

# Improved Kalman filter channel estimation method for OFDM systems in fast time-varying environment

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**Abstract:** Under analyzing several characteristics of frequency-selective fast fading channels, such as large Doppler spread and multi-path interference, a low-dimensional Kalman filter method based on pilot signals is presented for the channel estimation of orthogonal frequency division multiplexing (OFDM) systems. For simplicity, a one-dimensional autoregressive (AR) process is used to model the time-varying channel, and the least square (LS) algorithm based on pilot signals is adopted to track the time-varying channel fading factor  $a$ . The low-dimensional Kalman filter estimator greatly reduces the complexity of the high-dimensional Kalman filter. To utilize the relationship of fading channel in frequency domain, a minimum mean-square-error (MMSE) combiner is used to refine the estimation results. The simulation results in the frequency band of 5.5 GHz show that the proposed method achieves a good symbol error rate (SER) performance close to the theoretical bound of ideal channel estimation.

**Key words:** channel estimation; orthogonal frequency division multiplexing (OFDM); least square (LS); minimum mean-square-error (MMSE)

OFDM has been proposed to be a potential candidate for next generation systems due to a set of characteristics such as robustness against frequency-selective fading channels, high bandwidth efficiency, and expandability<sup>[1–3]</sup>. However, correlation demodulation at receiver requires the knowledge of channel state information, thus research on the channel estimation is prevalent<sup>[4–7]</sup>. In a fast time-varying environment, there exist inter-carrier interference (ICI) and additive white Gaussian noise (AWGN), which reduce the precision of channel estimation seriously. Refs. [8–10] have proposed several methods adopting the Kalman filter for blind channel estimation of OFDM systems in fast time-varying channels. Due to the complexity and imprecision of many blind channel estimation methods, we develop, in this paper, a channel estimation method for OFDM systems in fast time-varying channels based on the Kalman filter and pilot tracking to eliminate AWGN and ICI. To reduce the complexity of the high-dimensional Kalman filter, we propose to use a one-dimensional Kalman filter based on pilot signals and a tracker in an LS sense on each subcarrier. Fur-

ther, we propose to utilize an MMSE combiner to refine the results of the channel estimation, which can utilize the relationship of fading channel in the frequency domain. Both theoretical analysis and simulation results prove that the proposed channel estimation method can offer a considerable gain in signal-to-noise (SNR) of the system.

## 1 System Model

The OFDM system under consideration is depicted in Fig. 1, which employs the Kalman-filter channel estimator.

At the transmitter, source symbols are converted to  $X_k$  after mapping and serial-to-parallel conversion. Processed by an IFFT, transmit signals can be obtained as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp\left(j \frac{2\pi}{N} kn\right) \quad 0 \leq n \leq N-1 \quad (1)$$

At the receiver, after demodulation, sampling and cyclic-prefix (CP) removing, receive signals can be written as

$$y_n = \sum_{l=0}^{L-1} h_{n,l} x_{n-l} + w_n = \mathbf{x}_n^T \mathbf{h}_n + w_n \quad 0 \leq n \leq N-1 \quad (2)$$

where  $\mathbf{x}_n = \{x_n, x_{n-1}, \dots, x_{n-L+1}\}^T$ ,  $\mathbf{h}_n = \{h_{n,0}, h_{n,1}, \dots, h_{n,L-1}\}^T$  is the  $L \times 1$  vector representing the length  $L$  channel impulse response at time index  $n$ ,  $h_{n,l}$  is sample value of channel impulse response, and  $w_n$  is additive Gaussian random variable with zero mean and variance  $\sigma_n^2$ .

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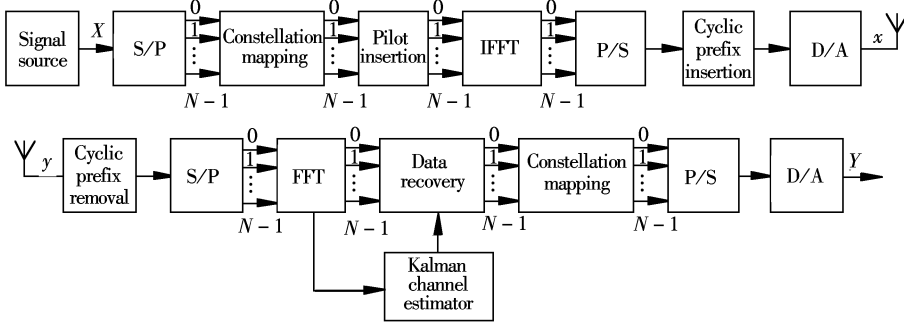


Fig. 1 System model

After FFT, we obtain the sample from the  $k$ -th subcarrier of the  $n$ -th OFDM symbol

$$Y_k = X_k H_k + I_k + W_k = X_k H_k + Z_k \quad (3)$$

where, to simplify notation, we will omit the symbol index  $n$ , and  $k$  is the sub-carrier index.  $H_{k,l} = \frac{1}{N} \sum_{n=0}^{N-1} h_{n,l} e^{-j2\pi nk/N}$ ,  $H_k = \sum_{l=0}^{L-1} H_{0,l} e^{-j2\pi lk/N}$ ,  $I_k = \sum_{m=0}^{N-1} \sum_{\substack{l=0 \\ m \neq k}}^{L-1} X_m H_{k-m,l} e^{-j2\pi ml/N}$ ,  $W_k = \text{FFT}(w_n)$ , and both  $I_k$  and  $W_k$  are high-frequency part of channel signal.

Assuming that the channel is Rayleigh fading, we use Jakes' model<sup>[11]</sup> for the power spectral density and Doppler spectrum of the fading process

$$E[H_{l,n} H_{l,k}^*] = r_l(n-k) = J_0(2\pi f_d T_s(n-k)) \quad (4)$$

where  $H_{l,n}$ ,  $H_{l,k}$  denote the channel gain of the  $l$ -th subcarrier of the  $n$ -th and the  $k$ -th OFDM symbols;  $f_d$  is the Doppler maximum frequency shift;  $T_s$  is the OFDM symbol duration;  $J_0(\cdot)$  is the zeroth-order Bessel function of the first kind.

Since there are several methods of pilot assignment, here comb pattern<sup>[4]</sup> is adopted.  $N_p$  subcarriers are used to transmit pilot signals. Assuming that  $N/N_p$  is an integer, we have

$$X_n(k) = X_n \left( m \frac{N}{N_p} + i \right) = \begin{cases} \text{pilot} & i=0 \\ \text{data} & i=1, 2, \dots, N/N_p - 1 \end{cases} \quad (5)$$

$0 \leq m \leq N_p - 1$

where  $m$  is the pilot index; pilot denotes the pilot signal. Then we propose to use the same pilot signals to reduce the system complexity.

Assume that  $N_p \times 1$  channel gain vector of pilot subcarriers of the  $n$ -th OFDM symbol is

$$H_n^p = \{H_{0,n}^p, H_{1,n}^p, \dots, H_{N_p-1,n}^p\}^T \quad (6)$$

According to the expression of received signal of OFDM, the signal from the  $k$ -th pilot subcarrier of the  $n$ -th OFDM symbol can be expressed as

$$Y_{k,n}^p = X_{k,n}^p H_{k,n}^p + Z_{k,n}^p \quad 0 \leq k \leq N_p - 1 \quad (7)$$

where  $X_{k,n}^p$  and  $Z_{k,n}^p$  are respectively pilot signal value and Gaussian white noise random variable with zero mean at the  $k$ -th pilot sub-carrier of the  $n$ -th OFDM symbol, and  $H_{k,n}^p$ ,  $Z_{k,n}^p$  i. i. d.

## 2 Proposed Channel Estimation Method

In this paper, assuming that the channel is time-variant, the two key points of our proposed method are discussed in detail as follows: the low-dimensional Kalman channel estimator based on pilots, the channel tracker in an LS sense. The estimate process of the  $k$ -th subcarrier based on a low-dimensional Kalman channel estimator with an LS tracker is depicted in Fig. 2.

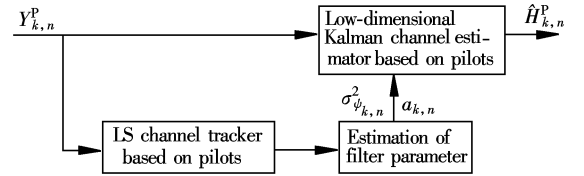


Fig. 2 Channel estimation method

### 2.1 Low-dimensional modified Kalman estimator on pilots

To reduce the complexity of the traditional high-dimensional Kalman filter channel estimator, the implementation of the low-dimensional Kalman filter at each sub-carrier side by side is presented. Channel response dynamics can be modeled by AR process as

$$H_{n,k}^p = \sum_{i=1}^p a_{i,k} H_{n-i,k}^p + \Psi_{n,k}^p \quad (8)$$

where  $p$  and  $a_{i,k}$  are order and coefficient of AR process,  $\Psi_{n,k}^p$  is the Gaussian white noise with zero mean and variance  $\sigma_{\psi}^2$ , and  $H_{n,k}^p$  is the  $k$ -th pilot sub-channel frequency response of the  $n$ -th OFDM symbol. For simplicity, we use the first-order AR process, i. e.,  $p = 1$ .

According to Eq. (8) and OFDM system model, state space can be described as

$$H_{k,n}^p = a_k H_{k,n-1}^p + \Psi_{k,n}^p \quad (9)$$

$$Y_{k,n}^p = X_{k,n}^p H_{k,n}^p + Z_{k,n}^p \quad (10)$$

where  $Z_{k,n}^p$  is the Gaussian white noise with zero mean and variance  $\sigma_z^2$ , and  $\Psi_{k,n}^p$  denotes system state variance noise due to channel response dynamics.

According to the Kalman algorithm at a per-sub-channel fashion proposed by Chen et al. <sup>[8]</sup>, the low-

dimensional pilot assisted Kalman filter iterative equation of each pilot sub-carrier can be obtained:

$$e_{k,n} = Y_{k,n}^P - \hat{Y}_{k,n}^P = Y_{k,n}^P - X_{k,n}^P \hat{H}_{k,n-1}^P \quad (11)$$

$$P'_{k,n} = |a_k|^2 P_{k,n-1} + Q_{k,n} \quad (12)$$

$$r_{k,n} = \sigma_z^2 + X_{k,n}^P P'_{k,n} (X_{k,n}^P)^H \quad (13)$$

$$K_{k,n} = P_{k,n} (X_{k,n}^P)^H r_{k,n}^{-1} \quad (14)$$

$$\hat{H}_{k,n}^P = a_k \hat{H}_{k,n-1}^P + K_{k,n} e_{k,n} \quad (15)$$

$$P_{k,n} = (1 - K_{k,n} X_{k,n}^P) P'_{k,n} \quad (16)$$

where  $Q_{k,n}$  is the covariance matrix of system state noise  $\Psi_{n,k}^P$ ;  $\hat{H}_{k,n}^P$  is the Kalman estimation of  $H_{n,k}^P$ , and is initialized zero;  $X_{k,n}^P$  is the pilot signal value;  $P_{k,0}$  is the stationary covariance of  $\hat{H}_{k,n}^P$ , which can be generated by Eq. (4).

## 2.2 LS track algorithm

Based on Jakes' channel fading model, AR process coefficient  $a_k$  can be generated by Yule-Walker equation, where  $a_k = J_0(2\pi f_d T_s)$ . Variance of process noise  $\sigma_\psi^2$  can also be obtained by  $\sigma_\psi^2 = 1 - J_0^2(2\pi f_d T_s)$ . However, in fact, at the receiver,  $f_d T_s$  is unknown, and the  $a_{k,n}$  is variable. Thus we implement LS estimator based on pilot to track the process coefficient  $a_{k,n}$  varying with the time index  $n$ .

The LS estimate of OFDM pilot sub-carrier frequency response is

$$\tilde{H}_{n,k}^P = H_{n,k}^P + \frac{Z_{n,k}^P}{X_{n,k}^P} \quad (17)$$

Because  $a_{k,n}$  denotes the transfer relation of channel frequency response, it can be real-time estimated by using LS estimate results based on pilots, then we can obtain parameter vector  $\mathbf{A}_n = \{a_{0,n}, a_{1,n}, \dots, a_{k,n}, \dots, a_{N_p-1,n}\}$ , where  $a_{k,0}$  is initialized one,

$$a_{k,n} = \frac{\tilde{H}_{k,n+1}^P}{\tilde{H}_{k,n}^P} \quad k=0, 1, \dots, N_p-1; n \geq 1 \quad (18)$$

Then the process noise variance  $\sigma_{\psi_{k,n}}^2$  can be expressed as

$$\sigma_{\psi_{k,n}}^2 = 1 - a_{k,n}^2 \quad (19)$$

Although these two steps have efficiently tracked the time domain dynamics of channel, we propose to refine the estimate results by an MMSE<sup>[8,12]</sup> linear combiner. Assume that merge matrix is  $\mathbf{T}$ ,  $\mathbf{H}_n^P$  is the  $N_p \times 1$  sub-channel gain vector and  $\hat{\mathbf{H}}_n^P$  is the  $N_p \times 1$  sub-channel gain estimation vector of the Kalman-filter estimator, thus refined result is

$$\tilde{\mathbf{H}}_n^P = \mathbf{T} \cdot \hat{\mathbf{H}}_n^P \quad (20)$$

where  $\mathbf{T}$  obeys MMSE principle, i. e., minimizing  $E[\|\mathbf{H}_n^P - \tilde{\mathbf{H}}_n^P\|^2]$ .

After differentiating  $E[\|\mathbf{H}_n^P - \tilde{\mathbf{H}}_n^P\|^2]$  with  $\mathbf{T}^H$ , we have

$$\mathbf{T} = E[\mathbf{H}_n^P (\hat{\mathbf{H}}_n^P)^H] \{E[\hat{\mathbf{H}}_n^P (\hat{\mathbf{H}}_n^P)^H]\}^{-1} \quad (21)$$

where  $\hat{\mathbf{H}}_n^P$  is the Kalman filter estimate result of  $\mathbf{H}_n^P$ ,

and then

$$\hat{\mathbf{H}}_n^P = \mathbf{H}_n^P + \mathbf{W}_n^P \quad (22)$$

where  $\mathbf{W}_n^P$  is the Gaussian whiten noise with zero mean, and is independent of  $\mathbf{H}_n^P$ .

Therefore, Eq. (21) can be written as

$$\mathbf{T} = E[\mathbf{H}_n^P (\mathbf{H}_n^P)^H] \{E[\mathbf{H}_n^P (\mathbf{H}_n^P)^H] + E[\mathbf{W}_n^P (\mathbf{W}_n^P)^H]\}^{-1} \quad (23)$$

Define  $\mathbf{R}(0) = E[\mathbf{H}_n^P (\mathbf{H}_n^P)^H]$ , and  $\mathbf{R}(0)$  can be precalculated.  $\mathbf{P} = E[\mathbf{W}_n^P (\mathbf{W}_n^P)^H]$ , and  $\mathbf{P}$  is an  $N_p \times N_p$  diagonal matrix. The  $(k, k)$  value at matrix diagonal is  $P_{k,n}$ , which is covariance of  $\hat{H}_{k,n}^P$ . We can further obtain

$$\mathbf{T} = \mathbf{R}(0) [\mathbf{R}(0) + \mathbf{P}]^{-1} \quad (24)$$

After processing the results from Eq. (20) with quadratic interpolation, we can obtain the frequency response estimates of  $N$  sub-carriers of channel.

## 3 Simulation

To evaluate the performance of our proposed method, simulations are carried out for the cases where the channels state information is assumed to be known. We consider a BPSK system with carrier frequency of 5.5 GHz and bandwidth of 20 MHz. We use a symbol spaced tap-delay-line channel model of 17 paths, where each channel tap is a Rayleigh fading process independently generated with the Doppler spectrum based on the Jakes' model. The average power distribution of the time-variant multipath channel model is given in Eq. (25). The simulation is carried out with different maximum Doppler frequencies 366.67 Hz and 1018.5 Hz. The maximum time delay is 4  $\mu$ s. The total number of all subcarriers is  $N = 1024$ , the number of pilot subcarriers is  $N_p = 256$  and the CP length is 81.

$$\mathbf{h} = \{1, 0.7499, 0.5623, 0.4217, 0.3162, 0.2371, 0.1778, 0.1334, 0.1, 0.075, 0.0562, 0.0422, 0.0316, 0.0237, 0.0178, 0.0133, 0.01\} \quad (25)$$

In Fig. 3, we compare the symbol error rate (SER) performance of our proposed method without LS tracker where  $f_d T_s$  is assumed to be known at the

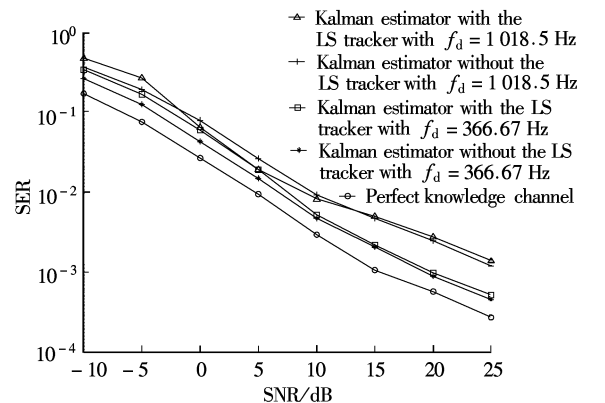


Fig. 3 Comparisons of SER

receiver, and the proposed method with LS tracker where  $f_d T_s$  is unknown. As shown in Fig. 3 using the proposed OFDM channel estimation based on the low-dimensional Kalman filter and pilots has a good performance close to the theoretical bound of ideal channel estimation. And the SER performance of the LS track algorithm is close to the performance when  $f_d T_s$  is known at the receiver.

#### 4 Conclusion

A channel estimation method for OFDM systems in fast time-varying channels based on the Kalman filter and pilot tracking to eliminate AWGN and ICI is developed. For reducing the complexity of the high-dimensional Kalman filter, we propose to use a one-dimensional Kalman filter based on pilot signals and a tracker in an LS sense on each subcarrier. Further, we utilize an MMSE combiner to refine the results of the channel estimation. Both theoretical analysis and simulation results prove that the proposed channel estimation method can offer a considerable gain in SNR of the system.

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## 一种快衰落信道中 OFDM 系统的 Kalman 信道估计方法

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**摘要:** 针对频率选择性快衰落信道的多径干扰和较大的多普勒频率扩展, 提出了一种基于导频的低维 Kalman 滤波算法用于正交频分复用 (OFDM) 系统信道估计。为了简化计算, 采用一阶自回归 (AR) 过程对时变信道进行建模。利用复杂度大大降低的一维 Kalman 滤波算法进行单个子载波的并行信道估计, 并采用基于导频的最小二乘 (LS) 算法估计时变的信道衰减因子  $a$ 。为了同时跟踪信道的频域相关性, 采用了最小均方误差 (MMSE) 线性合并器对 Kalman 信道估计结果进行修正。在 5.5 GHz 频段上的仿真表明了这种基于导频的低维 Kalman 信道估计方法, 降低了传统的 Kalman 滤波结构的复杂度, 能够跟踪信道的时频变化, 并且在一定程度上可以接近于理想信道估计的误码率性能。

**关键词:** 信道估计; 正交频分复用; 最小平方; 最小均方误差

**中图分类号:** TN914.3