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Noise-improved information transmission in a nonlinear threshold array for Gaussian mixture noise

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Abstract: To discuss further the dependence of stochastic resonance on signals, nonlinear systems and noise, especially on noise, the binary input signal buried in Gaussian mixture noise through a nonlinear threshold array is studied based on mutual information. It is obtained that Gaussian mixture noise can improve the information transmission through the array. Both stochastic resonance (SR) and suprathreshold stochastic resonance (SSR) can be observed in the single threshold system and in the threshold array. The parameters in noise distribution affect the occurrence of SR and SSR. The efficacy of information transmission can be significantly enhanced as the number of threshold devices in the array increases. These results show further the dependence of SR and SSR on the noise distribution, and also extend the applicability of SR and SSR in information transmission.

Key words: stochastic resonance; suprathreshold stochastic resonance; mutual information; Gaussian mixture noise

Sometimes, noise can improve signal processing or signal transmission. This phenomenon is called stochastic resonance (SR)[1]. Most studies of SR usually involve a subthreshold signal (signals cannot cross the threshold without the assistance of noise) through a single nonlinear system. The nonlinear system can produce a stronger beneficial response with the addition of noise[1 – 8]. Very recently, suprathreshold stochastic resonance (SSR) was introduced through a parallel array of threshold devices[9 – 13]. Without noise, the suprathreshold signal (signals can cross the threshold without the assistance of noise) usually elicits the same output from each of the threshold devices. However, when different noises are independently added on every device of the array, every device will produce a distinct response. When all these responses are summed up, the global response can be more efficient than the response in each of the threshold devices. SR and SSR are two distinct forms of improvement by noise. SR and SSR are usually characterized by signal-to-noise ratio (SNR), mutual information (MI), cross-correlation coefficient (CCC), and so on[1 – 13]. Here, we discuss further SR and SSR in information transmission based on MI through the parallel array for Gaussian mixture noise. Both SR and SSR can be observed not only in the single threshold system but also in the array. The parameters in noise distribution affect the occurrence of SR and SSR. The efficacy of information transmission can be significantly enhanced as the number of threshold devices in the array increases. These results show further the dependence of SR and SSR on noise distribution and also extend the applicability of SR and SSR in information transmission.

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1 An Array of Threshold Devices and Mutual Information

An array of N threshold devices is considered. When $N = 1$, the array is a single threshold system. A noise η_i, independent of the input binary signal $x \in \{0, 1\}$, can be added to x before quantization by the threshold device i, which is with a threshold level u_i. This gives the output $y_i = T(x + \eta_i - u_i)$, $i = 1, 2, \ldots, N$, where $T(x) = 1$ if $x > 0$ and is zero otherwise. We consider that the noises $\eta_i (i = 1, 2, \ldots, N)$ are mutually independent and identically distributed. The response of the array is obtained by summing up the outputs of all the devices $y = \sum_{i=1}^{N} y_i$. Such conditions, with arrays of threshold devices, are especially relevant for existing and future multi-sensor networks having to cope with limited time and resources for data processing, storage, communication, and energy supply[9]. Because threshold distribution has little effect on the performance of the array, especially at larger noise intensity[13], we now also assume that all the thresholds share the same value u, as in Refs. [9 – 13]. By similar discussion[9 – 12], the average mutual information I can be written as

$$
\begin{aligned}
I = {} & H(y) - H(y \mid x) = \\
& -\sum_{n=0}^{N} P(y=n)\log_2 P(y=n) - \\
& \Big[-\sum_{k=0}^{1} P(x=k) \sum_{n=0}^{N} P(y=n \mid x=k) \cdot \\
& \log_2 P(y=n \mid x=k) \Big] = \\
& -\sum_{n=0}^{N} P(y=n)\log_2 P^*(y=n) - \\
& \Big\{ -N\sum_{k=0}^{1} P(x=k) \sum_{j=0}^{1} P(y_i=j \mid x=k) \cdot \\
& \log_2 P(y_i=j \mid x=k) \Big\}
\end{aligned} \tag{1}
$$

where $H(y)$ is the information content (or entropy) of y and $H(y \mid x)$ can be interpreted as the amount of encoded information lost in the transmission of the signal; $P(x=k)$ is the probability of input signal x; $P(y=n \mid x=k)$ and $P(y_i=j \mid x=k)$ are conditional probabilities; $P(y=n)$ is the probability of the output y,

$$P(y=n) = \sum_{k=0}^{1} P(x=k)P(y=n \mid x=k)$$

$$P(y=n \mid x=k) = \binom{N}{n} P^n(y_i=1 \mid x=k)P^{N-n}(y_i=0 \mid x=k)$$

$$P^*(y=n) = \frac{P(y=n)}{\binom{N}{n}}$$

where $\binom{N}{n}$ is the binomial coefficient.

2 SR and SSR

Given the probability density function (PDF) of the threshold noise, we can derive $P(y_i=j \mid x=k)$ and then discuss the effect of noise on mutual information I. To demonstrate various conditions where the mutual information I can be improved by adding noise η_i, we consider the case where η_i is chosen in the class of Gaussian mixture noise with PDF:

$$
\begin{aligned}
f_\eta(x) = {} & \frac{1}{\sqrt{2\pi(\sigma^2 - m^2)}} \Big[\alpha\exp\Big(-\frac{(x-m)^2}{2(\sigma^2 - m^2)} \Big) + \\
& (1-\alpha)\exp\Big(-\frac{(x+m)^2}{2(\sigma^2 - m^2)} \Big) \Big] \\
& \sigma \geqslant m, 0 < \alpha < 1
\end{aligned} \tag{2}
$$

When the parameter $m=0$, the Gaussian mixture noise is just the Gaussian noise. Gaussian mixture noise is widely used to model ocean acoustic noise and has been discussed in the research of SR[5 – 7]. Because the parameter α only affects the efficacy of information transmission and does not affect the occurrence of SR and SSR, we can choose $\alpha = 1/2$. We then have

$$P(y_i=0 \mid x=0) = \frac{1}{2} + \frac{1}{4}\Big[\mathrm{erf}\Big(\frac{u-m}{\sqrt{2(\sigma^2 - m^2)}} \Big) + \mathrm{erf}\Big(\frac{u+m}{\sqrt{2(\sigma^2 - m^2)}} \Big) \Big]$$

$$P(y_i=1 \mid x=0) = \frac{1}{2} - \frac{1}{4}\Big[\mathrm{erf}\Big(\frac{u-m}{\sqrt{2(\sigma^2 - m^2)}} \Big) + \mathrm{erf}\Big(\frac{u+m}{\sqrt{2(\sigma^2 - m^2)}} \Big) \Big]$$

$$P(y_i=0 \mid x=1) = \frac{1}{2} + \frac{1}{4}\Big[\mathrm{erf}\Big(\frac{u-1-m}{\sqrt{2(\sigma^2 - m^2)}} \Big) + \mathrm{erf}\Big(\frac{u-1+m}{\sqrt{2(\sigma^2 - m^2)}} \Big) \Big]$$

$$P(y_i=1 \mid x=1) = \frac{1}{2} - \frac{1}{4}\Big[\mathrm{erf}\Big(\frac{u-1-m}{\sqrt{2(\sigma^2 - m^2)}} \Big) + \mathrm{erf}\Big(\frac{u-1+m}{\sqrt{2(\sigma^2 - m^2)}} \Big) \Big]$$

Figs. 1 to 4 show variations of the mutual information I of Eq. (1), as a function of the noise intensity (root-mean-squared (rms) amplitude) σ of the threshold noises η_i in some typical conditions: $m=2$ and $P(x=0)=0.5$. In Fig. 1 ($N=1$) and Fig. 2 ($N=20$), with the increase of threshold level, not only in the single threshold system but also in the array, SSR and SR occur for $u=0.5$ and $u=1.5$. SR disappears for $u=2$ and $u=3$, and SR occurs again for $u=3.1$ and $u=3.3$. SSR existing in a single threshold system is not observed in previous references[9 – 12]. Fig. 1 and Fig. 2 also show that the array can dramatically improve the efficacy of information transmission. Fig. 3 and Fig. 4 show the relation of SR or SSR with the noise parameter m in the single threshold system for two representative threshold levels (subthreshold and suprathreshold for input signal). When the input signal is suprathreshold ($u=0.5$), Fig. 3 shows that SSR does not exist for $m=0$ and $m=0.5$. But SSR occurs for $m=1$ and $m=2$. When the input signal is subthreshold ($u=1.5$), Fig. 4 shows that SR occurs for $m=0$, SR disappears for $m=0.5$ and $m=1$, and SR occurs again for $m=2$. For the Gaussian noise ($m=0$), when the signal is subthreshold, SR exists. However, when the signal is suprathreshold, SSR does not exist. This result agrees with previous results about SR[1 – 8], and may be generalized to unimodal noises.

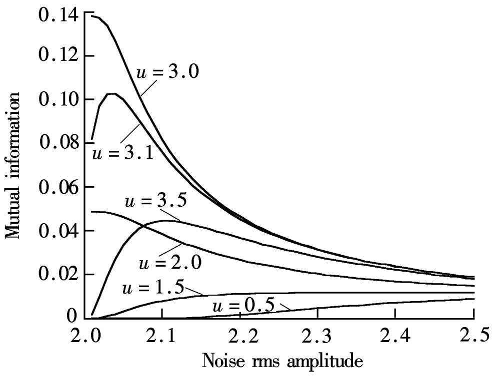

Fig. 1 Mutual information I in the single threshold system for different threshold levels u

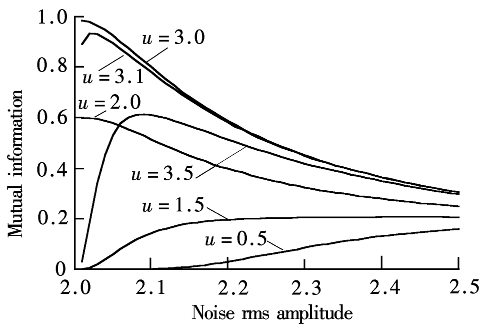

Fig. 2 Mutual information I in the array of 20 threshold devices for different threshold levels u

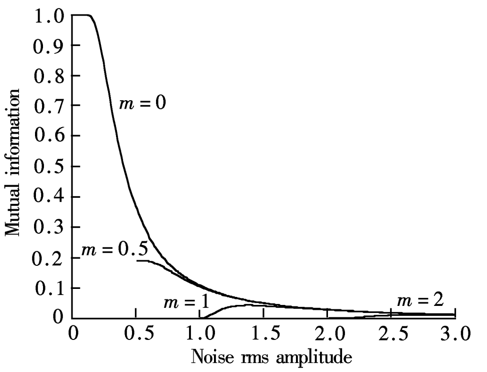

Fig. 3 Mutual information I in the single threshold system for different parameters m ($u = 0.5$)

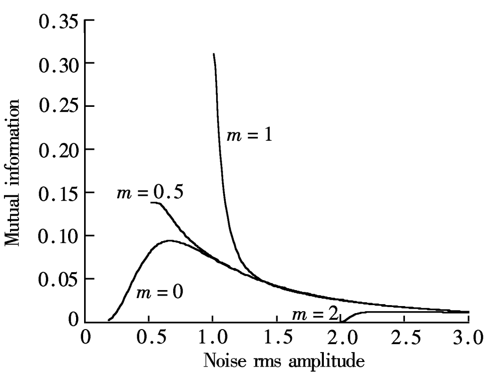

Fig. 4 Mutual information I in the single threshold system for different parameters m ($u = 1.5$)

3 Conclusion

In this paper, we discuss mutual information in a parallel array of threshold devices (especially, when $N = 1$, the array is a single threshold system), the input is a discrete binary signal and the threshold noise is Gaussian mixture noise. Both SR and SSR can be observed not only in the single threshold system but also in the parallel array. The parameter in noise distribution affects the occurrence of SR and SSR. The efficacy of information transmission can be significantly enhanced as the number of threshold devices in the array increases. Some of the above results are similar to previous results about SR[1-8]. And some results are not observed in previous references, especially, that SSR exists in a single nonlinear system. These results show further the dependence of SR and SSR on noise distribution. The effect of SR or SSR may be brought by noise, signal and nonlinearity, and especially by the cooperative action of the three.

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高斯混合噪声下非线性门限阵列中噪声改善信息的传输

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摘要:为了探讨随机谐振现象的发生对信号、非线性系统和噪音的依赖性,特别是对噪音的依赖性,以互信息量为测度研究了二进制信号在非线性门限阵列的传输问题. 在高斯混合噪音下计算了单门限系统和门限阵列的互信息量. 通过研究得到:高斯混合噪音能改善信息的传输;随机谐振和超门限随机谐振现象不但在单门限系统中而且在门限阵列中存在;噪声分布中的参数影响随机谐振和超门限随机谐振现象的出现;随着门限阵列中门限单元数的增加,门限阵列能大大地改善信息的传输效果. 这些结果进一步说明随机谐振和超门限随机谐振现象对噪声分布特性的依赖,同时也拓展了随机谐振和超门限随机谐振在信息传输中的应用.

关键词:随机谐振;超门限随机谐振;互信息;高斯混合噪声
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