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Monte Carlo numerical simulation and its application in probability analysis of long span bridges

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Abstract: To get the probability of long span bridges under the influence of external random factors, the Monte Carlo method using Latin hypercube sampling is applied. Combined with the condition assessment system on Runyang Suspension Bridge, which is the longest suspension bridge in China, the structural probabilities in normal and damaged situations are calculated with the external random factors considered including environmental temperature, wind load, load of vehicles, etc. The main assessment items contain the maximal vertical displacement of girder, the maximal stress of cables, the maximal horizontal displacement of towers etc. Finally, the probabilities and their cumulative distribution functions are provided. The analysis results can be plotted on line in a clear and vivid way, so the efficiency of assessment is increased and the decision-making of maintenance is more objective and accurate.

Key words: condition assessment; health monitoring; structure probability; Monte Carlo method

According to the Chinese national code, the unified standard for reliability design of highway structure (GB/T50283—1999), the probabilistic theory has been introduced and the probabilistic design of limit state has been specified to be the general principle of structural design, which is a breakthrough in design theory. However, in the field of condition assessment, especially in condition assessment on long-span bridges, there is no perfect method that can take those nondeterministic factors into consideration accurately[1 – 3], so the influence of those uncertain factors can have hardly been analyzed. Till now, most of these analyses depend upon qualitative estimates by experts; however, for people who undertake daily bridge maintenance, it is hard for them to make quick and reliable judgment upon these uncertain and fuzzy descriptions.

To consider the effects caused by arbitrary factors quantificationally and to provide a reliable basis for structural health monitoring and condition assessment, in the following research, the probability analysis based upon Monte Carlo simulation is used to calculate the structural probability under different situations. The output of analysis results is clear and vivid, so both the efficiency and accuracy of assessment are increased.

1 Monte Carlo Numerical Simulation

1.1 Principle of the Monte Carlo method

The Monte Carlo method is a way to solve practical problems by setting up a mathematical or physical model using statistical sampling theory[4, 5]. Generally, a probabilistic model should be set up first, which is related in some way with the practical problems. Then computers are used to generate enough values of input variables that follow specified distributions, which means enough experiments are made. Because there are similarities between the two sides, then the generated values can be an indication of the variation of random factors in the practical problem. Finally, the calculated eigenvalues of the model (mean value or standard deviation of output variables, etc.) can provide an approximate solution to practical problems.

1.2 Generation of stochastic values

To describe the random input variables, the following distribution functions are specified in Monte Carlo sampling: Gaussian (normal) distribution, truncated Gaussian distribution, lognormal distribution, triangular distribution, uniform distribution, exponential distribution, Beta distribution, Gamma distribution, and Weibull distribution, etc.

When the random values are to be generated, for example, there is a random variable X following Gaussian (normal) distribution, $X \sim N(m, \sigma^2)$, the random values in region $(-\infty, +\infty)$ can be generated by

$$X = m + \sigma\phi^{-1}[\mathrm{rand}(0, 1)] \tag{1}$$

The random values in sub-region (a, b) can be generated by

$$X = m + \sigma\phi^{-1}\left\{\phi\left(\frac{a-m}{\sigma}\right) + \left[\phi\left(\frac{b-m}{\sigma}\right) - \phi\left(\frac{a-m}{\sigma}\right)\right]\mathrm{rand}(0, 1)\right\} \tag{2}$$

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where $\phi(\cdot)$, $\phi^{-1}(\cdot)$ are the cumulative distribution functions of the Gaussian (normal) distribution and its inverse function, respectively.

1.3 Sampling method

According to the magnitude of probability of failure and the complexity of different problems, there are several sampling methods available.

1.3.1 Direct Monte Carlo sampling

The direct Monte Carlo sampling technique is not the most efficient way, due to the fact that the sampling process has no "memory", so we may get a cluster of two (or more) sampling points that occur close to each other (see Fig. 1), which provides no new information.

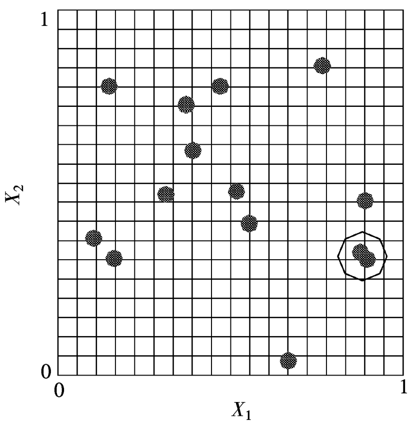

Fig. 1 Direct Monte Carlo sampling

1.3.2 Latin Hypercube sampling (LHS)

Compared to the direct Monte Carlo sampling, the Latin hypercube sampling (LHS) is a more advanced and efficient form. In an LHS, a sample "memory" is used, meaning it avoids repeating samples that have been evaluated before (see Fig. 2).

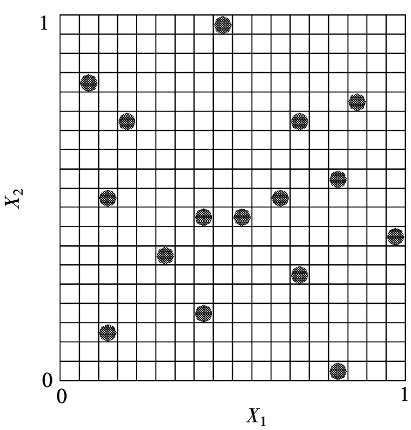

Fig. 2 Latin hypercube sampling

1.3.3 Important sampling

When the probability of failure is small, using the direct Monte Carlo sampling would require too many times of simulations, so we should increase the probability of failure to make the sampling more efficient, meaning that more sampling points should be chosen in the failure region. This is called the important sampling, in this case, the probability of failure is shown in Eq. (3)

$$P_f = \int_{-\infty}^{+\infty} \frac{I[g(v)]f_x(v)}{P_v(v)} P_v(v)\,\mathrm{d}v \tag{3}$$

where $f_X(v)$ is the probability density function of variable V; $P_v(v)$ is the density function of important sampling; $I[g(v)] = \begin{cases} 1 & g(v) \leqslant 0 \\ 0 & g(v) > 0 \end{cases}$. The precision of important sampling depends on the definition of failure region, if the region is not properly defined, the precision of sampling can hardly be achieved.

2 Application in Probability Analysis of Runyang Yangtse River Bridge

Runyang Yangtse River Bridge is a long-span bridge composed of a suspension bridge (with a main span of 1 490 m in length) and a cable stayed bridge (with three spans of 176 m, 406 m, and 176 m, respectively), of which the suspension bridge is the longest one in China and the third in the world. Doing research on it and its condition assessment system will surely make great sense and have practical use.

2.1 Process of probability analysis using Monte Carlo method

The probability analysis of Runyang Yangtse River Bridge using the Monte Carlo method can be done according to the following process, which is shown in Fig. 3.

① Build the model parametrically;

② Assign to parameters the quantities that will be used as random input variables and random output parameters;

③ Specify any correlations between the variables;

④ Specify random output parameters;

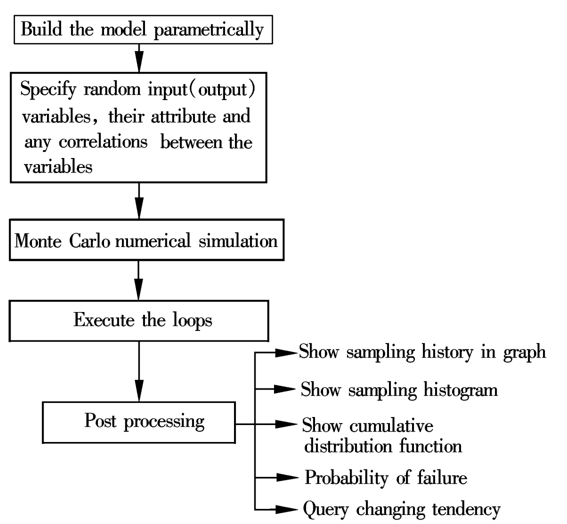

Fig. 3 Process of probability analysis

⑤ Choose the probabilistic design tool or method;

⑥ Execute the loops required for the probabilistic analysis;

⑦ Review results of the probabilistic analysis.

2.2 Specification of input variables

In general, the random input variables can be divided into two types, the load or load effect and the resistance. The first type of input variables used in the condition assessment system of Runyang Bridge (the load or load effect) and their statistical characters are listed in Tab. 1.

Tab. 1 Statistical property of load or load effect

Type of load	Type of load effect	Type of distribution	Mean value/ characteristic value	Deviation factor V	Distribution function
Weight		Gaussian	1.021 2	0.046 2	$F_G(x) = \frac{1}{0.047\,2r_k\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left[-\frac{(u-1.021\,2r_k)^2}{0.004\,5r_k^2}\right]du$
Load of vehicles (dense state)	Moment	Gaussian	0.788 2	0.11	$F_{OM}(x) = \frac{1}{0.085\,3S_{QK}\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left[-\frac{(u-0.788\,2S_{QK})^2}{0.014\,6S_{QK}^2}\right]du$
	Shear		0.709 6	0.10	$F_{OV}(x) = \frac{1}{0.068\,4S_{QK}\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left[-\frac{(u-0.709\,6S_{QK})^2}{0.009\,4S_{QK}^2}\right]du$
Dynamic load of vehicles		Extremum I	1.078	0.030 40	$F(x) = \exp\{-\exp[-\alpha(x-\beta)]\}$
Wind load		Extremum I	1.171	0.162	$F_{WT}(x) = [F_{W_y}(x)]^{100} = \exp\left\{-\exp\left[-\frac{(x-1.086W_K)}{0.148W_K}\right]\right\}$
Temperature effect (southern China)	Highest	Extremum I	34.0 ℃ (mean value)	0.022	$F(x) = \exp\{-\exp[-(x-33.7)/0.58]\}$
	Lowest	Extremum I	−8.5 ℃ (mean value)	0.175	$F(x) = \exp\{-\exp[-(x-7.8)/1.16]\}$

In the same way, the second type of input variables can be specified, including material property, geometrical parameters and analysis mode, etc.

Fig. 4 and Fig. 5 show the characters of samplings and the sampling times-history of variable "structure deadweight". Using the Latin hypercube sampling, altogether there are 30 times of samplings, and all these samplings followed the Gaussian (normal) distribution, which is specified beforehand.

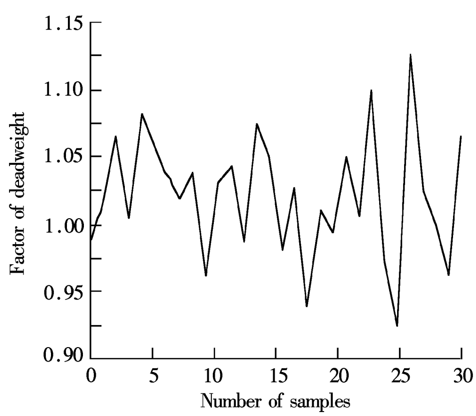

Fig. 4 Sampling histogram Ⅰ of variable "structure deadweight"

2.3 Reliability analysis in normal situations

According to Kececioglu[6], after N times of samplings and calculations, the lower confidence limit of a $1-\alpha$ confidence interval can be determined from Eqs. (4) and (5).

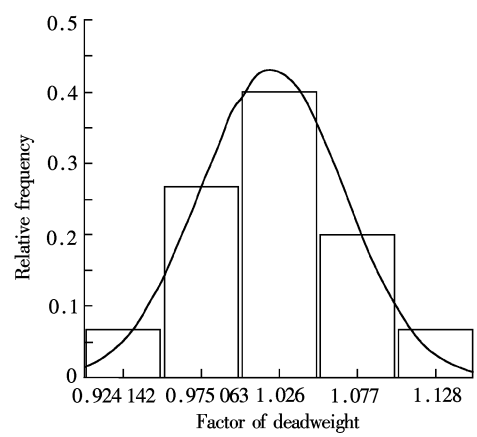

Fig. 5 Sampling histogram Ⅱ of variable "structure deadweight"

$$\sum_{1}^{n}\frac{n!}{(n-k)!\,k!}\langle F_i\rangle_{\frac{\alpha}{2}}^{k}(1-\langle F_i\rangle_{\frac{\alpha}{2}})^{n-k} = \frac{\alpha}{2} \quad (4)$$

$$\sum_{1}^{n}\frac{n!}{(n-k)!\,k!}\langle F_i\rangle_{1-\frac{\alpha}{2}}^{k}(1-\langle F_i\rangle_{1-\frac{\alpha}{2}})^{n-k} = 1-\frac{\alpha}{2} \quad (5)$$

Till now, the cumulative distribution function of sampled data can only be given at the individual sampled values $x_1, x_2, \ldots, x_i, x_{i+1}, \ldots, x_n$. Hence, the evaluation of the probability that the random variable is less than or equal to an arbitrary value x requires an interpolation between the available data points.

If x is between x_i and x_{i+1}, then the probability that the random variable X is less than or equal to x is

$$P(X \leqslant x) = F_i + (F_{j+1} - F_i)\frac{x - x_i}{x_{i+1} - x_i} \quad (6)$$

The confidence interval for the probability $P(X \pm x)$ can be evaluated by interpolating on the confidence interval curves using the same approach.

According to the probability analysis approach above, under the combined actions of deadweight, vehicle load, wind load and temperature effect, several output parameters are chosen and their effects are calculated, when the bridge is in normal situations, including the maximal stress of box girder, the maximal vertical displacement, the maximal stress of suspension cables, the maximal stress of central buckle, and the maximal displacement of towers (see Tab. 2). According to Tab. 2, under the normal circumstance, those important parameters of the bridge are all acceptable, meaning that the bridge has enough reliability. Fig. 6 and Fig. 7 are presented here as analyses reports, showing the details of analyses on the assessment item "maximal stress of box girder" and its cumulative distribution function. These pictures show the final results in a clear and vivid way. For technicians, especially those in charge of the daily maintenance, it renders a great sense of guidance.

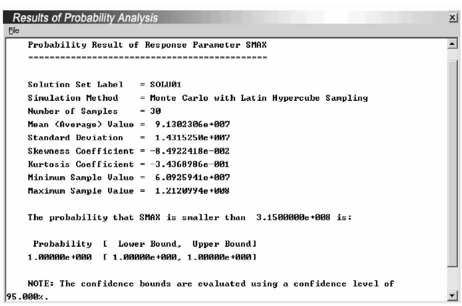

Fig. 6 Details of analysis results on "maximal stress of girder"

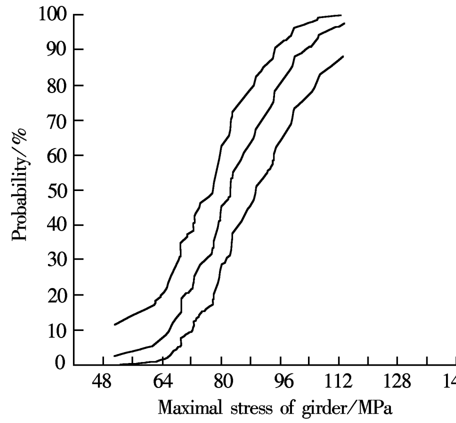

Fig. 7 Cumulative distribution function of "maximal stress of girder"

2.4 Probability analysis in damaged situations

The availability of the assessment program also depends on some other factors, for example, the finite element model should meet the need of analysis and in the module of damage identification all the damage must be found and be put into the module of assessment. Because no damage has occurred up to now (the bridge is newly built), the input damage is only a numerical simulation. On the assumption that there is a 3% to 5% weakening of cables detected after N years of service, because of rust, several important structural items are evaluated and the final results are reported in Tab. 3. Known by Tab. 3 even in the 3% case of a damage situation, the bridge is still reliable, though all the evaluation items are worse than those in a normal situation. However, when there is a 5% occurrence of weaning in cables (see Tab. 4), the probability of girder displacement decreases to 96.2%, no longer meeting the need of applicability. Meanwhile, the probability of girder stress decreases to 96.2%, the capacity of cable is not satisfactory any more.

Tab. 2 Probability analysis output in normal situations during the design working life of bridge

Assessment item	Mean value	Standard deviation	Maximal value	Minimal value	Limit value	Probability/%
Maximal stress of girder/MPa	91.3	14.3	121.2	60.9	315	100
Maximal vertical displacement of girder/m	0.981 4	0.654 3	2.4	0.04	3.725	100
Maximal stress of cables/MPa	578.5	24.2	627.4	530.5	835	100
Maximal stress of central buckle/MPa	158	20.827	210.6	107.1	315	100
Maximal horizontal displacement of tower/m	0.096 95	0.045 8	0.220 5	0.037 6	0.338	100

Tab. 3 Probability analysis output in damaged situations (3%)

Assessment item	Mean value	Standard deviation	Maximal value	Minimal value	Limit value	Probability/%
Maximal stress of girder/MPa	91.8	14.1	120.3	62.9	315	100
Maximal vertical displacement of girder/m	1.52	1.06	3.48	0.2	3.48	100
Maximal stress of cables/MPa	734	31.2	796.6	673.1	835	100

Tab. 4 Probability analysis output in damaged situations (5%)

Assess item	Mean value	Standard deviation	Maximal value	Minimal value	Limit value	Probability/%
Maximal stress of girder/MPa	91.9	14.6	120.9	63.2	315	100
Maximal vertical displacement of girder/m	1.95	1.21	3.87	0.05	3.725	96.2
Maximal stress of cables/MPa	734	35.2	835.3	663.7	835	97.7

3 Conclusion

If there are enough times of samplings, the Monte Carlo method can be regarded as a precise approach. After the bridge has been put into use, with the continuous collection and analysis on monitoring figures, the evaluation items will get continuous updating and be more precise[7,8]. Using the Monte Carlo numerical simulation method introduced here, the effects on bridge structure caused by arbitrary variables can be analyzed quantificationally, and all the results can be plotted in a vivid and clear way, making the decision-making of maintenance more objective and accurate.

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蒙特卡罗数值模拟及其在大跨桥梁可靠度分析中的应用

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摘要:为了定量地研究大跨桥梁在多种随机因素下的可靠度,采用了蒙特卡罗法中的拉丁超立方体抽样法,对润扬悬索桥(中国跨度最大的悬索桥)在温度、风、车辆荷载、车辆冲击等随机变量作用下的结构可靠度进行了计算.通过对大桥在正常运营和损伤状态下的可靠度分析,获得了大桥主缆应力、吊索应力、桥塔位移以及钢箱梁线形等重要指标的可靠度和累积分布函数,分析结果形象直观,提高了评估的效率,使维护决策的制定更加客观准确.

关键词:状态评估;健康监测;结构可靠度;蒙特卡罗法

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