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Novel method for exactly evaluating the energy of Catmull-Clark subdivision surfaces

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Abstract: A novel method is produced to evaluate the energy of the Catmull-Clark subdivision surface including extraordinary points in the control mesh. A closed-form analytic formula for thin plate energy of the Catmull-Clark subdivision surface of arbitrary topology is derived through translating the Catmull-Clark subdivision surface into bi-cubic B-spline surface pieces. Using this method, both the membrane energy and the thin plate energy can be evaluated without requiring recursive subdivision. Therefore, it is more efficient and more accurate than the existing methods for calculating the energy of the Catmull-Clark subdivision surface with arbitrary topology. The example of surface fairing demonstrates that this method is efficient and successful for evaluating the energy of subdivision surfaces.

Key words: Catmull-Clark subdivision; energy of surface; fairing of surface

Energy of subdivision surfaces has been widely used in geometrics, computer graphics, physics-based dynamic modeling and scientific visualization[1]. However the existing methods have many defects. First, energy evaluations of patch subdivision surfaces with an extraordinary point needs recursive subdivision and numerical integrals. Secondly, they obtain only approximate results of energy of patches with an extraordinary point[2]. The aim of this paper is to solve these questions.

There are many schemes to express the energy of surface, such as elastically deformable energy[3], data-dependent energy[4] and so on. In this paper, the surface energy is composed of a membrane energy and a thin plate energy[5]. This form energy has clear physics meanings.

$$E = \mu E_{\mathrm{m}} + \xi E_{\mathrm{p}} \tag{1}$$

where μ and ξ are the characteristic coefficients of the material, usually μ and ξ are all equal to 1,

$$E_{\mathrm{m}} = \iint (\|S_u(u,w)\|^2 + \|S_w(u,w)\|^2)\,\mathrm{d}u\mathrm{d}w$$

$$E_{\mathrm{p}} = \iint (\|S_{uu}(u,w)\|^2 + 2\|S_{uw}(u,w)\|^2 + \|S_{ww}(u,w)\|^2)\,\mathrm{d}u\mathrm{d}w$$

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Although there are many surface subdivision schemes, the Catmull-Clark subdivision[6] is a widely used subdivision scheme. The energy evaluation of the Catmull-Clark subdivision surface is analyzed in this paper. Our method may easily be adapted for computing the energy of other subdivision schemes.

1 Catmull-Clark Surfaces

Catmull and Clark have shown that the rules expressed for the cubic B-spline subdivision not only work for arbitrary rectangular meshes, but also can be extended to meshes with arbitrary topology.

The number of edges adjacent to a vertex is called its valence. A vertex whose valence is equal to four is called an ordinary vertex; otherwise, it is called an extraordinary vertex. The new control mesh of a subdivision surface is reconnected by the following new points by subdividing the initial mesh[7].

- Face point

The new face points are generated from each face of the initial mesh. The position of the new face point generated is the average of all the original points, that is

$$V_{\mathrm{F}} = \frac{1}{n}\sum_{i=1}^{n} V_i$$

- Edge point

The new edge points are generated from each edge of the initial mesh. The position of the new edge point is the average of the mid points of the original edge with the two new face points of the faces adjacent to the edge.

$$V_E = \frac{1}{4}(V_i + V_j + F_1 + F_2)$$

- Vertex point

The new vertex points are generated from each vertex of the initial mesh. The position of the new vertex points can be calculated as

$$V_V = \alpha_N V + \beta_N \sum_{i=1}^{N} V_{2i} + \gamma_N \sum_{i=1}^{N} V_{2i+1}$$

where $\beta_N = \frac{3}{2N^2}, \gamma_N = \frac{1}{4N^2}, \alpha_N = 1 - N(\beta_N + \gamma_N)$.

The Catmull-Clark subdivision masks are shown in Fig. 1.

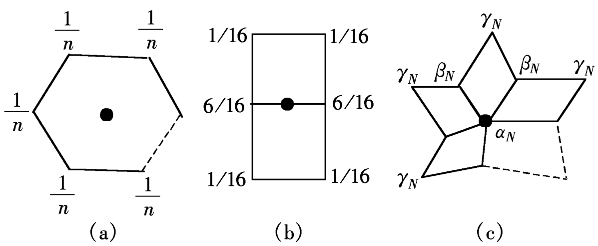


Fig. 1 Catmull-Clark masks. (a) Face point; (b) Edge point, $N=4$; (c) Vertex point

This subdivision scheme produces surfaces that are C^2 continuity everywhere except at extraordinary vertices where they are C^1 continuity.

2 Energy Analysis of Catmull-Clark Subdivision Surfaces

2.1 Parameterization of Catmull-Clark subdivision surfaces

Regular patch of the Catmull-Clark subdivision without extraordinary vertices is actually uniform bicubic B-spline surface, as shown in Fig. 2. The surface can be represented as[6]

$$\boldsymbol{S}(u, w) = \boldsymbol{C}_0^{\mathrm{T}} \boldsymbol{b}(u, w) \qquad 0 \leqslant u \leqslant 1, 0 \leqslant w \leqslant 1$$

where $\boldsymbol{C}_0 = \{c_{0,1}, c_{0,2}, \ldots, c_{0,16}\}^{\mathrm{T}}$, and $\boldsymbol{b}(u, w) = \{b_1(u,w), b_2(u, w), \ldots, b_{16}(u, w)\}^{\mathrm{T}} = \boldsymbol{B}$ (see Ref. [8]).

The control vertex structure near an extraordinary

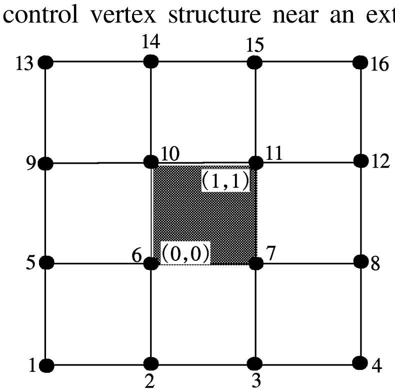


Fig. 2 A bicubic B-spline defined by 16 control vertices

vertex is not a simple rectangular grid. A patch that contains extraordinary vertices cannot be evaluated as uniform B-spline, as is shown in Fig. 3. It should be subdivided into many sub-patches. We assume that the initial mesh has been subdivided at least twice, isolating the extraordinary vertices, so that each sub-patch is a quadrilateral and contains at most one extraordinary vertex[8].

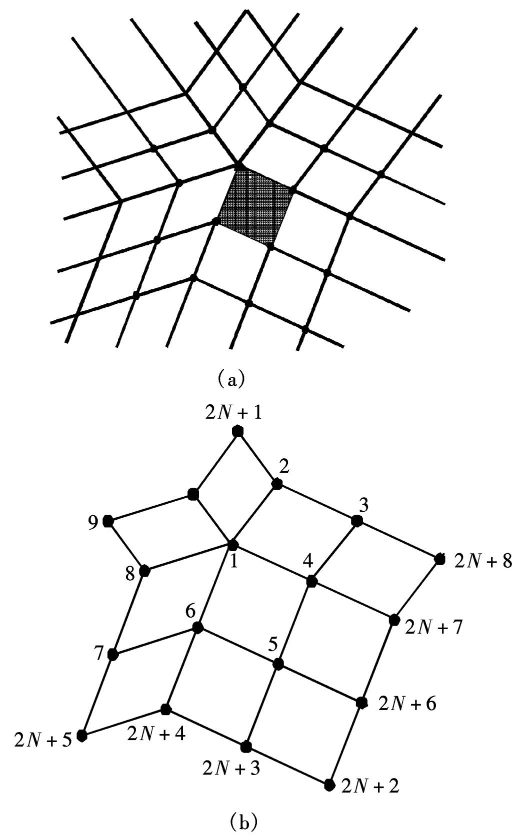


Fig. 3 Irregular surface patch. (a) Surface patch contained an extraordinary vertex; (b) Ordering of the control vertices

Let N denote the valence of the extraordinary vertex, $K = 2N + 8$ be the number of vertices for calculating the patch. A new set of $M = K + 9$ vertices can be generated by subdivision. The new set of vertices is shown as circles superimposed on the initial vertices in Fig. 4. Subsets of these new vertices are the control vertices of three uniform B-spline patches. Therefore, three-quarters of our surface patch is parameterized. Denote this new set of vertices by

$$\boldsymbol{C}_1 = \{c_{1,1}, c_{1,2}, \ldots, c_{1,16}\}^{\mathrm{T}}$$

$$\overline{\boldsymbol{C}}_1 = \{c_{1,1}, c_{1,2}, \ldots, c_{1,K}, c_{1,K+1}, \ldots, c_{1,M}\}^{\mathrm{T}}$$

With these matrices, the subdivision step is a multiplication by an $M \times K$ (extended) subdivision matrix $\boldsymbol{A}$:

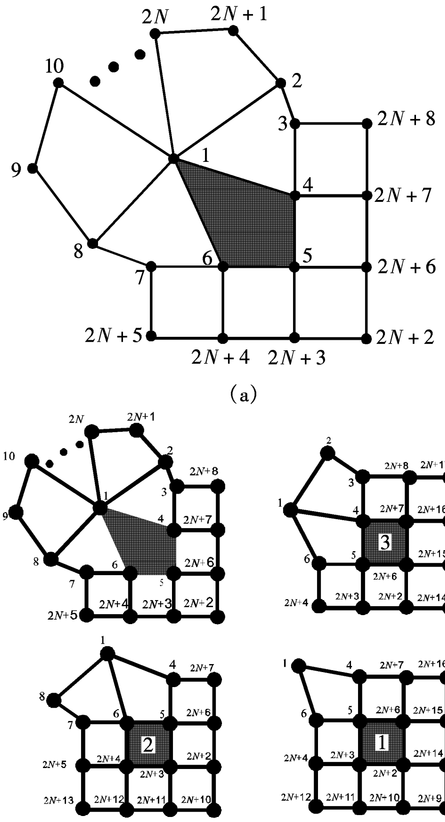
$$\boldsymbol{C}_1 = \boldsymbol{A}\boldsymbol{C}_0$$

The additional points needed to evaluate the three B-spline patches are defined using a bigger matrix $\overline{\boldsymbol{A}}$ of size $M \times K$:

$$\overline{\boldsymbol{C}}_1 = \overline{\boldsymbol{A}} \quad \overline{\boldsymbol{C}}_0$$

The matrices $\boldsymbol{A}$ and $\overline{\boldsymbol{A}}$ are defined in Ref. [8].

$$\overline{\boldsymbol{C}}_n = \overline{\boldsymbol{A}}\boldsymbol{C}_{n-1} = \overline{\boldsymbol{A}}\boldsymbol{A}^{n-1}\boldsymbol{C}_0 = \overline{\boldsymbol{A}}\boldsymbol{V}\boldsymbol{\Lambda}^{n-1}\boldsymbol{V}^{-1}\boldsymbol{C}_0$$



(a) (b)

Fig. 4 A patch of Catmull-Clark subdivision with an extraordinary vertex. (a) Control mesh with an extraordinary vertex; (b) Sub-patches after one subdivision

For each subdivision level n, a subset of the vertices of $\overline{\boldsymbol{C}}_n$ becomes the control vertices of three B-spline patches. These control vertices can be defined by selecting 16 control vertices from $\overline{\boldsymbol{C}}_n$ and storing them in picking matrices[8] $\boldsymbol{P}_k$.

Each sub-patch corresponding to each control vertex is defined as

$$S_{k,n}(u, w) = \overline{\boldsymbol{C}}_n^{\mathrm{T}} \boldsymbol{P}_k^{\mathrm{T}} \boldsymbol{b}(u, w)$$

where $(u, w) \in \Omega$ and $k = 1, 2, 3$. The unit square Ω can be parted into an infinite set of tiles Ω_k^n, as shown in Fig. 5.

$$\Omega_1^n = [2^{\frac{1}{n}}, 2^{\frac{1}{n-1}}] \times [0, 2^{\frac{1}{n}}]$$

$$\Omega_2^n = [2^{\frac{1}{n}}, 2^{\frac{1}{n-1}}] \times [2^{\frac{1}{n}}, 2^{\frac{1}{n-1}}]$$

$$\Omega_3^n = [0, 2^{\frac{1}{n}}] \times [2^{\frac{1}{n}}, 2^{\frac{1}{n-1}}]$$

A parameterization for $S(u, w)$ is constructed by defining its restriction to each tile Ω_k^n.

$S(u, w)$ is equal to B-spline patch defined by the control vertices.

$$S(u, w)\,|_{\Omega_k^n} = S_{k,n}(t_{k,n}(u, w)) = \overline{\boldsymbol{C}}_n^{\mathrm{T}}\boldsymbol{P}_k^{\mathrm{T}}\boldsymbol{b}(t_{k,n}(u, w)) =$$

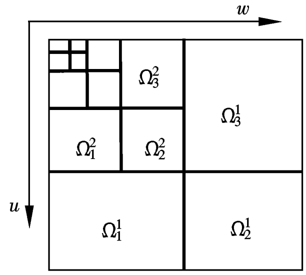


Fig. 5 Partition of unit square into an infinite family of tiles

$$\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_k^{\mathrm{T}}\boldsymbol{b}(t_{k,n}(u, w)) \tag{2}$$

where $\boldsymbol{X}_k = \boldsymbol{P}_k\overline{\boldsymbol{A}}\boldsymbol{V}$.

The transformation $t_{k,n}$ maps the tile Ω_k^n onto the unit square Ω.

$$t_{1,n}(u, w) = (2^n u - 1, 2^n w)$$

$$t_{2,n}(u, w) = (2^n u - 1, 2^n w - 1)$$

$$t_{3,n}(u, w) = (2^n u, 2^n w - 1)$$

2.2 Exact energy analysis of regular patch of Catmull-Clark subdivision

As regular patch of the Catmull-Clark subdivision is uniform bi-cubic B-spline surface, its energy E_{I} can be evaluated as

$$E_{\mathrm{Im}} = \int_0^1\int_0^1 (\| S_u(u, w) \|^2 + \| S_w(u, w) \|^2)\,\mathrm{d}u\mathrm{d}w$$

$$= \boldsymbol{C}_0^{\mathrm{T}}\Big[\int_0^1\int_0^1 (\boldsymbol{B}_u\boldsymbol{B}_u^{\mathrm{T}} + \boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}})\,\mathrm{d}u\mathrm{d}w\Big]\boldsymbol{C}_0$$

where

$$\boldsymbol{B}_u = \boldsymbol{b}_u(u, w) = \{b_{u,1}(u, w), b_{u,2}(u, w), \ldots, b_{u,16}(u, w)\}^{\mathrm{T}}$$

$$b_{u,1}(u, w) = \frac{1}{36}(-3 + 6u - 3u^2)(1 - 3w + 3w^2 - w^3)$$

Similarly, $\boldsymbol{B}_w, \boldsymbol{B}_{uu}, \boldsymbol{B}_{uw}, \boldsymbol{B}_{ww}$ can also be written.

Energy of regular patch of Catmull-Clark Subdivision E_{I} can be calculated as

$$E_{\mathrm{I}} = E_{\mathrm{Im}} + E_{\mathrm{Ip}} = \boldsymbol{C}_0^{\mathrm{T}}\,\boldsymbol{Q}_{\mathrm{I}}\boldsymbol{C}_0 \tag{3}$$

where $\boldsymbol{Q}_{\mathrm{I}} = \Big[\int_0^1\int_0^1 (\boldsymbol{B}_u\boldsymbol{B}_u^{\mathrm{T}} + \boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}} + \boldsymbol{B}_{uu}\boldsymbol{B}_{uu}^{\mathrm{T}} + 2\boldsymbol{B}_{uw}\boldsymbol{B}_{uw}^{\mathrm{T}} + \boldsymbol{B}_{ww}\boldsymbol{B}_{ww}^{\mathrm{T}})\,\mathrm{d}u\mathrm{d}w\Big]_{16\times16}$.

2.3 Exact energy analysis of patch of Catmull-Clark subdivision with an extraordinary point

Although the Catmull-Clark subdivision surface with an extraordinary point cannot be a parameterization, the energy of a patch of the Catmull-Clark subdivision with an extraordinary point can be evaluated by a sub-patch. As an example, the following shows how to get an exact value of $\iint \| S_u(u, w) \|^2\mathrm{d}u\mathrm{d}w$ of the patch with an extraordinary point shown in Fig. 4. After one subdivision, the patch is divided into four sub-patches. The sub-patch is subdivided recursively. The area of the sub-patch that contains an extraordinary point becomes smaller and smaller. Obviously, the integral may be done in three parts: Ω_1^n, Ω_2^n and Ω_3^n, as shown in Fig. 5.

According to $S(u,w)|_{\Omega_k^n} = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_k^{\mathrm{T}}\cdot\boldsymbol{b}(t_{k,n}(u,w))$, on Ω_1^n:

$$S_\alpha = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_k^{\mathrm{T}}\boldsymbol{b}_\alpha(2^n u-1, 2^n w)$$

$$S_\beta = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_k^{\mathrm{T}}\boldsymbol{b}_\beta(2^n u-1, 2^n w)$$

where $\alpha = 2^n u - 1, \beta = 2^n w$.

The integral of $\iint \| S_u(u,w)\|^2 \mathrm{d}u\mathrm{d}w$ on these $\Omega_1^n(n=1,2,3,\ldots)$ can be obtained by Eq. (2).

On Ω_1^1:

$$\iint_{\Omega_1^1} \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w = \iint_{\Omega_1^1} S_\alpha S_\alpha^{\mathrm{T}}\mathrm{d}u\mathrm{d}w =$$

$$\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-\mathrm{T}}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_1^{\mathrm{T}}\left(\int_{\frac{1}{2}}^{1}\int_0^{\frac{1}{2}}\boldsymbol{b}_\alpha(2u-1,2w)\cdot\right.$$

$$\left.\boldsymbol{b}_\alpha^{\mathrm{T}}(2u-1,2w)\mathrm{d}u\mathrm{d}w\right)\boldsymbol{X}_1\boldsymbol{\Lambda}^{n-1}\boldsymbol{V}^{-1}\boldsymbol{C}_0 =$$

$$\frac{1}{4}\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_1^{\mathrm{T}}\left(\int_0^1\int_0^1\boldsymbol{b}_\alpha(u,w)\cdot\right.$$

$$\left.\boldsymbol{b}_\alpha^{\mathrm{T}}(u,w)\mathrm{d}\alpha\mathrm{d}\beta\right)\boldsymbol{X}_1\boldsymbol{\Lambda}^{n-1}\boldsymbol{V}^{-1}\boldsymbol{C}_0 =$$

$$\frac{1}{4}\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_1^{\mathrm{T}}\left(\int_0^1\int_0^1\boldsymbol{b}_u(u,w)\cdot\right.$$

$$\left.\boldsymbol{b}_u^{\mathrm{T}}(u,w)\mathrm{d}u\mathrm{d}w\right)X_1\boldsymbol{\Lambda}^{n-1}\boldsymbol{V}^{-1}\boldsymbol{C}_0 =$$

$$\frac{1}{4}\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_1^{\mathrm{T}}\left(\int_0^1\int_0^1 B_u B_u^{\mathrm{T}}\,\mathrm{d}u\mathrm{d}w\right)\cdot$$

$$X_1\boldsymbol{\Lambda}^{n-1}\boldsymbol{V}^{-1}\boldsymbol{C}_0$$

On Ω_1^2:

$$\iint_{\Omega_1^2} \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w =$$

$$\frac{1}{4^2}\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^1\boldsymbol{X}_1^{\mathrm{T}}\left(\int_0^1\int_0^1\boldsymbol{B}_u\boldsymbol{B}_u^{\mathrm{T}}\,\mathrm{d}u\mathrm{d}w\right)\boldsymbol{X}_1\boldsymbol{\Lambda}^1\boldsymbol{V}^{-1}\boldsymbol{C}_0$$

On Ω_1^n:

$$\iint_{\Omega_1^n} \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w =$$

$$\frac{1}{4^n}\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\boldsymbol{\Lambda}^{n-1}\boldsymbol{X}_1^{\mathrm{T}}\left(\int_0^1\int_0^1\boldsymbol{B}_u\boldsymbol{B}_u^{\mathrm{T}}\,\mathrm{d}u\mathrm{d}w\right)X_1\boldsymbol{\Lambda}^{n-1}\boldsymbol{V}^{-1}\boldsymbol{C}_0$$

Thus, summing the above equations yields integral on Ω_1.

$$\iint_{\Omega_1} \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w =$$

$$\lim_{t_1\to\infty}\sum_{n=1}^{t_1}\left[\iint_{\Omega_1^n} \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w\right] =$$

$$\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\left[\frac{(\boldsymbol{D}_{u1})_{ij}}{4-\lambda_i\lambda_j}\right]\boldsymbol{V}^{-1}\boldsymbol{C}_0$$

where $\boldsymbol{D}_{u1} = \boldsymbol{X}_1^{\mathrm{T}}\left(\int_0^1\int_0^1\boldsymbol{B}_u\boldsymbol{B}_u^{\mathrm{T}}\,\mathrm{d}u\mathrm{d}w\right)\boldsymbol{X}_1$.

Similarly

$$\iint_{\Omega_k} \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-\mathrm{T}}\left[\frac{(\boldsymbol{D}_{uk})_{ij}}{4-\lambda_i\lambda_j}\right]\boldsymbol{V}^{-1}\boldsymbol{C}_0$$

where λ_i is the i-th eigenvalue of the subdivision matrix $\boldsymbol{A}$, $\boldsymbol{D}_{uk} = \boldsymbol{X}_k^{\mathrm{T}}\left(\int_0^1\int_0^1\boldsymbol{B}_u\boldsymbol{B}_u^{\mathrm{T}}\,\mathrm{d}u\mathrm{d}w\right)\boldsymbol{X}_k$.

Thus, summing the above equations yields integral of $\iint \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w$ on Ω.

$$\iint \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w =$$

$$\sum_{k=1}^{3}\left[\sum_{n=1}^{\infty}\left(\iint_{\Omega_k^n} \| S_u(u,w)\|^2\mathrm{d}u\mathrm{d}w\right)\right] =$$

$$\sum_{k=1}^{3}\left(\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{V}^{-T}\left[\frac{(D_{uk})_{ij}}{4-\lambda_i\lambda_j}\right]\boldsymbol{V}^{-1}\boldsymbol{C}_0\right) = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{Q}_u\boldsymbol{C}_0$$

where $\boldsymbol{Q}_u = \sum_{k=1}^{3}\left[\boldsymbol{V}^{-\mathrm{T}}\left(\frac{(\boldsymbol{D}_{uk})_{ij}}{4-\lambda_i\lambda_j}\right)\boldsymbol{V}^{-1}\right]$.

Similarly, $\iint\|S_w(u,w)\|^2\mathrm{d}u\mathrm{d}w$, $\iint\|S_{uu}(u,w)\|^2\mathrm{d}u\mathrm{d}w$, $\iint\|S_{uw}(u,w)\|^2\mathrm{d}u\mathrm{d}w$ and $\iint\|S_{ww}(u,w)\|^2\mathrm{d}u\mathrm{d}w$ can be exactly evaluated, thus the energy of the patch with an extraordinary point E_{II} can be obtained.

$$E_{\mathrm{II}} = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{Q}_{\mathrm{II}}\boldsymbol{C}_0 \tag{4}$$

$$\boldsymbol{Q}_{\mathrm{II}} = \sum_{k=1}^{3}\left[\boldsymbol{V}^{-\mathrm{T}}\left(\frac{(\boldsymbol{D}_k)_{ij}}{4-\lambda_i\lambda_j}\right)\boldsymbol{V}^{-1}\right]$$

where $\boldsymbol{D}_k = (\boldsymbol{P}_k\bar{\boldsymbol{A}}\boldsymbol{V})^{\mathrm{T}}\left[\int_0^1\int_0^1(\boldsymbol{B}_u\boldsymbol{B}_u^{\mathrm{T}}+\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}+\boldsymbol{B}_{uu}\boldsymbol{B}_{uu}^{\mathrm{T}}+2\boldsymbol{B}_{uw}\boldsymbol{B}_{uw}^{\mathrm{T}}+\boldsymbol{B}_{ww}\boldsymbol{B}_{ww}^{\mathrm{T}})\mathrm{d}u\mathrm{d}w\right](\boldsymbol{P}_k\bar{\boldsymbol{A}}\boldsymbol{V})$, $\boldsymbol{D}_k$ is independent of parameter (u, w), and it may be pre-computed.

2.4 Total energy evaluation of Catmull-Clark subdivision surfaces

According to energy analysis in sections 2.2 and 2.3, the energy can be evaluated on both regular and non-regular patches of the Catmull-Clark subdivision surface. The total energy of the Catmull-Clark subdivision surfaces E can be evaluated as

$$E = \sum_{i=1}^{d}((\boldsymbol{R}_i\boldsymbol{C})^{\mathrm{T}}\boldsymbol{Q}_i(\boldsymbol{R}_i\boldsymbol{C})) = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C} \tag{5}$$

where $\boldsymbol{C} = \{V_1, V_2, \ldots, V_m\}^{\mathrm{T}}$ is the column vector composed of m control points of the Catmull-Clark subdivision surfaces; d is the number of patches composed of m control points; $\boldsymbol{R}_i$ is a $K\times m$ picking matrix, in which each row has only one non-zero entry that equals one and entries all equal zero; $\boldsymbol{Q}_i$ is a $K\times K$ matrix, $\boldsymbol{Q}_{\mathrm{II}}$ or $\boldsymbol{Q}_{\mathrm{I}}$; $\boldsymbol{Q}$ is an $m\times m$ matrix.

3 Application

In this part, the minimum energy of the Catmull-

Clark subdivision surfaces is used as a fairness norm for blending patches.

There are many methods to blend patches[9]. Here, the subdivision surface is adopted to blend patches.

It is supposed that the base patches are bi-cubic B-spline patches which are defined by an $s \times t$ control net. The k-th patch is denoted by P^k and its control vertex is denoted by $\{v_{ij}^k\}(i = 1, 2, \ldots, s; j = 1, 2, \ldots, t)$. The directions of i and j are shown in Fig. 6(a). The blending problem can be described as constructing a Catmull-Clark subdivision surface to join B-spline patches.

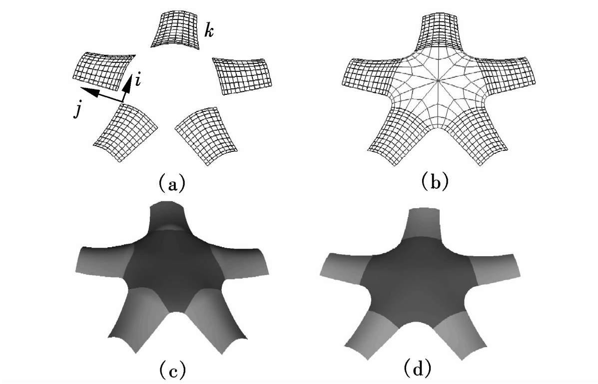


Fig. 6 Blending 5 patches. (a) B-spline control net of 5 base patches; (b) Initial mesh of blending subdivision surface; (c) Blending effect without optimizing; (d) Blending effect by optimizing

3.1 Creation of initial mesh of subdivision surface

In order to generate a fairing subdivision blending surface, it is required to construct the initial mesh of subdivision surfaces meeting the fairing conditions[10]. Firstly, according to C^2 continuity of joining with each patch P^k, let the initial mesh contain the inner three rows of the control vertex of P^k(i. e., $\{v_{ij}^k\}$, $i = 1, 2, 3; j = 1, 2, \ldots, t$). Next, add some new vertices $(\ldots, v_2^k, v_1^k, v_0, v_1^{k-1}, v_2^{k-1}, \ldots)$ that formed a subdivision surface patch with the smallest energy.

3.2 Optimizing of new vertices of initial mesh

If there is no fairing restriction, the new vertex can be chosen freely. In this paper, a fair blending subdivision surface is guaranteed by norm of minimizing the energy of the subdivision surface. For the convenience of description, renumber the control vertices as $V_1, V_2, \ldots, V_m$. By the analysis in section 2, the target function of fair blending subdivision surface is expressed as

$$\min: E = q_{11} V_1^2 + q_{12} V_1 V_2 + q_{13} V_1 V_3 + \ldots + q_{ii} V_i^2 + \ldots + q_{mm} V_m^2$$

Additional constraints: the positions of the outer three rows of the new filling vertices are the same as the positions of inner three rows of the control vertices of the base patches.

According to $\partial E / \partial V_i = 0$ and additional constraints, the liner system that contains the new control vertices is obtained. The reasonable positions of new control vertices of fair blending subdivision surfaces are obtained through resolving this linear system.

3.3 Example of fair blending subdivision surface

Fig. 6 illustrates the process of blending 5 bi-cubic B-spline patches which are defined by 13×9 control net (see Fig. 6(a)). Two rows of the new control vertices of subdivision surfaces are inserted; i. e., the control net adds 66 new control vertices (see Fig. 6 (b)). The blending effect without optimizing is shown in Fig. 6(c). Contrastively, the optimization blending effect shown in Fig. 6(d) is better than in Fig. 6(c). In Fig. 6(d), the positions of these 66 new vertices are calculated by the energy optimization of subdivision surfaces. The control vertices that are contained in the energy optimizing system are shown in Fig. 6(b). The blending subdivision surface is built by removing boundary subdivision approach[10].

4 Conclusion

Unlike the existing methods in which the evaluation of subdivision surface energy diverges when the patch of subdivision surfaces with an extraordinary point, the new method presented in this paper has given analytical formulae for computing the energy of the Catmull-Clark subdivision surfaces, so it not only always converges but can also obtain exact results in evaluating the energy of the subdivision surfaces, even if the subdivision surface contains extraordinary vertices.

The application result of energy computing of the Catmull-Clark subdivision surfaces has demonstrated that the new method proposed in this paper is efficient. In addition, the new method may easily be adapted for computing the energy of other types of subdivision surfaces such as loop subdivisions and so on.

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Catmull-Clark 细分曲面能量精确分析方法

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摘要:提出了一种新的精确计算包含奇异顶点的 Catmull-Clark 细分曲面能量的方法. 通过把 Catmull-Clark 细分曲面片转化成双 3 次 B 样条曲面片,导出了任意拓扑结构的细分曲面完整的能量计算公式. 该方法不需要对细分曲面进行递归细分,就能计算出细分曲面的膜能量和薄板能量,与现有方法相比,该方法能快速准确地计算出细分曲面的能量. 曲面光顺实例表明,用该方法计算细分曲面能量高效可行.

关键词: Catmull-Clark 细分;曲面能量;曲面光顺

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