Journal of Southeast University (English Edition) Vol. 21, No. 4, pp. 495 – 499 Dec. 2005 ISSN 1003—7985

Behaviors of pile-supported embankments in highway engineering

Shi Minglei Pang Feng Zhang Bo

(College of Transportation, Southeast University, Nanjing 210096, China)

Abstract: Based on the variational approach for pile groups embedded in soil modeled using a load-transfer curve method, a practical method was conducted to estimate the settlement of symmetric pile group supported embankments. The working mechanism of composite foundations improved by rigid or semi-rigid columns is analyzed by this method. Under equivalent strain conditions, the pile-soil stress ratio approaches the pile-soil modulus ratio up to a limited value of pile stiffness ($R_m < 10$); in the subsequent stages of high pile stiffness ($R_m > 10$), a further increase in the pile-soil modulus ratio cannot lead to a significant increase of stress transferred to the columns in composite foundations. The major influencing factor of the stress concentration from soil to pile in a high pile-soil modulus ratio is the padding stiffness. For the composite foundation improved by cement mixing columns, the effective column length is about 15 to 20 m and it is a more economical and effective design when the column length is less than 15 m.

Key words: variational approach; pile-supported embankment; composite foundation improved by columns; settlement calculation

The use of pile-supported embankments is not a new technology. Embankments on relief piles have been used for more than 60 years, and “modern” stone pile technology was first implemented in Europe in the 1960s. However, the new method of considering total direct and indirect project costs along with newer more economical designs has dramatically increased the usage of pile-supported embankments worldwide. Specifications for designing modern pile-supported embankments have been implemented in the United Kingdom, Sweden, and Germany. Modern pile and soil reinforcement technology, pile-supported embankments will frequently provide the best combination of economy, construction speed, settlement performance, reliability, installation/QC simplicity, and environmental compliance than that of any other ground improvement methods or bridge construction technique.

Pile-supported embankments have been designed and built extensively in Europe and Asia, in which the cushion’s modulus is larger than that of ground soil and embankment soil. So the interaction between the cushion and pile should be considered when calculating the settlement. By now, many theories about the interaction between cap and group pile have been promoted. Butterfield and Banerjee[1] promoted the boundary element method based on the Mindlin solution to analyze group-pile foundation, in which interaction between cap and foundation was considered. Ottaviani[2] did research on cap-group pile with the three-dimensional finite element method. Poulos and Davis[3] systematically summarized the calculation methods of pile-soil interaction under a grid cap. Chow[4] promoted a method based on a finite element method to analyze the group pile in unhomogeneous foundations, and Shi et al. [5,6] used this method to calculate the settlement of large diameter bored piles (LDBPs) which took into consideration the grid cap coupling and the piles’ reinforcement behavior. But, what talked above are all based on the hypothesis of a grid cap board, it cannot be suitable for the calculation of settlement of a pile-supported embankment directly, and these methods are involved in considering the dispersion as the direction of pile body, so the calculation magnitude is limited. From the literature review, it has been found that piles or columns have been successfully used in combination with the cushion reinforced with geosynthetics to support embankments over soft soil. The purpose of the cushion over piles is to improve load transfer from soil to piles, reduce total and differential settlements, and increase slope stability[7].

Shen[8] depicted the strain and pile shaft shear stress of the group pile system by two finite series separately, in which the foundation’s stress-strain relation was assumed elastic in isotropic semi-infinite solids, and the interaction between group and cap through the soil medium was also simulated based on the Mindlin

Received 2005-04-03.

Foundation item: The Transportation Scientific Foundation of Jiangsu Province (No. 02Y015).

Biography: Shi Minglei (1962—), male, doctor, associate professor, Mingleish@263.net.

solution. Based on the overall potential energy equation of group piles, the variational approach was used to analyze the group pile system, and a stiffness matrix of pile-soil-cap and a load-displacement grid matrix were created. In this paper, based on what Shen has done[8], the pile-soil stress ratio is assumed first, then the average soil element stress and the grid matrix of pile-soil system are obtained. The use of a finite series to depict group pile strain and shaft shear stress avoids the pile dispersion, which reduces the calculation magnitude dramatically. Furthermore, the symmetry of the road central line is considered for the group pile system, and the calculation magnitude is reduced.

1 Calculation Theory

Based on Ref. [8], the potential energy of a pile group can be expressed as

$$\pi_{\mathrm{p}} = \sum_{i=1}^{n_{\mathrm{p}}} \frac{1}{2}\iiint_V E_{\mathrm{p}}\left(\frac{\partial \boldsymbol{w}_{zi}}{\partial z}\right)^2 \mathrm{d}V + \frac{1}{2}\iint_s \boldsymbol{w}_z^{\mathrm{T}}\boldsymbol{\tau}_z \mathrm{d}s + \frac{1}{2}\iint_A \boldsymbol{w}_{\mathrm{b}}^{\mathrm{T}}\boldsymbol{\sigma}_{\mathrm{b}}\mathrm{d}A + \frac{1}{2}\iint_B \boldsymbol{w}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{p}_{\mathrm{s}}\mathrm{d}B - \boldsymbol{w}_{\mathrm{t}}^{\mathrm{T}}\boldsymbol{P}_{\mathrm{t}} \quad (1)$$

Because of the symmetry, the behavior of the load and displacement of the pile-soil system covered by shadow shown in Fig. 1 can reflect the whole embankment. When a load is applied, only the stress occurs, and the lateral displacement is zero in the boundary of the pile-soil system covered by shadowing. So the general potential energy of the partial pile-soil system can be given first, and then the pile-supported embankment can be analyzed by the variational method. The general potential energy of a partial pile-soil system is

$$\pi_{\mathrm{p}} = \sum_{i=1}^{m_{\mathrm{p}}} \frac{m_{\mathrm{p}i}}{2}\iiint_V E_{\mathrm{p}}\left(\frac{\partial \boldsymbol{w}_{zi}}{\partial z}\right)^2 \mathrm{d}V + \sum_{i=1}^{m_{\mathrm{p}}} \frac{m_{\mathrm{p}i}}{2}\iint_s \boldsymbol{w}_{zi}\boldsymbol{\tau}_{zi}\mathrm{d}s + \sum_{i=1}^{m_{\mathrm{p}}} \frac{m_{\mathrm{p}i}}{2}\iint_A \boldsymbol{w}_{\mathrm{b}i}\boldsymbol{\sigma}_{\mathrm{b}i}\mathrm{d}A + \sum_{i=1}^{m_{\mathrm{s}}} \frac{m_{\mathrm{s}i}}{2}\iint_A \boldsymbol{w}_{\mathrm{s}i}\boldsymbol{\sigma}_{\mathrm{s}i}\mathrm{d}A - \sum_{i=1}^{m_{\mathrm{p}}} m_{\mathrm{p}i}\boldsymbol{w}_{\mathrm{t}i}\boldsymbol{p}_{\mathrm{t}i} \quad (2)$$

where m_{p} is the piles covered by shadow; $m_{\mathrm{p}i}$ is the ratio of the i-th pile's head area to the cross section area; m_{s} is the soil element number of the shadowing part; $m_{\mathrm{s}i}$ is the ratio of the i-th soil element covered by shadow to the soil element.

When solving the load-displacement relation of partial group piles based on the Mindlin solution, it is different from that of the total group pile system. So what should be considered is only the interaction of pile-pile and pile-soil in the partial pile group area. The settlement at depth z of the partial group pile system can be expressed as

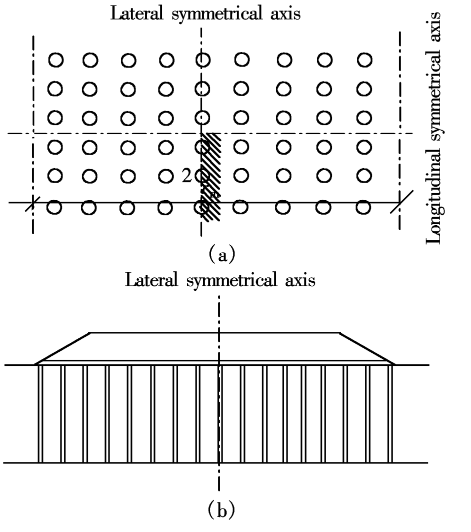

Fig. 1 Symmetrical embankment. (a) Pile groups plan layout; (b) Pile groups section

$$\boldsymbol{w}_z = \sum_{i=1}^{n_{\mathrm{p}}}\left(\iint_s f(z, z_i)\boldsymbol{\tau}_{zi}\mathrm{d}s + \iint_A f(z, z_{\mathrm{b}i})\boldsymbol{\sigma}_{\mathrm{b}i}\mathrm{d}A\right) + \sum_{e=1}^{n_{\mathrm{s}}}\iint_B f(z, z_{\mathrm{s}e})\mathrm{d}B \times \boldsymbol{p}_{\mathrm{s}e} \quad (3)$$

where n_{p} is the total number of the piles. Using the method proposed by Shen, we can obtain the soil displacement in Gauss-point and the pile tip displacement in the partial group pile system. The displacement matrix in the surface of ground is

$$\begin{Bmatrix} \boldsymbol{w}_{\mathrm{p}} \\ \boldsymbol{w}_{\mathrm{s}} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{f}_{\mathrm{pp}} & \boldsymbol{f}_{\mathrm{ps}} \\ \boldsymbol{f}_{\mathrm{sp}} & \boldsymbol{f}_{\mathrm{ss}} \end{bmatrix}\begin{Bmatrix} \boldsymbol{q}_{\mathrm{p}} \\ \boldsymbol{q}_{\mathrm{s}} \end{Bmatrix} \quad (4)$$

The matrix rank is $(m_{\mathrm{p}} \times (n_{\mathrm{g}} + 1) + m_{\mathrm{s}}) \times (n_{\mathrm{p}} \times (k_2 + 1) + n_{\mathrm{s}})$.

Because of symmetry, the symmetrical element of pile in the vector $\boldsymbol{q}_{\mathrm{p}}$ and soil in the vector $\boldsymbol{q}_{\mathrm{s}}$ are equal. In the coefficient matrix of Eq. (4), the corresponding elements can be added to each other, and Eq. (4) can be written as

$$\begin{Bmatrix} \boldsymbol{w}_{\mathrm{p}} \\ \boldsymbol{w}_{\mathrm{s}} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{g}_{\mathrm{pp}} & \boldsymbol{g}_{\mathrm{ps}} \\ \boldsymbol{g}_{\mathrm{sp}} & \boldsymbol{g}_{\mathrm{ss}} \end{bmatrix}\begin{Bmatrix} \boldsymbol{t}_{\mathrm{p}} \\ \boldsymbol{t}_{\mathrm{s}} \end{Bmatrix} \quad (5)$$

The matrix rank is $(m_{\mathrm{p}} \times (n_{\mathrm{g}} + 1) + m_{\mathrm{s}}) \times (m_{\mathrm{p}} \times (k_2 + 1) + m_{\mathrm{s}})$.

Following the same procedures in Ref. [8], first vary Eq. (2), then transform Eq. (5). The stiffness matrix corresponding to the load and displacement of pile heads and interface between padding and ground can be expressed as

$$\begin{Bmatrix} \boldsymbol{P}_{\mathrm{t}} \\ \boldsymbol{P}_{\mathrm{s}} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{k}_{\mathrm{tpp}} & \boldsymbol{k}_{\mathrm{tps}} \\ \boldsymbol{k}_{\mathrm{tsp}} & \boldsymbol{k}_{\mathrm{tss}} \end{bmatrix}\begin{Bmatrix} \boldsymbol{w}_{\mathrm{t}} \\ \boldsymbol{w}_{\mathrm{s}} \end{Bmatrix} \quad (6)$$

2 Analyses of Calculation Results

Now, in order to control the intolerable total and differential settlements of embankment over soft soils effectively, the rigid and semi-rigid columns were used in the highway foundation improvement. This is included the widening of existing roads. But it is not clear what the interaction between the padding and the composite foundation treated by rigid columns or semi-rigid columns was. So, the procedure proposed in this paper was used to research the settlement of a composite foundation improved by columns.

Fig. 2 shows the relationship of pile-soil modulus ratio R_m and settlement S under a different pile-soil stress ratio R_s, where R_m is the pile shaft material modulus/foundation soil modulus; R_s is the vertical stress applied on the pile head/the vertical stress on foundation soils between piles. As shown in Fig. 2, the increase in the pile-soil modulus can reduce the settlement. But while the pile-soil modulus ratio exceeds a certain value, defined as the maximum effective pile-soil modulus ratio, the increase in the stiffness of the pile shaft cannot reduce the settlement of the composite foundation. The larger pile-soil stress ratio promotes the higher value of the maximum effective pile-soil modulus ratio. That is, the reduction of the settlement of soft ground improved by rigid columns or semi-rigid columns can be improved by an increase in the pile-soil stress ratio. Under the same condition, the magnitude of the pile-soil stress ratio tends to be greater, the thicker the padding and the higher its stiffness. If the padding' s stiffness does not improve dramatically, such as in general sand padding, the high pile-soil modulus ratio (rigid pile or semi-rigid pile) cannot lead to a significant reduction in the settlement of a composite foundation, which is rather important for the composite foundation improved by large distances between rigid piles.

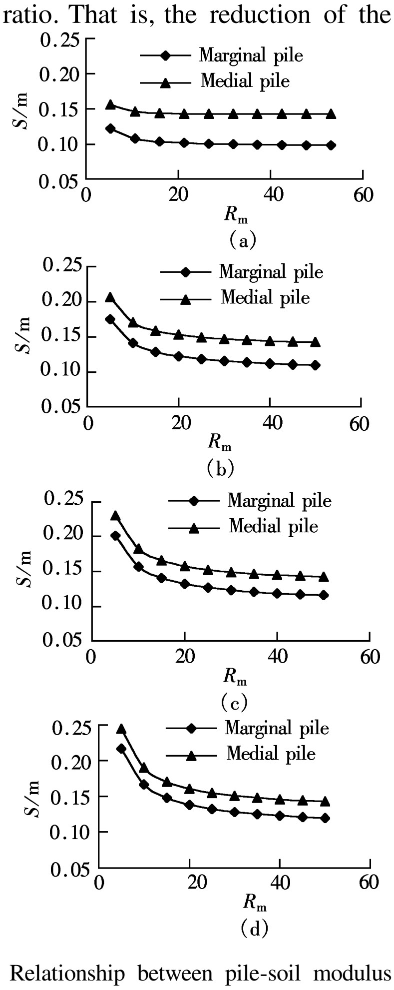

Fig. 2 Relationship between pile-soil modulus ratio and settlement. (a) $R_s=5$; (b) $R_s=10$; (c) $R_s=15$; (d) $R_s=20$

Fig. 3 indicates the relationship of the pile-soil stress ratios R_s and settlement under different pile-soil modulus ratios R_m. As shown in Fig. 3, for a conventional pile-supported earth platform, the pile-soil stress ratio increases with the increase of the pile-soil modulus ratio under equivalent strain conditions, but the former' s increase is lower than latter' s, as is shown in Tab. 1. To provide a measure of the performance, the factor λ is introduced as

$$\lambda = R_s / R_m \tag{7}$$

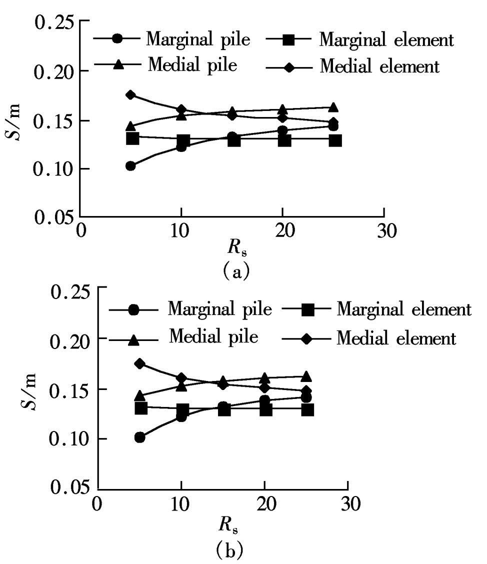

Fig. 3 Relationship of pile-soil stress ratio and settlement. (a) $R_m=10$; (b) $R_m=20$

Tab. 1 Pile-soil stress ratio under equivalent strain

R_m	R_s of margined pile	R_s of medial pile	Average R_s
5	5	5	5
10	8	9	9
20	12	14	13
30	16	18	17
40	20	22	21
50	22	24	23

Under equivalent strain conditions, the pile-soil stress ratio has the following rule. When the pile-soil stress ratio is relatively small ($R_m < 10$), the pile-soil stress ratio approaches the pile-soil modulus ratio; that is, the factor λ is equal to 1, which coincides with the assumption of the composite foundation theory. But, with the further increase in the pile-soil modulus ($R_m > 10$), under equivalent strain conditions, the pile-soil stress ratio is lower than the pile-soil modulus ratio, that is, the factor λ is less than 1, as shown in Fig. 4.

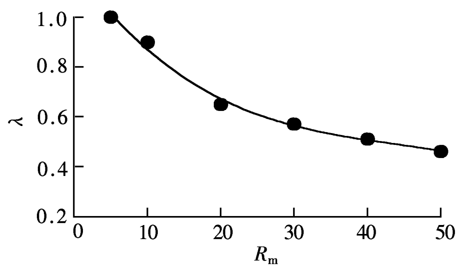

Fig. 4 Factor λ vs. pile soil modulus R_m

Some conclusions similar to those proposed above are also obtained. As for rigid columns applied in a composite foundation, if a high stiffness padding is not used in the design, the pile-soil stress ratio will be markedly lower than the pile-soil modulus ratio, which can reduce the stress concentration from foundation soil to pile head, and the rigid pile composite foundation's function cannot be brought into play adequately. So, when a rigid pile system is used in a soft foundation, if the special padding is not designed to improve the padding's stiffness, a pronounced stress concentration on the vertical columns cannot occur.

The research on effective pile length is always one of the key problems for composite foundations. Through calculation, we can get some basic results, as shown in Fig. 5. When the pile-soil modulus ratio R_m ranges from 10 to 50, and the length-diameter ratio ranges from 30 to 40, the increase of the length-diameter ratio can slightly reduce the settlement. So we can know that when the diameter is 50 cm, the effective length of columns used in a composite foundation is about 15 to 20 m, and when the pile-soil modulus ratio R_m is between 10 and 50, the increase in the pile-soil modulus ratio has little effect on the pile's effective length. So, it is more economical and effective for cement mixing piles when the columns lengths are less than 15 m.

Additionally, analysis of the pile space-diameter ratio, defined as s_p/D which is the space between columns and the diameter of the column, shows that, the higher the pile-soil modulus ratio is, the greater the effect of the factor s_p/D on the reduction of a composite foundation is. When the pile space-diameter ratio

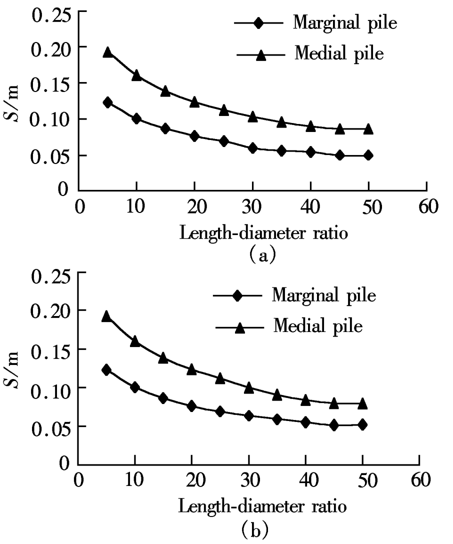

Fig. 5 Pile length-diameter ratio and settlement. (a) $R_m = 10$; (b) $R_m = 50$

s_p/D is less than 2.5, it is not useful to reduce the settlement of a composite foundation by a decrease in pile distance. In summary, when a lower pile space-diameter ratio is designed, a high modulus of columns is more reasonable.

3 Conclusions

Based on the variational approach for pile groups in soil modeled using load-transfer curves, a practical solution for estimating the settlement of a symmetric pile group system supported embankment is presented. This method is capable of treating rigid and semi-rigid pile groups. Analysis data indicate some important conclusions:

1) A reduction in the settlement of a composite foundation improved by columns as the pile-soil modulus ratio increases is evident at the low stage of column stiffness, and remains at a plateau at the subsequent high stages of column stiffness.

2) With the increase in column stiffness up to a limited value ($R_m < 10$), the pile-soil stress ratio increases rapidly and approaches the pile-soil modulus ratio; that is, the factor λ is equal to 1. Then with the further increase in column stiffness ($R_m > 10$), the factor λ decreases gradually to a value less than 0.5 and the increased amount of stress transferred to the columns is slight.

3) There is a specific combination of stiffness of pile and soil beyond which further increase in the pile-soil modulus ratio neither increases the pile-soil stress ratio nor reduces the settlement of the composite foundation.

4) Padding stiffness is rather important in enhan-

cing the increase in the pile-soil stress ratio when rigid piles or semi-rigid piles are used in the improvement of a soft foundation.

5) For column in a composite foundation, especially for cement mixing columns, it is more economical and effective when the column length is less than 15 m.

References

[1] Batterfield R, Banerjee P K. The elastic analysis of compressible piles and pile groups [J]. *Geotechnique*, 1971, **21** (1): 43 − 60.

[2] Ottaviani M. Three dimensional finite element analysis of vertically loaded pile groups [J]. *Geotechnique*, 1975, **25** (2): 159 − 174.

[3] Poulos H G, Davis E H. *Pile foundation analysis and design* [M]. New York: John Wiley and Sons, 1980.

[4] Chow Y K. Analysis of vertically loaded pile groups [J]. *International Journal for Numerical and Analytical Mechanics*, 1986, **10**(1): 59 − 72.

[5] Shi Minglei, Deng Xuejun, Liu Songyu. An analysis of settlement of a single pile in cohesive soils [J]. *Journal of Highway and Transportation Research and Development*, 2002, **19**(6): 81 − 87. (in Chinese)

[6] Shi Minglei, Deng Xuejun, Liu Songyu. Study of restrain effect of among piles [J]. *Journal of Southeast University* (*Natural Science Edition*), 2003, **33**(3): 343 − 347. (in Chinese)

[7] Han J, Gabr M A. A numerical study of load transfer mechanisms in geosynthetic reinforced and pile supported embankments over soft soil [J]. *Journal of Geotechnical and Geoenvironmental Engineering*, *ASCE*, 2002, **128**(1), 44 − 53.

[8] Shen W Y. A variational approach for the analysis of pile group-pile cap interaction [J]. *Geotechnique*, 2000, **50**(4): 349 − 357.

公路工程中桩承路堤特性研究

石名磊　庞　峰　张　波

（东南大学交通学院，南京 210096）

摘要：基于土中群桩分析变分原理，结合桩间土荷载传递法，建立了实用群桩沉降计算方法，并应用于对称桩承路堤结构分析. 采用该计算方法，分析了刚性、半刚性竖向加固体复合地基的工作机理. 在等应变条件下，桩土模量比相对较低时（$R_m < 10$），桩土应力比与桩土模量比相近；在桩土模量比较高时（$R_m > 10$），桩身模量提高对复合地基应力集中的影响减弱. 垫层刚度直接影响桩土模量增加对应力集中的作用效果. 水泥搅拌桩有效桩长为 15 ~ 20 m，采用小于 15 m 桩长的经济效益和加固效率更加.

关键词：变分方法；桩承垫层路堤；竖向加固体复合地基；沉降计算

中图分类号：TU473. 1