Journal of Southeast University (English Edition) Vol. 21, No. 4, pp. 459 – 462 Dec. 2005 ISSN 1003—7985

Theoretical research on structural damage alarming of long-span bridges using wavelet packet analysis

Ding Youliang Li Aiqun Miao Changqing

(College of Civil Engineering, Southeast University, Nanjing 210096, China)

Abstract: The state equation and observation equation of the structural dynamic systems under various analysis scales are derived based on wavelet packet analysis. The time-frequency properties of structural dynamic response under various scales are further formulated. The theoretical analysis results reveal that the wavelet packet energy spectrum (WPES) obtained from wavelet packet decomposition of structural dynamic response will detect the presence of structural damage. The sensitivity analysis of the WPES to structural damage and measurement noise is also performed. The transfer properties of the structural system matrix and the observation noise under various analysis scales are formulated, which verify the damage alarming reliability using the proposed WPES with preferable damage sensitivity and noise robusticity.

Key words: structural damage alarming; wavelet packet analysis; wavelet packet energy spectrum; long-span bridge

Recently the damage identification using structural dynamic responses based on the wavelet packet energy spectrum (WPES) has received growing attention in the field of structural health monitoring of civil engineering structures[1-7]. Numerical simulation results show that the wavelet packet energy spectrum can represent structural damage status effectively and efficiently through energy variations of structural dynamic responses decomposed by wavelet packet analysis; i. e., such tiny structural damages are often discernible from wavelet packet analysis, although they are not sensitive to existing methods such as the modal frequency approach. However, the theoretical basis and innate characteristics of the structural damage identification based on the wavelet packet energy spectrum have never been formulated, which will undoubtedly result in adverse applications in the structural damage alarming of long-span bridges. In present work, the systematic study has developed the theoretical basis for the structural damage alarming by wavelet packet energy spectrum of the decomposed structural dynamic responses using wavelet packet analysis, which is outlined in brief as follows:

1) Applicability analysis of damage alarming: Whether the WPES will detect the occurrence of the structural damage sensitively or not?

2) Sensitivity analysis of damage alarming: Which of the wavelet packet decomposition levels can most sensitively represent the structural damage status?

3) Noise sensitivity analysis of damage alarming: What influence of the measurement noise will be exerted on the WPES?

1 Wavelet Packet Energy Spectrum

In this section the state equation and observation equation of dynamic structures under various analysis scales are derived, firstly, based on wavelet packet analysis[8]. Assuming that one accelerometer is installed on the structure to measure its acceleration response, the state equation and observation equation of the structure with n degrees of freedom can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{F}(t) \tag{1}$$

$$\boldsymbol{f}(t)=\boldsymbol{T}\ddot{\boldsymbol{x}}(t)+\boldsymbol{v}(t) \tag{2}$$

where $\boldsymbol{M}\in\mathbf{R}^{n\times n}$, $\boldsymbol{C}\in\mathbf{R}^{n\times n}$ and $\boldsymbol{K}\in\mathbf{R}^{n\times n}$ are the mass, damping and stiffness matrices of the structure, respectively; $\boldsymbol{x}\in\mathbf{R}^{n\times 1}$ and $\boldsymbol{F}\in\mathbf{R}^{n\times 1}$ are the displacement vector and the force vector, respectively; $\boldsymbol{T}\in\mathbf{R}^{p\times n}$ and $\boldsymbol{v}\in\mathbf{R}^{p\times 1}$ are the sensor location vector and the measurement noise vector, respectively; $\boldsymbol{f}(t)$ is the measured acceleration output at the sensor location. These equations can be rewritten in the state-space form as

$$\dot{\bar{\boldsymbol{x}}}(t)=\boldsymbol{A}\bar{\boldsymbol{x}}(t)+\boldsymbol{B}\bar{\boldsymbol{F}}(t) \tag{3}$$

$$\boldsymbol{f}(t)=\bar{\boldsymbol{C}}\bar{\boldsymbol{x}}(t)+\bar{\boldsymbol{D}}\bar{\boldsymbol{F}}(t)+\boldsymbol{v}(t) \tag{4}$$

where $\bar{\boldsymbol{x}}(t)=\begin{Bmatrix}\boldsymbol{x}(t)\\ \dot{\boldsymbol{x}}(t)\end{Bmatrix}$, $\boldsymbol{A}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C}\end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ -\boldsymbol{M}^{-1} & \boldsymbol{0}\end{bmatrix}$, $\bar{\boldsymbol{C}}=[-\boldsymbol{T}\boldsymbol{M}^{-1}\boldsymbol{K}\quad -\boldsymbol{T}\boldsymbol{M}^{-1}\boldsymbol{C}]$, $\bar{\boldsymbol{D}}=$

Received 2005-03-01.

Foundation item: The National Natural Science Foundation of China (No. 50378017).

Biographies: Ding Yonliang (1979—), male, graduate; Li Aiqun (corresponding author), male, doctor, professor, aiqunli@ public1. ptt. js. cn.

$[\boldsymbol{TM}^{-1} \quad \boldsymbol{0}], \overline{\boldsymbol{F}}(t) = \begin{Bmatrix} \boldsymbol{F}(t) \\ \boldsymbol{0} \end{Bmatrix}$. The state-space Eq. (3) and Eq. (4) can be expressed in the discrete form as

$$\overline{\boldsymbol{x}}(N,k+1) = \overline{\boldsymbol{A}}(N)\overline{\boldsymbol{x}}(N,k) + \overline{\boldsymbol{B}}(N)\overline{\boldsymbol{F}}(N,k) \qquad k \in \mathbf{Z}^+ \tag{5}$$

$$\boldsymbol{f}(N,k) = \overline{\boldsymbol{C}}(N)\overline{\boldsymbol{x}}(N,k) + \overline{\boldsymbol{D}}(N)\overline{\boldsymbol{F}}(N,k) + \boldsymbol{v}(N,k) \qquad k \in \mathbf{Z}^+ \tag{6}$$

where $\overline{\boldsymbol{A}} = \mathrm{e}^{A\Delta t}$, $\overline{\boldsymbol{B}} = \int_{t-\Delta t}^{t} \mathrm{e}^{A(t-\tau)}\boldsymbol{B}\mathrm{d}\tau = \int_0^{\Delta t} \mathrm{e}^{A\tau}\boldsymbol{B}\mathrm{d}\tau$, Δt is the time step increment and N is the corresponding analysis scale.

Assume that the state equation and observation equation of the structure under wavelet packet analysis scale i ($1 \leqslant i \leqslant N$) are expressed as

$$\overline{\boldsymbol{x}}(i,k+1) = \overline{\boldsymbol{A}}(i)\overline{\boldsymbol{x}}(i,k) + \overline{\boldsymbol{B}}(i)\overline{\boldsymbol{F}}(i,k) \qquad k \in \mathbf{Z}^+ \tag{7}$$

$$\boldsymbol{f}(i,k) = \overline{\boldsymbol{C}}(i)\overline{\boldsymbol{x}}(i,k) + \overline{\boldsymbol{D}}(i)\overline{\boldsymbol{F}}(i,k) + \boldsymbol{v}(i,k) \qquad k \in \mathbf{Z}^+ \tag{8}$$

Through wavelet packet multiscale decomposition from scale i to scale $i-1$, the state equation and observation equation of the structure on coarse scale space $\boldsymbol{V}_{i-1}$ (low-frequency band) can be expressed as

$$\begin{aligned}\overline{\boldsymbol{x}}_V^i(i-1,k+1) &= \sum_l h(l)\overline{\boldsymbol{x}}(i,2k-l+2) = \\ &\sum_l h(l)\overline{\boldsymbol{A}}(i)\overline{\boldsymbol{x}}(i,2k-l+1) + \\ &\sum_l h(l)\overline{\boldsymbol{B}}(i)\overline{\boldsymbol{F}}(i,2k-l+1) = \overline{\boldsymbol{A}}_V^i(i-1) \cdot \\ &\overline{\boldsymbol{x}}_V^i(i-1,k) + \overline{\boldsymbol{B}}_V^i(i-1)\overline{\boldsymbol{F}}_{V,x}^i(i-1,k)\end{aligned} \tag{9}$$

$$\begin{aligned}\boldsymbol{f}_V^i(i-1,k) &= \sum_l h(l)\boldsymbol{f}(i,2k-l) = \\ &\sum_l h(l)[\overline{\boldsymbol{C}}(i)\overline{\boldsymbol{x}}(i,2k-l) + \overline{\boldsymbol{D}}(i)\overline{\boldsymbol{F}}(i,2k-l) + \\ &\boldsymbol{v}(i,2k-l)] = \overline{\boldsymbol{C}}_V^i(i-1)\overline{\boldsymbol{x}}_V^i(i-1,k) + \\ &\overline{\boldsymbol{D}}_V^i(i-1)\overline{\boldsymbol{F}}_{V,f}^i(i-1,k) + \boldsymbol{v}_V^i(i-1,k)\end{aligned} \tag{10}$$

where $h(l)$ are quadrature mirror filters associated with the scaling function, the subscript $\boldsymbol{V}$ denotes the projection of the state vector $\overline{\boldsymbol{x}}(i,k)$ on the coarse scale space $\boldsymbol{V}_{i-1}$ and the superscript i denotes that Eq. (9) and Eq. (10) are obtained through the wavelet packet decomposition from analysis scale i.

$$\overline{\boldsymbol{A}}_V^i(i-1) = \overline{\boldsymbol{A}}(i)\overline{\boldsymbol{A}}(i), \quad \overline{\boldsymbol{B}}_V^i(i-1) = \overline{\boldsymbol{B}}(i) \tag{11}$$

$$\overline{\boldsymbol{F}}_{V,x}^i(i-1,k) = \overline{\boldsymbol{A}}(i)\sum_l h(l)\overline{\boldsymbol{F}}(i,2k-l) + \sum_l h(l)\overline{\boldsymbol{F}}(i,2k-l+1) \tag{12}$$

$$\overline{\boldsymbol{C}}_V^i(i-1) = \overline{\boldsymbol{C}}(i), \quad \overline{\boldsymbol{D}}_V^i(i-1) = \overline{\boldsymbol{D}}(i) \tag{13}$$

$$\left.\begin{aligned}\overline{\boldsymbol{F}}_{V,f}^i(i-1,k) &= \sum_l h(l)\overline{\boldsymbol{F}}(i,2k-l) \\ \boldsymbol{v}_V^i(i-1,k) &= \sum_l h(l)\boldsymbol{v}(i,2k-l)\end{aligned}\right\} \tag{14}$$

Similarly, the state equation and observation equation of the structure on the fine scale space $\boldsymbol{W}_{i-1}$ (high-frequency band) can be expressed as

$$\begin{aligned}\overline{\boldsymbol{x}}_W^i(i-1,k+1) &= \sum_l g(l)\overline{\boldsymbol{x}}(i,2k-l+2) = \\ &\overline{\boldsymbol{A}}_W^i(i-1)\overline{\boldsymbol{x}}_W^i(i-1,k) + \overline{\boldsymbol{B}}_W^i(i-1)\overline{\boldsymbol{F}}_{W,x}^i(i-1,k)\end{aligned} \tag{15}$$

$$\begin{aligned}\boldsymbol{f}_W^i(i-1,k) &= \sum_l g(l)\boldsymbol{f}(i,2k-l) = \\ &\overline{\boldsymbol{C}}_W^i(i-1)\overline{\boldsymbol{x}}_W^i(i-1,k) + \overline{\boldsymbol{D}}_W^i(i-1) \cdot \\ &\overline{\boldsymbol{F}}_{W,f}^i(i-1,k) + \boldsymbol{v}_W^i(i-1,k)\end{aligned} \tag{16}$$

where $g(l)$ are quadrature mirror filters associated with the wavelet function and the subscript $\boldsymbol{W}$ denotes the projection of the state vector $\overline{\boldsymbol{x}}(i,k)$ on the fine scale space $\boldsymbol{W}_{i-1}$.

$$\overline{\boldsymbol{A}}_W^i(i-1) = \overline{\boldsymbol{A}}(i)\overline{\boldsymbol{A}}(i), \quad \overline{\boldsymbol{B}}_W^i(i-1) = \overline{\boldsymbol{B}}(i) \tag{17}$$

$$\overline{\boldsymbol{F}}_{W,x}^i(i-1,k) = \overline{\boldsymbol{A}}(i)\sum_l g(l)\overline{\boldsymbol{F}}(i,2k-l) + \sum_l g(l)\overline{\boldsymbol{F}}(i,2k-l+1) \tag{18}$$

$$\overline{\boldsymbol{C}}_W^i(i-1) = \overline{\boldsymbol{C}}(i), \quad \overline{\boldsymbol{D}}_W^i(i-1) = \overline{\boldsymbol{D}}(i) \tag{19}$$

$$\left.\begin{aligned}\overline{\boldsymbol{F}}_{W,f}^i(i-1,k) &= \sum_l g(l)\overline{\boldsymbol{F}}(i,2k-l) \\ \boldsymbol{v}_W^i(i-1,k) &= \sum_l g(l)\boldsymbol{v}(i,2k-l)\end{aligned}\right\} \tag{20}$$

From the aforementioned structural multiscale decomposition, it can be seen that after performing wavelet packet decomposition of structural dynamic response $\boldsymbol{f}(N,k)$, a series of component signals $\boldsymbol{f}(i,k)$ ($1 \leqslant i \leqslant N$) will indicate the dynamic properties of the structural system in various frequency bands. Therefore, the structural damage will suppress or enhance some component signals in special frequency bands. Hence, the energy distribution of component signals in various frequency bands contains ample information of structural damage; i.e., the structural damage can cause energy increase of some component signals or energy decrease of other component signals. Accordingly, the energy variation of one or several component signals can provide an effective indication for changes in the structural system. Hence, the energy distribution of structural dynamic responses decomposed using wavelet packet analysis in various frequency bands, namely, the WPES, is extracted as the index vector for structural damage alarming[4-6]. Let $\boldsymbol{f}$ denote the original signal of structural dynamic response, and then it can be expressed as

$$\boldsymbol{f} = \sum_k \boldsymbol{f}_{j,k} = \boldsymbol{f}_{j,0} + \boldsymbol{f}_{j,1} + \ldots + \boldsymbol{f}_{j,2^j-1} \qquad k = 0,1,2,\ldots,2^j-1 \tag{21}$$

where $\boldsymbol{f}_{j,k}$ are the component signals with orthogonal frequency width $\boldsymbol{f}_{\mathrm{m}}/2^j$ (here $\boldsymbol{f}_{\mathrm{m}}$ denotes the highest analytical frequency of the original signal $\boldsymbol{f}$); j indicates the wavelet packet decomposition level. The energy of these component signals $\boldsymbol{f}_{j,k}$ is obtained as

$$E_{j,k} = \int |\boldsymbol{f}_{j,k}(t)|^2 \mathrm{d}t \tag{22}$$

Hence, wavelet packet energy spectrum $\boldsymbol{E}_j$ of structural dynamic response $\boldsymbol{f}$ at decomposition level j can be expressed as

$$\boldsymbol{E}_j = \{E_{j,k}\} \qquad k = 0, 1, 2, \dots, 2^j - 1 \tag{23}$$

2 Damage Sensitivity Analysis

In this section, a damage sensitivity analysis is performed on the proposed wavelet packet energy spectrum. Assuming that structural damage only causes the stiffness loss in one or more elements of a structure, but not a loss in the mass, the stiffness loss $\Delta\boldsymbol{K}$ of a structure is expressed as

$$\Delta\boldsymbol{K} = \sum_{j=1}^{n_e} \alpha_j \boldsymbol{K}_j \tag{24}$$

where $\boldsymbol{K}_j$ is the stiffness matrix of the j-th element, α_j is the stiffness reduction factor ranging between -1 and 0 for the j-th element, and n_e is the number of structural elements.

Hence the system matrix $\overline{\boldsymbol{A}}^{\mathrm{d}}(N)$, control matrix $\overline{\boldsymbol{B}}^{\mathrm{d}}(N)$ and output matrix $\overline{\boldsymbol{C}}^{\mathrm{d}}(N)$ in Eq. (5) and Eq. (6) of a damaged structure can be rewritten as

$$\overline{\boldsymbol{A}}^{\mathrm{d}}(N) = \mathrm{e}^{(A+\Delta A)\Delta t}, \quad \overline{\boldsymbol{B}}^{\mathrm{d}}(N) = \int_0^{\Delta t} \mathrm{e}^{(A+\Delta A)\tau} \boldsymbol{B} \mathrm{d}\tau \tag{25}$$

$$\overline{\boldsymbol{C}}^{\mathrm{d}}(N) = [-\boldsymbol{T}\boldsymbol{M}^{-1}(\boldsymbol{K}+\Delta\boldsymbol{K}) \quad -\boldsymbol{T}\boldsymbol{M}^{-1}\boldsymbol{C}] \tag{26}$$

where

$$\Delta\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\Delta\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \tag{27}$$

As discussed in section 3, when the structure is multiscale decomposed by wavelet packet analysis from scale i to scale $i-1$, the control matrices $\overline{\boldsymbol{B}}(i-1)$, $\overline{\boldsymbol{D}}(i-1)$ and output matrix $\overline{\boldsymbol{C}}(i-1)$ under analysis scale $i-1$ are the same as those under analysis scale i, while the system matrix $\overline{\boldsymbol{A}}(i-1)$ under analysis scale $i-1$ is changed as

$$\overline{\boldsymbol{A}}_V^i(i-1) = \overline{\boldsymbol{A}}(i)\overline{\boldsymbol{A}}(i), \quad \overline{\boldsymbol{A}}_W^i(i-1) = \overline{\boldsymbol{A}}(i)\overline{\boldsymbol{A}}(i) \tag{28}$$

Thus, the system matrix $\overline{\boldsymbol{A}}^{\mathrm{d}}(i)$ of the damaged structure under analysis scale i can be obtained through the following iterations:

$$\overline{\boldsymbol{A}}^{\mathrm{d}}(i) = \overline{\boldsymbol{A}}^{\mathrm{d}}(i+1)\overline{\boldsymbol{A}}^{\mathrm{d}}(i+1) = \dots = \mathrm{e}^{2^{(N-i)}(A+\Delta A)\Delta t} \tag{29}$$

Through the derivation of Eq. (29) to $\Delta\boldsymbol{A}$, the sensitivity coefficients of $\overline{\boldsymbol{A}}^{\mathrm{d}}(i)$ can be obtained as

$$\frac{\mathrm{d}\overline{\boldsymbol{A}}^{\mathrm{d}}(i)}{\mathrm{d}\Delta\boldsymbol{A}} = \frac{\mathrm{d}\mathrm{e}^{2^{(N-i)}(A+\Delta A)\Delta t}}{\mathrm{d}\Delta\boldsymbol{A}} = \frac{\mathrm{d}\mathrm{e}^{2^{(N-i)}(A+\Delta A)\Delta t}}{\mathrm{d}\Delta t}\frac{\mathrm{d}\Delta t}{\mathrm{d}\Delta\boldsymbol{A}} = 2^{(N-i)}(\boldsymbol{A}+\Delta\boldsymbol{A})\mathrm{e}^{2^{(N-i)}(A+\Delta A)\Delta t}\frac{\mathrm{d}\Delta t}{\mathrm{d}\Delta\boldsymbol{A}} \tag{30}$$

Similarly, the sensitivity coefficients of $\overline{\boldsymbol{A}}^{\mathrm{d}}(N)$ to $\Delta\boldsymbol{A}$ can be expressed as

$$\frac{\mathrm{d}\overline{\boldsymbol{A}}^{\mathrm{d}}(N)}{\mathrm{d}\Delta\boldsymbol{A}} = \frac{\mathrm{d}\mathrm{e}^{(A+\Delta A)\Delta t}}{\mathrm{d}\Delta\boldsymbol{A}} = (\boldsymbol{A}+\Delta\boldsymbol{A})\mathrm{e}^{(A+\Delta A)\Delta t}\frac{\mathrm{d}\Delta t}{\mathrm{d}\Delta\boldsymbol{A}} \tag{31}$$

Noticing from Eq. (30) and Eq. (31), it can be seen that the system matrix becomes more sensitive to structural damage with the decrease of analysis scale i. Hence a series of component signals $\boldsymbol{f}_{j,k}$ decomposed from structural response signal $\boldsymbol{f}$ using wavelet packet analysis are more capable of catching the changes in the structural system with the increase of the decomposition level j. Therefore, the wavelet packet energy spectrum is sensitive to tiny structural damage, which is suitable for detecting the initial damage in various engineering structures.

3 Noise Sensitivity Analysis

Assuming that the measurement noise $\boldsymbol{v}(i,k)$ of the structure under analysis scales i is zero-mean stationary Gaussian white noise process, which meets the following statistical characteristics

$$E[\boldsymbol{v}(i,k)] = 0, \quad E[\boldsymbol{v}(i,k)\boldsymbol{v}^{\mathrm{T}}(i,j)] = \boldsymbol{R}(i)\delta_{kj} \tag{32}$$

Through wavelet packet decomposition of $\boldsymbol{v}(i,k)$ from scale i to scale $i-1$, the measurement noise on coarse scale space $\boldsymbol{V}_{i-1}$ and fine scale space $\boldsymbol{W}_{i-1}$ are $\boldsymbol{v}_V^i(i-1,k)$ and $\boldsymbol{v}_W^i(i-1,k)$, respectively. Auto-correlation of component signal $\boldsymbol{v}_V^i(i-1,k)$ can be obtained as[9]

$$E[(\boldsymbol{v}_V^i(i-1,k))(\boldsymbol{v}_V^i(i-1,j))^{\mathrm{T}}] = \frac{1}{2}\boldsymbol{R}(i)\delta_{kj} \tag{33}$$

Similarly, auto-correlation of component signal $\boldsymbol{v}_W^i(i-1,k)$ can be expressed as[9]

$$E[(\boldsymbol{v}_W^i(i-1,k))(\boldsymbol{v}_W^i(i-1,j))^{\mathrm{T}}] = \frac{1}{2}\boldsymbol{R}(i)\delta_{kj} \tag{34}$$

Cross-correlation of component signal $\boldsymbol{v}_V^i(i-1,k)$ and $\boldsymbol{v}_W^i(i-1,j)$ can be determined as[9]

$$E[(\boldsymbol{v}_V^i(i-1,k))(\boldsymbol{v}_W^i(i-1,j))^{\mathrm{T}}] = 0 \tag{35}$$

Noticing from Eq. (33) and Eq. (34), with the increase of the wavelet packet decomposition level, the amplitude of noise in the measurement is effectively reduced. In addition, component signal $\boldsymbol{v}_V^i(i-1,k)$ is uncorrelated with the signal $\boldsymbol{v}_W^i(i-1,j)$ according to Eq. (35). Hence, the wavelet packet energy decomposition of measurement noise has two basic properties. On the one hand, the energy of measurement noise is evenly distributed in all frequency bands; on the other hand, the noise energies in all frequency bands are reduced effectively with the increase of the wavelet packet decomposition level. Therefore, the proposed wavelet packet energy spectrum is sensitive to structural damage but insensitive to measurement noise, which makes it suitable and viable to develop a robust scheme for structural damage alarming.

4 Conclusion

In this paper a thorough investigation on the theoretical applicability of wavelet packet energy spectrum for the purpose of structural damage alarming has been conducted based on the theoretical formulation of wavelet packet multiscale decomposition of structural dynamic systems. The sensitivity analysis of WPES to structural damage and measurement noise is further formulated, which verifies that with the increase of the decomposition level the WPES is more sensitive to tiny structural damage and the energy of measurement noise is effectively reduced and evenly distributed in all frequency bands. Therefore, the proposed wavelet packet energy spectrum is sensitive to structural damage but insensitive to measurement noise, which makes it suitable and viable to develop a robust scheme for detecting the initial damage in long-span bridges.

References

[1] Kim H, Melhem H. Damage detection of structures by wavelet analysis [J]. *Engineering Structures*, 2004, **26**(3): 347 − 362.

[2] Hou Z, Noori M, St Amand R. Wavelet-based approach for structural damage detection [J]. *Journal of Engineering Mechanics*, 2000, **126**(7): 677 − 683.

[3] Wu Y, Du R. Feature extraction and assessment using wavelet packets for monitoring of machining processes [J]. *Mechanical System and Signal Processing*, 1996, **10**(1): 29 − 53. (in Chinese)

[4] Ding Y L, Li A Q, Han X L. Numerical research on real-time structural damage alarming based on wavelet packet analysis [J]. *Journal of Southeast University* (*Natural Science Edition*), 2003, **33**(5): 643 − 646. (in Chinese)

[5] Yan Y J, Yam L H. Online detection of crack damage in composite plates using embedded piezoelectric actuators/sensors and wavelet analysis [J]. *Composite Structures*, 2002, **58**(1): 29 − 38.

[6] He Y, Shen S, Ying H Q, et al. Application of wavelet packet decomposition and its energy spectrum on the fault diagnosis of reciprocation machinery [J]. *Journal of Vibration Engineering*, 2001, **14**(1): 72 − 75. (in Chinese)

[7] Sun Z, Chang C C. Structural damage assessment based on wavelet packet transform [J]. *Journal of Structural Engineering*, 2002, **128**(10): 1354 − 1361.

[8] Chui C K. *An introduction to wavelets* [M]. San Diego: Academic Press, 1992. 1 − 22.

[9] Wen Chenglin. Research on theory and method of multi-resolution estimation [D]. Xi' an: Department of Automatic Control of Northwestern Polytechnical University, 1999. (in Chinese)

基于小波包分析的大跨桥梁结构损伤预警的理论研究

丁幼亮　李爱群　缪长青

(东南大学土木工程学院,南京 210096)

摘要:基于小波包分析推导了结构动力系统在不同分析尺度上的状态方程和观测方程,在此基础上研究了结构动力响应在不同分析尺度上的时-频特性.理论分析证明,采用小波包分解结构的动力响应得到小波包能量谱,将可以表征结构损伤的发生.在此基础上进行了小波包能量谱关于结构损伤和观测噪声的敏感性研究,分析了结构系统矩阵和测量噪声在各个分析尺度上的递推性.结果表明,采用结构动力响应的小波包能量谱进行结构损伤预警具有较好的损伤敏感性和噪声鲁棒性.

关键词:结构损伤预警;小波包分析;小波包能量谱;大跨桥梁

中图分类号:TU311.41