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# Finite element model updating and validating of Runyang Suspension Bridge based on SHMS

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**Abstract:** Based on the finite element (FE) program ANSYS, a three-dimensional model for the Runyang Suspension Bridge (RSB) is established. The structural natural frequency, vibration mode, stress and displacement response under various load cases are given. A new method of FE model updating is presented based on the physical meaning of sensitivity and the penalty function concept. In this method, the structural model is updated by modifying the parameters of design, and validated by structural natural vibration characteristics, stress response as well as displacement response. The design parameters used for updating are bounded according to measured static response and engineering judgment. The FE model of RSB is updated and validated by the measurements coming from the structural health monitoring system (SHMS), and the FE baseline model reflecting the current state of RSB is achieved. Both the dynamic and static results show that the method is effective in updating the FE model of long span suspension bridges. The results obtained provide an important research basis for damage alarming and health monitoring of the RSB.

**Key words:** suspension bridge; finite element; model updating; model validating; baseline model; structural health monitoring system (SHMS)

The Runyang Suspension Bridge (RSB) is a single-span hinged and simply supported steel box girder bridge with a main span of 1 490 m as shown in Fig. 1. It is the longest suspension bridge in China and the third in the world. In addition, the central buckle is for the first time used in the suspension bridge in China. The research and establishment of the structural health monitoring system (SHMS) for the RSB will certainly show important significance[1]. In order to ensure the SHMS function well, a baseline model of the target bridge is indispensable. A baseline finite element model must be a full three-dimensional model that can reflect the current state comprehensively and correctly. In addition, it must be a model validated by field tests. However, it is often difficult to get an FE model that can truly reflect all aspects of the structure. A number of model updating methods in structural dynamics have been proposed[2 – 4]. In the direct method, the resulting updated matrices reproduce the measured structural modal properties exactly, but generally they cannot maintain connectivity and the corrections suggested are not always physically meaningful. The iterative parameter updating method involves using the sensitivity of the parameters to update the model. This sensitivity-based parameter updating approach has the advantage of identifying parameters that can directly affect the dynamic characteristics of the structure.

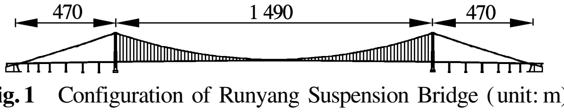


**Fig. 1** Configuration of Runyang Suspension Bridge (unit: m)

The main motive of this study is to use the updated FE model in the SHMS so that we can predict the static and dynamic responses of the RSB through the updated FE model. The traditional method used only the free vibration frequencies and modes which came from ambient vibration measurements in FE model updating[5], so the updated FE model could not be validated by other kinds of structural response, such as response of force, stress, displacement, etc. In this paper, a new simple and practical method of model updating is proposed based on the physical meaning of sensitivity. In this method, the structural model is updated by modifying the parameters of design, and validated by structural natural vibration characteristics, stress response, as well as, displacement response of the girder derived from the SHMS of the RSB. It is expected that the outcome of this study will be interesting and useful to researchers and professionals involved in complicated civil engineering.

## 1 Structural Modeling and Analysis

### 1.1 FE modeling

Based on the finite element program ANSYS, a

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three-dimensional model for the RSB is established according to the design. In the FE model, the deck, the central buckle and the towers were simulated by beam elements (BEAM4) with six degrees of freedom (DOFs) for each node. As a traditional deck spine model, the suspenders and the deck were linked with massless rigid elements placed perpendicular to the spine. The main cables and the suspenders were simulated by three-dimensional linear elastic truss elements (LINK10) with three DOFs for each node. The main cables and the deck were meshed according to the nodes of the suspenders. The pavement and the railings on the steel box girder were simulated by mass elements (MASS21) without rigidity. The material properties and real constants of the structure were strictly calculated and assigned to the simulating elements. The nonlinear stiffness characteristic of the back cables due to gravity effect was approximately simulated by linearizing the cable stiffness using the Ernst equation of equivalent modulus of elasticity[6].

The deck and the corresponding crossbeams of the towers were coupled in three DOFs including vertical displacement (UY), lateral displacement (UZ) and rotation around longitudinal direction (ROTX). As was first used in China, the central buckle was precisely simulated and coupled with the deck and the main cables according to the design. The main cables are fixed on the top of the towers. The bottom of both the back cables and the towers are fixed at the bases.

### 1.2 FE analysis

The suspension bridge is a flexible structure and the gravity rigidity caused by dead load is of great significance; therefore, the geometry nonlinearity must be considered. Based on the model of the RSB, the nonlinear static analysis was carried out at first and the initial stress was stiffened. The stress rigidity was thus formed and added into the geometry rigidity matrix. Then the natural vibration properties were calculated on the basis of the nonlinear static equilibrium equation. As a result, the natural frequencies and vibration modes which were obtained. Modal analysis results show that the coupling vibrations of the RSB appear more frequently than other long span suspension bridges, because of the adoption of the central buckle.

A part of the calculated natural frequencies and their vibration modes are summarized in Tab. 1. Some calculated mode shapes are illustrated in Fig. 2. In order to compare the calculated results with ambient test results more easily, the RSB's vibration modes were classified into the deck-dominate modes and the tower-dominate modes. Next, the deck-dominate modes were classified into vertical bending (DV), lateral bending (DL), and torsional (DT) modes. At the same time, the tower-dominate modes were classified into longitudinal bending (TLO), lateral bending (TLA), and torsional (TT) modes.

**Tab. 1** Modal analysis summary of the RSB

| Calculated frequency/Hz | Ambient vibration test | | MAC value/% | $\frac{f_c - f_m}{f_m}$ /% * | Nature of modes of vibration ** | |
|---|---|---|---|---|---|---|
| | Frequency/Hz | Damping ratio/% | | | | |
| 0.053 7 | 0.058 6 | 2.20 | 94.2 | −8.36 | DL1 | Sym |
| 0.117 3 | 0.122 1 | 0.97 | 97.0 | −3.93 | DV1 | Sym |
| 0.140 2 | 0.144 0 | 1.13 | 94.5 | −2.64 | DV2 | Anti-sym |
| 0.146 6 | 0.158 7 | 1.18 | 89.6 | −7.62 | DL2 | Anti-sym |
| 0.162 1 | 0.168 5 | 1.65 | 95.4 | −3.80 | DV3 | Sym |
| 0.192 2 | 0.188 0 | 1.10 | 89.7 | 2.23 | DV4 | Anti-sym |
| 0.223 8 | 0.239 8 | 1.59 | 79.3 | −6.67 | DT1 | Sym |
| 0.255 4 | 0.280 0 | 0.36 | 90.0 | −8.78 | DV5 | Sym |
| 0.287 4 | 0.309 8 | 1.01 | 83.2 | −7.23 | DT2 | Anti-sym |
| 0.338 3 | 0.341 8 | 0.66 | 87.4 | −1.02 | DV6 | Anti-sym |
| 0.344 2 | 0.371 1 | 0.80 | 77.6 | −7.25 | DT3 | Sym |
| 0.660 2 | 0.683 8 | 0.99 | 92.2 | −3.45 | TLO1 | |
| 0.364 9 | 0.351 7 | 1.02 | 94.4 | 3.75 | TLA1 | |
| 0.956 3 | 1.066 3 | 0.76 | 88.6 | −11.50 | TT1 | |
| 1.612 1 | 1.524 1 | 0.73 | 85.0 | 5.77 | TLO2 | |
| 1.618 2 | 1.418 5 | | 92.3 | 12.34 | TLA2 | |
| 2.171 6 | 2.157 4 | 1.04 | 97.2 | 0.66 | TLA3 | |
| 2.290 8 | 2.423 6 | | 90.8 | −5.48 | TLO3 | |
| 2.693 7 | 2.631 9 | | 85.1 | 2.35 | TLA4 | |
| 2.703 4 | 2.772 0 | | 82.7 | −2.47 | TLO4 | |

* $f_c$ represents the calculated frequency; $f_m$ represents the measured frequency.

** Sym represents the mode shape is symmetrical; anti-sym represents the mode shape is anti-symmetrical.

As we can see from Tab. 1, the lowest vibration frequency of the RSB is 0.053 7 Hz when the vibration mode is a lateral bending mode of the deck. The first tower-dominate vibration mode is an LA mode with a frequency of 0.364 9 Hz. It is noted that some deck-dominate modes often couple with the tower-dominate vibration modes to a certain extent.

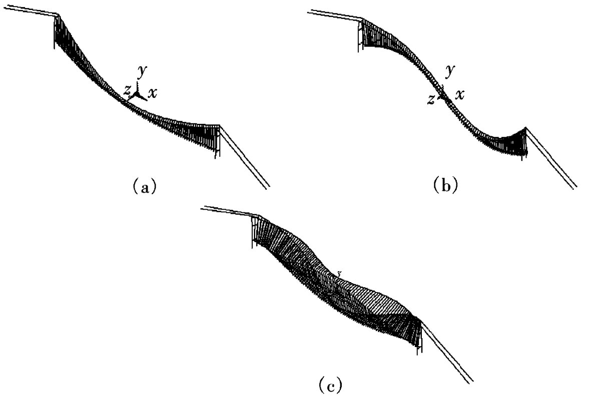


**Fig. 2** Some mode shapes of the RSB. (a) The 1st DL mode; (b) The 1st DV mode; (c) The 1st DT mode

## 2 Ambient Vibration Test and Modal Parameter Identification

The ambient vibration test can provide an accurate and reliable description of dynamic characteristics for a structure. Compared with traditional forced vibration tests, the ambient vibration test using environmental excitations induced by traffic, winds, and pedestrians is superior in many aspects[7]. However, the course of testing need not be interrupted when using this technique and relatively long records of response measurements are required. In addition, the signal levels are considerably low in the ambient vibration test.

Just before the official opening of the RSB, the ambient vibration tests for the bridge were carried out in January, 2005. The equipment used for the tests included high-sensitivity accelerometers, signal cables, a 16-channel data acquisition system, a signal amplifier and a portable computer. The theory of the ambient vibration test has been previously reported[8].

In order to measure the dynamic characteristics of the deck, measurement points were selected at locations near the joints of suspenders and deck. The high-sensitivity accelerometers were placed on both sides of the deck in the vertical and transverse directions. As for the measurement of the towers, only the south tower was considered because of the structural similarity of the two towers. Fig. 3 shows the ten selected measurement points of the south tower. Accelerometers were placed at these ten locations in the vertical and transverse directions.

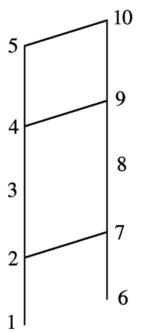


**Fig. 3** Ten measurement points of the south tower

Because the input excitations are not measured, the ambient vibration measurements cannot be calculated by FRFs. All of the measured data were analyzed by using the conventional FFT-based modal parameter identification technique. A total of 20 frequencies, mode shapes and damping ratios were obtained in the ambient vibration tests. These modes contain 11 deck-dominate modes and nine tower-dominate modes. All of the measured frequencies, damping ratios and mode natures are listed in Tab. 1.

The mode shapes identified from field ambient vibration measurements are paired with those obtained from the initial FE model of the RSB. The modal assurance criteria (MAC) are used to evaluate the correlation of mode shapes. The initial FE and the measured modal properties of the bridge are compared in Tab. 1. As we can see from Tab. 1, the frequency correlations of the DV bending modes are better than the DT and DL bending modes. The maximum error between the measured and the computed frequencies is 12. 34% in the TLA2 bending mode. As for the correlation of mode shapes, the tower-dominate modes seem obviously better than the deck-dominate ones. The minimum MAC value is 77. 6% in the DT3 bending mode. Both the frequency and the mode correlations show that the simulation of torsional rigidity in the long span suspension should be paid particular attention to.

## 3 Model Updating

During model updating, the measured modal properties are looked as the real behaviors of the structure. Therefore, the measured data can be used to update the initial FE model.

The sensitivity-based parameter updating approach is a traditional method for the FE model updating. In this method, sensitivity analysis of the selected structural parameters is carried out to see which ones are the relative sensitive parameters. These relative sensitive parameters used for FE model updating are listed in Tab. 2. The model updating procedures are finally transformed into optimum problems. Furthermore, a good updating approach is often transformed into a multi-objective nonlinear optimum problem under constraints of equations and inequations. Generally, the constrained optimization problem can be formed as follows:

$$\min J = J(x) \tag{1}$$

Subjected to

$$\begin{aligned}
&\underline{x}_i \leqslant x_i \leqslant \bar{x}_i && i = 1, 2, \ldots, N \\
&\underline{g}_j(x) \leqslant \bar{g}_j && j = 1, 2, \ldots, m_1 \\
&\underline{h}_k \leqslant h_k(x) && k = 1, 2, \ldots, m_2 \\
&\underline{w}_l \leqslant w_l(x) \leqslant \bar{w}_l && l = 1, 2, \ldots, m_3
\end{aligned}$$

where $x_i$ are the design variables; $g_j$, $h_k$ and $w_l$ represent the state variables; $N$ is the number of design variables and $m_1 + m_2 + m_3$ is the number of state variables. The under bar and over bar of the variables represent lower and upper bounds, respectively. In this paper, the design variables used for updating are bounded according to measured static results and engineering judgments. The allowable errors between the measured and the computed frequency are set to be less than ±5% for the first lateral, vertical and torsional dominated modes. For other modes, the errors are set to be less than ±10%.

**Tab. 2** Parameters of the RSB before and after updating

| Parameters | | Initial value | Updated value | Change/% |
|---|---|---|---|---|
| Steel girder deck | Mass density*/(kg·m$^{-3}$) | 15.26×10$^3$ | 16.41×10$^3$ | 7.54 |
| | Elastic modulus/GPa | 210 | 214 | 1.90 |
| | Vertical moment of inertia/m$^4$ | 2.08 | 2.16 | 3.85 |
| | Lateral moment of inertia/m$^4$ | 139.30 | 154.79 | 11.13 |
| | Torsional moment of inertia/m$^4$ | 5.90 | 6.34 | 7.46 |
| Main cable | Mass density/(kg·m$^{-3}$) | 7.86×10$^3$ | 8.68×10$^3$ | 10.43 |
| | Elastic modulus/GPa | 200 | 186 | −7.00 |
| Back cable | Elastic modulus/GPa | 200 | 172 | −14.00 |
| Tower | Mass density/(kg·m$^{-3}$) | 2.60×10$^3$ | 2.64×10$^3$ | 1.54 |
| | Elastic modulus/GPa | 35.0 | 38.4 | 9.71 |

* The densities of the paver and the railings on the deck are added into the mass density of the deck.

There are several techniques available to solve the constrained optimization problem. The first order optimization method and the penalty function concept are utilized in this paper. With regard to this optimization method, the constrained problem statement expressed in Eq. (1) is transformed into an unconstrained one using penalty functions. The unconstrained form of Eq. (1) is expressed as

$$Q(x,q)=\frac{J}{J_0}+\sum_{i=1}^{N}P_x(x_i)+q\left[\sum_{j=1}^{m_1}P_g(g_j)+\sum_{k=1}^{m_2}P_h(h_k)+\sum_{l=1}^{m_3}P_w(w_l)\right] \tag{2}$$

where $Q(x,q)$ is the unconstrained objective function; $J_0$ is the reference objective function value that is selected from the current group of design sets; $P_x$, $P_g$, $P_h$ and $P_w$ represent penalties of the constrained design and state variables respectively. The optimization iteration formula is expressed as

$$x^{(j+1)}=x^{(j)}+s_j d^{(j)} \tag{3}$$

where $s_j$ is the line search parameter and $d^{(j)}$ is the search direction vector which leads to the minimum value of $Q(x,q)$. Various slope and direction searches are performed during each iteration until the convergence is reached.

$$\left|J^{(j)}-J^{(j-1)}\right|\leqslant\tau,\quad \left|J^{(j)}-J^{(\mathrm{b})}\right|\leqslant\tau \tag{4}$$

where $J^{(j)}$, $J^{(j-1)}$ and $J^{(\mathrm{b})}$ refer to the current, previous and best objective function values, respectively; and $\tau$ is the objective function tolerance.

The values of parameters before and after updating are listed in Tab. 2. The errors between the measured and the computed frequency of both the initial and the updated FE models are compared in Fig. 4.

Fig. 4 shows that the frequency errors reduce obviously after updating and the maximum error changes from 12.34% to 7.38%. In order to evaluate the correlation of mode shapes after updating, the mode shapes from the updated FE model are also paired with those from field ambient vibration measurements. Results show that most of the MAC values rise and all of them become larger than 80% after updating.

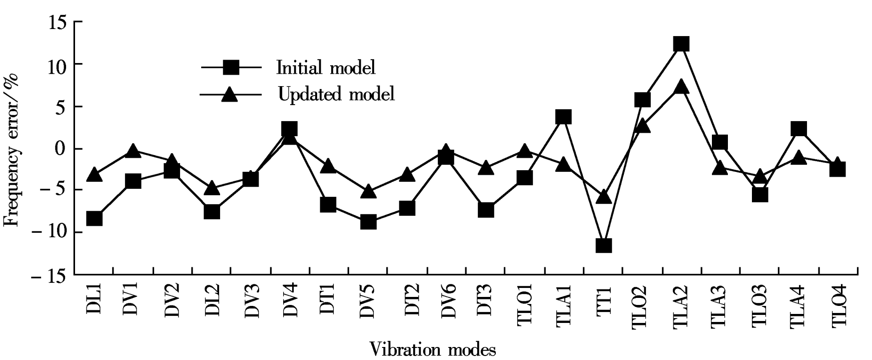


**Fig. 4** Errors between the measured and the computed frequencies

## 4 Validating the Updated Model

Static analysis on the stress and displacement response of the RSB is carried out based on the initial and updated FE model respectively. The computed displacement response values of key sections under eight kinds of load cases are directly obtained from the static analysis results above, and the measured ones come from the location measurement system, GPS. The number and the location of the load trucks in each load case are clearly described in Ref. [9]. In this paper, the selected key sections are $L/4$, $L/2$ and $3L/4$ section of the steel box girder ($L$ is the main span of the RSB) and the top of the south tower.

The values of vertical displacement response of the selected girder sections under four kinds of load cases are listed in Tab. 3. Because of the significance of displacement response on top of the tower, its longitudinal values under six kinds of load cases are listed in Tab. 4. As for the displacement responses, the northern and the up values are set to be positive, and the southern and the down ones are set to be negative. We can see from Tab. 3 that in most of the four load cases and the three selected key sections, the displacement errors between the measured and the computed values become fewer after updating.

**Tab. 3** Vertical displacement values of key sections of the steel box girder m

| Load case | Value of $L/4$ section | | | Value of $L/2$ | | | Value of $3L/4$ section | | |
|---|---|---|---|---|---|---|---|---|---|
| | Measured | Initial | Updated | Measured | Initial | Updated | Measured | Initial | Updated |
| 1 | −2.826 | −2.828 | −2.820 | 0.170 | 0.074 | 0.106 | 1.268 | 1.247 | 1.243 |
| 2 | 0.062 | 0.058 | 0.061 | −2.384 | −2.414 | −2.400 | 0.069 | 0.054 | 0.062 |
| 3 | 1.229 | 1.247 | 1.241 | 0.104 | 0.076 | 0.108 | −2.814 | −2.828 | −2.821 |
| 4 | 0.091 | 0.043 | 0.052 | −1.7835 | −1.831 | −1.7869 | 0.0553 | 0.040 | 0.048 |

**Tab. 4** Longitudinal displacement values on top of the south tower mm

| Load case | Measured | Initial | Updated |
|---|---|---|---|
| 1 | 51 | 64 | 58 |
| 2 | 76 | 85 | 80 |
| 3 | 61 | 63 | 62 |
| 4 | 52 | 54 | 53 |
| 5 | 18 | 26 | 22 |
| 6 | 18 | 20 | 19 |

In order to compute the stresses of the steel girder deck, the spatial FE model of the segment steel box girder is particularly established by the shell element, as shown in Fig. 5.

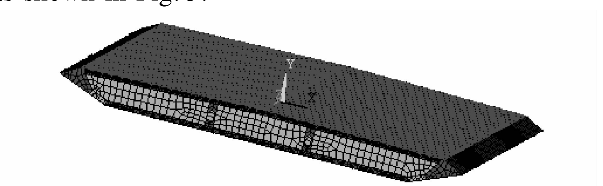

**Fig. 5** Spatial FE model of the steel box girder

The FE model with a length of 32.2 m is composed of two standard segments of the steel box girder. In the model, the pavement on the steel deck was simulated by raising the density of the upper deck. The real constants and the material properties of the shell elements are strictly calculated according to the design. After the boundary constraints and the relative forces are applied, the key section stress of the steel box girder is then calculated based on the theory of elastic mechanics[10]. In the calculation, the stresses induced by gravity, the load trucks and the temperature are considered together. Fig. 6 shows some of the stress response values of the $L/4$ section of the steel box girder under load case 3, which is the most disadvantageous load case for the $L/4$ section. In Fig. 6, the $x$-axis represents the distance of measured points to section center along the transverse girder and the $y$-axis is the corresponding stress value. Ref. [9] showed the accurate location of each measured points of the $L/4$ section.

As we can see from Fig. 6, the computed stress values cannot completely match the measured ones after updating due to measurement errors and limitations of the calculation method, but the stress errors between the measured and the computed values become fewer. At the same time, the stress distribution trend is more similar after updating.

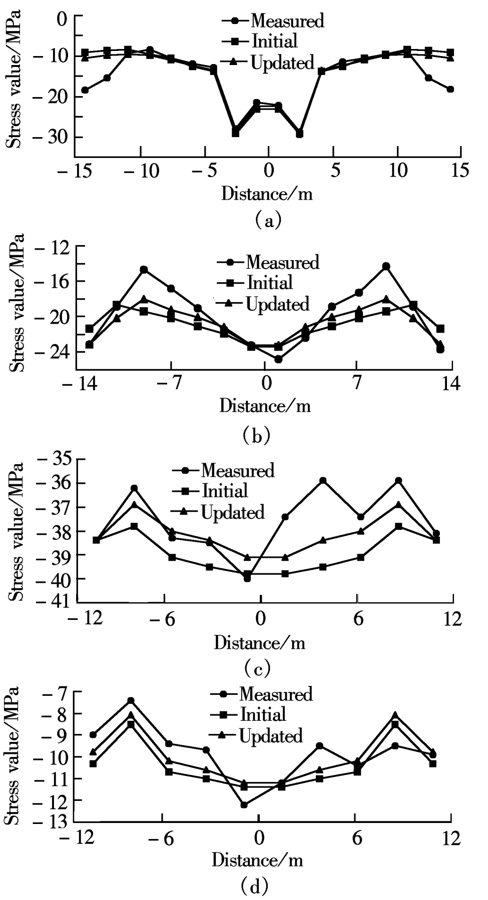


**Fig. 6** Stress values of $L/4$ section of the steel box girder. (a) Upper board; (b) "U" girder under the upper board; (c) Bottom board; (d) Intersection of "U" girder and the upper board

## 5 Conclusions

1) This paper presents a sensitivity-based finite

element model updating method for the RSB using the signals coming from the SHMS. Both the dynamic and static results show that the method is effective in updating the FE model of long span suspension bridges. Successful FE model updating of the RSB demonstrates that, even for large and complex structures, the updating method presented in this paper is practical.

2) Generally, the design parameters used for updating are bounded according to engineering judgments only. In this method, the static values are used to bind the design parameters in order that the computed static values match the measured ones better after updating. Therefore, multi-objective updating is achieved by the single objective updating method.

3) Since the first order optimization method is used to solve the unconstrained objective function in this updating method, the obtained results are very accurate. But this time-consuming method is possible to converge in a local minimum. Therefore, appropriate initial values and objective function tolerance for the updating are required, and the zero order optimization method can be used instead when converging in a local minimum.

4) Both the natural frequencies and the static response become much closer to the measured ones by using the updated FE model. However, the errors between the computed and measured values cannot be reduced by further iteration calculation, which suggests that the FE model refining and the modal identification technique are the future study emphases.

## References


[1] Li Aiqun, Miao Changqing, Li Zhaoxia, et al. Health monitoring system for the Runyang Yangtse River Bridge [J]. *Journal of Southeast University (Natural Science Edition)*, 2003, **33**(5): 544 − 548. (in Chinese)

[2] Mottershead J E, Friswell M I. Model updating in structural dynamics: a survey [J]. *Journal of Sound and Vibration*, 1993, **167**(2): 347 − 375.

[3] Zhang Qiwei, Sun Limin. FE model updating of suspension bridge based on vibration measurements [A]. In: Li Zhongxian, Ru Jiping, eds. *Proc of the Seventh International Symposium on Structural Engineering for Young Experts*[C]. Beijing: Science Press, 2002. 793 − 800.

[4] He Xuhui. Study on the structural health monitoring of Nanjing Yangtze River Bridge and its key technologies [D]. Changsha: School of Civil Engineering and Architecture of Central South University, 2004. (in Chinese)

[5] Wu J R, Li Q S. Finite element model updating for a high-rise structure based on ambient vibration measurements [J]. *Engineering Structures*, 2004, **26**(7): 979 − 990.

[6] Ernst J H. Der E-modul von seilen unter berucksichtigung des durchhanges [J]. *Der Bauingenieur*, 1965, **40**(2): 52 − 55.

[7] Jaishi B, Ren W X. Structural finite element model updating using ambient vibration test results [J]. *Journal of Structural Engineering*, 2005, **131**(4): 617 − 628.

[8] Wang Hao, Qiao Jiandong. Finite element analysis and experimental study on dynamic characteristics of bridge structures [J]. *Journal of Highway and Transportation Research and Development*, 2004, **21**(6): 78 − 80. (in Chinese)

[9] Li Aiqun. Report on field measurement of Runyang Yangtse River Bridge [R]. Nanjing: College of Civil Engineering, Southeast University, 2005. (in Chinese)

[10] Gia Lijun, Sun Bin, Liu Yu, et al. Analysis of preliminary design program of Hongkong Tsing Lung Bridge [J]. *Journal of Tongji University (Natural Science Edition)*, 2001, **29**(1): 89 − 93. (in Chinese)


# 基于结构健康监测的润扬悬索桥有限元模型修正与验证


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**摘要:**采用大型有限元分析软件 ANSYS 建立了润扬悬索桥的三维有限元模型并对其进行了自振特性分析,同时计算了各种工况下大桥的应力及位移响应. 基于灵敏度的物理意义以及罚函数的思想,提出了一种结构有限元模型修正的新方法. 该方法仍以自振特性为目标函数,以结构设计参数为待修正参数,但设计参数的上下限根据测试所得静力响应值和理论值的对比以及工程经验来确定,因此静力响应可用于验证修正后的有限元模型. 采用结构健康监测系统数据及提出的方法对大桥有限元模型进行了修正与验证,得到了能够较好地反映大桥整体动静力性能的有限元基准模型,为润扬悬索桥的健康监测和损伤预警提供了研究基础.



**关键词:**悬索桥;有限元;模型修正;模型验证;基准模型;结构健康监测系统

**中图分类号:**U442;U448