971, **19**(4): 492 - 500.

[2] Tomiuk B R, Beaulieu N C, Abu-Dayya A A. General forms for maximal ratio diversity with weighting errors [J]. *IEEE Trans Commun*, 1999, **47**(4): 488 - 492.

[3] Cai X, Giannakis G B. Adaptive PSAM accounting for channel estimation and prediction errors [J]. *IEEE Trans Wireless Commun*, 2005, **4**(1): 246 - 256.

[4] Ma Y, Schober R, Pasupathy S. Effect of imperfect channel estimation on MRC diversity in fading channels [A]. In: *IEEE ICC* 2004 [C]. Paris, 2004. 3163 - 3167.

[5] Cavers J K. An analysis of pilot symbol assisted modulation for Rayleigh fading channels [J]. *IEEE Trans Veh Technol*, 1991, **40**(4): 686 - 693.

[6] Brennan D G. Linear diversity combining techniques [J]. *Proc IRE*, 1959, **46**: 1075 - 1102.

[7] Haykin S. *Adaptive filter theory*. 4rd ed [M]. Englewood Cliffs: Prentice-Hall, 1996.

[8] Jakes W C. *Microwave mobile communications* [M]. New York: Wiley, 1974.

[9] Proakis J G. *Digital communications*. 3rd ed [M]. New York: McGrawHill, 1995.

非理想最大比合并的误码率下界

盛　彬　　尤肖虎

(东南大学移动通信国家重点实验室,南京 210096)

摘要: 研究了信道估计的均方误差,利用无限长、非因果的维纳滤波器推导出了它的下界. 通过对不同衰落信道多普勒谱的积分,求出了相应的信道估计均方误差下界的闭合表达式,并在此基础上得到了分集系统在独立同分布的衰落信道中采用最大比合并时的误码率下界. 数值分析结果表明,非理想信道估计下最大比合并的误码率性能是以理想信道估计的结果为下界,并随着最大多普勒频移的增加或信道估计信噪比的减小而逐渐劣化.

关键词: 下界;误码率;最小均方误差;信道估计;最大比合并

中图分类号: TN929