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Test analysis on relationship between anti-vibration performance and chaos characteristics of vehicle suspension

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Abstract: Based on the primary principle of anti-vibration on vehicles, a chaos description on the vibration in suspensions is put forward. The vibration curve of the suspensions of test vehicles is obtained based on the data from a test rig for vehicle braking vs. suspension anti-vibration efficiency. The system parameters such as first inherent frequency and damp rate, as well as the chaos parameters such as the minimum embedding dimension and correlation dimension, are calculated by the vibration curve. The relationship among anti-vibration performance, chaos parameters and system parameters of vehicle suspension is presented. The research results show that the minimum embedding dimension $M_{\min}$ can be used to estimate the change of the anti-vibration performance of the front suspension of the off-road jeep. The smaller $M_{\min}$ is, the worse anti-vibration performance is. The corresponding stiffness and damp of the front suspension of the off-road jeep is smaller. Correlation dimension D_2 can be used to identify different suspension types such as those of the off-road jeep and the car. The D_2 of the off-road jeep is larger than the one of the car.

Key words: vehicle; suspension; anti-vibration; chaos

The anti-vibration performance of vehicle suspension influences the smoothness, comfort and control stabilization directly. In order to make the suspension achieve better adaptation to different road conditions, the nonlinear stiffness of suspension and the nonlinear damp of the absorber are usually designed, yet up to now there is not a kind of nonlinear criterion which can be a token of anti-vibration performance. In this paper, based on the chaos properties of unstable and nonlinear dynamic systems, the anti-vibration performance of vehicle suspensions is described by using the chaos characteristics. By testing, the chaos phenomena in signals of suspension vibration is found; by correlation analysis between anti-vibration performance and chaos behavior, the relationship among anti-vibration performance, chaos parameters and system parameters of vehicle suspension is presented. Therefore, it is possible to establish a kind of new criterion that is a token for the anti-vibration performance of vehicle suspensions.

1 Primary Principle of Anti-Vibration on Vehicle Suspension

The anti-vibration on vehicle suspension means setting some components with proper stiffness and damp in the transfer route between the vibration source and objects needed to anti-vibrate; that is, to cut off or prevent vibration transfer. As for the linear anti-vibration system, some rules are listed where f_0 is first inherent frequency, ξ is damp rate, H is transfer function and f is the frequency of exciting force: ① $f \in (0, \sqrt{2} f_0)$ and $H \geqslant 1$, there is not any anti-vibration performance in this value increasing domain; ② $f \geqslant \sqrt{2} f_0$ and $H < 1$, there is anti-vibration performance in this value decreasing domain and it can be designed by controlling f_0 and ξ. By increasing f_0, the value decreasing domain is moved up; by increasing ξ, the anti-vibration performance is decreased while the sympathetic vibration can be restrained effectively. The limitation of the linear anti-vibration system is that the length and performance of the value decreasing domain is almost solely dependent on f_0 and ξ. The anti-vibration effect is not symmetrical. Because of the limitation of structure, the linear anti-vibration cannot always reach the expectant effect on the vehicle suspension, which demands lower first inherent frequency and wider domain of anti-vibration.

The anti-vibration performance of vehicle suspension should be expressed in two ways: ① Based on *Mechanical vibration and shock-evaluation of human exposure to whole-body vibration—Part 1: general requirement*, in the situation that the "limit of tiredness decreasing work efficiency" becomes a lower limit, the driver's tiredness will not increase because of the anti-vibration performance of vehicle suspension after a long time of driving. ② In order to increase the ex-

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posed limit that the passengers can endure[1,2], the transfer rate of vehicle suspension vibration in the sensitive frequency band from 4 Hz to 8 Hz should be decreased. Meanwhile considering that the first inherent frequency of vehicle suspension in China is usually more than or equal to $\sqrt{2}$ Hz[1,3], the anti-vibration performance of vehicle suspension can be reviewed by the transfer rate of vibration in the frequency band from 2 Hz to 8 Hz.

2 Chaos Description on Vehicle Suspension Vibration

Chaos is more prevalent than orderliness in nature and its characteristic factors are unpredictability, inseparability and dense periodicity. Chaos was noticed by a scholar in 1963. When the American meteorologist E. N. Lorenz did the numerical calculations, he found that the non-periodic or chaos result was obtained whereas the parameters of a third degree invariable differential equation that was completely determinate were endued with certain values. Because the subsystems interact and variables interrelate in the complex nonlinear anti-vibration system of vehicle suspension, the output is easier to come into chaos. A time variable will cover the abundant information of all variables which are involved in the dynamic process, so the chaos characteristic in a time serial of suspension vibration signals can be analyzed, and the chaos space formed with some factors which influence the anti-vibration performance of vehicle suspension can be expressed quantificationally. The primary chaos characteristics parameters are minimum embedding dimension $M_{\min}$, correlation dimension D_2, Kolmogorov entropy and so on[4,5].

Reviewing the course of researching chaos, the track[6] is expressed as follows: ① The mechanism of chaotic motion→② The parameters and approaches which influence chaotic motion→③ The characters of chaos and the identification measures for them→④ The chaotic motion rule in different systems→⑤ The control of chaotic motion. Up to now, the research has been focused on the control and synchronization of chaos and complex system motions[7,8], yet the correlative report about the chaos description on the performance of the anti-vibration system has not been found. The anti-vibration device based on the nonlinear principle is researched thoroughly because of the dissatisfaction with linear anti-vibration system, but it is limited in the field of periodic motion. In spite of the fact that much work about act-vibration mechanism has been done by the chaos vibration[9], the anti-vibration problem often met in engineering has not been researched effectively based on the chaos analysis all along, including the anti-vibration performance on vehicle suspension.

The two chaos parameters, such as the minimum embedding dimension and correlation dimension, which are suitable for describing the chaos characteristics of vehicle suspension which belong to unstable and nonlinear dynamics systems, reflect the whole change of the strange attractor in a chaos system. These are chosen as tokens of the relationship between the performance of anti-vibration on vehicle suspension and chaos characteristics. Meanwhile, a new criterion of anti-vibration performance is gained.

3 Minimum Embedding Dimension $M_{\min}$ and Correlation Dimension D_2

The strange attractor in the chaos system is the result of interaction between the whole stability and the part instability, and it is the geometrical object in phase space of the motion orbit after puckering, twisting and extending infinitely, reflecting the degree of chaos. The strange attractor is with the truss of self-comparability; i. e., an infinitely subtle fraction structure that can be gained by re-constructing the phase space. If there is a time serial $\{x_i\}(i=1,2,\ldots,N_0)$, the elements in the re-constructed phase space $\mathbf{R}^M$ are expressed as

$$X_j(M,N_0,\tau)=\{x_j,x_{j+1},\ldots,x_{j+(M-1)\tau}\}$$
$$X_j\in\mathbf{R}^M;\ j=1,2,\ldots,p \tag{1}$$

where M is the dimension of the re-constructed phase space; $\tau=k\Delta t$ is the time delay, k is the natural number, Δt is the sampling interval; $p=N_0-(M-1)\tau$ is the vector number of time serial that embeds in phase space. All the elements of original time serial are involved in the vector aggregation $\{X_j\mid_{j=1,2,\ldots,p}\}$. According to the embedding theory, it can be found that several major characteristics of original space state orbit are reserved in the space state orbit that is formed by the vector aggregation $\{X_j\}$. Through vector aggregation $\{X_j\}$, orbit matrix X is constructed as

$$X=\frac{1}{\sqrt{p}}[X_1^{\mathrm{T}},X_2^{\mathrm{T}},\ldots,X_p^{\mathrm{T}}]^{\mathrm{T}} \tag{2}$$

The covariance matrix $S=X^{\mathrm{T}}X$ is constructed and it becomes a standard model by full-order linearity transform. The characteristic vector groups of S become a group of complete radices of re-constructed phase space. The minimum embedding dimension will be confirmed until the maximum characteristic value of covariance matrix S does not change when the phase space dimension M changes from small to large.

A norm vector X_i is selected arbitrarily from the p vectors in vector aggregation $\{X_j\mid_{j=1,2,\ldots,p}\}$, the distances from the other $p-1$ vectors to vector $\boldsymbol{X}_i$ are calculated as

$$r_{ij} = \| X_{ij} \| = \sum_{l=0}^{p-1} |\boldsymbol{X}_i - \boldsymbol{X}_l| \tag{3}$$

This process is repeated to all $\boldsymbol{X}_i (i=1, 2, \ldots, p)$, the correlation integral is obtained as

$$C_M(\varepsilon) = \frac{2}{p(p-1)} \sum_{i,j=1}^{p} \theta(\varepsilon - \| X_{ij} \|) \tag{4}$$

where $\theta(u)$ is the Heaviside function, $\theta(u) = \begin{cases} 1 & u \geqslant 0 \\ 0 & u < 0 \end{cases}$; ε is the unmarked observation scale, the process of $\varepsilon \to 0$ is attenuated with the constant α and $\alpha = \lim\limits_{n\to\infty} \frac{\varepsilon_n}{\varepsilon_{n+1}} = 2.502\,907\,875\ldots$, $n = 1, 2, 3, \ldots$. If ε is small enough, Eq. (4) approaches Eq. (5).

$$\ln C_M(\varepsilon) = \ln C + D(M) \ln \varepsilon \tag{5}$$

The correlation dimension D_2 of the strange attractor in $\mathbf{R}^M$ can be expressed as

$$D_2 = \lim_{M\to\infty,\, \varepsilon\to 0} \frac{\partial \ln C_M(\varepsilon)}{\partial \ln \varepsilon} \tag{6}$$

4 Experiments and Analysis of Results

The experiments are carried out by two off-road jeeps BJ212 and BJ2020S with leaf spring rigid axle suspension, and by two passenger cars SUZUKI 2.5 and HONDA 3.0 with coil spring independent suspension. The test rig for vehicle braking vs. suspension anti-vibration efficiency, which is co-produced by Tsinghua University and Italy Orient Company, is shown in Fig. 1. The vibration curve of the suspension

Fig. 1 4PLD test rig for vehicle braking vs. suspension anti-vibration efficiency

of the test vehicle at a velocity about 5 km/h is obtained (see Fig. 2) with an example of the off-road jeep BJ2020S. The average result is obtained after 10 tests are repeated under each test condition. The system parameters such as first inherent frequency and damp rate[3] can be obtained, meanwhile the chaos parameters such as the minimum embedding dimension and correlation dimension can be calculated by the vibration curve.

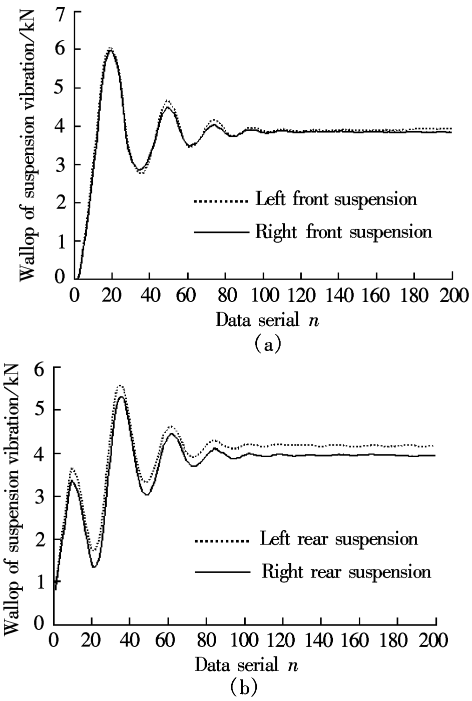

Fig. 2 Axis impulse vs. vibration response curve of BJ2020S off-road jeep suspension. (a) Front suspension vibration; (b) Rear suspension vibration

4.1 Test result and analysis while changing the numbers of leaves of front suspension

To the front suspension of BJ212, the response curve of vibration is obtained under the conditions preserving the numbers of six leaves that is in original condition, four leaves and two leaves, respectively. The stiffness and damp of the front suspension decrease at the same time.

Reviewing the data and test results under the same conditions, it is found that the system parameters and chaos parameters of the left front-suspension are almost coherent to the ones of the right, which means there is a symmetry between the left front-wheel and the right one of the test vehicle. Different damps are obtained by calculations on the vibration response curve with different overshoots, so the front suspension of the test vehicle is a nonlinear system[3]. After averaging the parameters between the left and the right front-suspensions, the system parameters and chaos parameters are obtained and expressed in Tab. 1.

Tab. 1 Parameters while changing the numbers of leaves of front suspension

Test conditions	Front suspension		System parameters		Chaos parameters	
(Numbers of leaves spring)	Stiffness and damp	Anti-vibration performance	f_0/Hz	ξ	$M_{\min}$	D_2
6	Original value	Original performance	1.275 7	0.355 4	67.82	0.581 1
4	Middle value	Worse performance	1.145 3	0.337 5	53.58	0.525 0
2	Minimum value	Worst performance	1.100 4	0.320 1	47.71	0.604 7

From Tab. 1, what can be learned are listed below:

1) When the numbers of leaves spring is decreased from 6 to 2, the first inherent frequency f_0 of the front suspension is decreased by 14%, meanwhile the damp rate ξ is decreased by 10%.

2) To the in-use vehicle or production vehicle, the parameters of the front suspension in original condition correspond to the better designed anti-vibration performance. After decreasing the numbers of leaves the minimum embedding dimension $M_{\min}$ would be decreased with the anti-vibration performance decreasing. This strong correspondence between the anti-vibration performance of the front suspension and the minimum embedding dimension $M_{\min}$ can be used to identify different anti-vibration performances of front suspensions of the same type vehicle.

3) The minimum embedding dimension $M_{\min}$ is decreased with the first inherent frequency f_0 and damp rate ξ of front suspension decreasing, but there is not a corresponding relationship between the correlation dimension D_2 and f_0 or ξ.

4.2 Test result and analysis for front suspension of different types of vehicles

It is shown by the test data that there is a symmetry between the left front-wheel and right one of the test vehicle. After averaging the parameters between the left and the right front-suspensions, the system parameters and chaos parameters are obtained and expressed in Tab. 2.

Tab. 2 Vibration parameters of front suspension of different types of vehicles (original condition)

Test vehicles	Vehicle types	Front suspension types	System parameters		Chaos parameters	
			f_0/Hz	ξ	$M_{\min}$	D_2
BJ212	Off-road jeep	Leaf spring rigid axle suspension	1.275 7	0.355 4	67.82	0.581 1
BJ2020S			1.358 0	0.292 2	136.56	0.551 6
SUZUKI 2.5	Car	Coil spring independent suspension	1.416 0	0.390 2	46.02	0.354 4
HONDA 3.0			1.310 0	0.371 9	33.64	0.316 0

From Tab. 2, what can be learned are listed below:

1) The first inherent frequency f_0 of front suspension of different types of vehicles is about 2 Hz corresponding to the test result presented in Ref. [3]. The anti-vibration domain is begun from about 2 Hz in which the most sensitive frequency band to the human body from 4 Hz to 8 Hz is involved.

2) The first inherent frequency f_0 of the same type vehicles can be compared with each other, such as the off-road jeeps BJ212 and BJ2020S, or the cars SUZUKI 2.5 and HONDA 3.0. Meanwhile the correlation dimension D_2 of the same type vehicles can also be compared with each other. For different types of vehicle suspension, for example, the suspension of the off-road jeep or the car, there is not any difference in the first inherent frequency f_0 of suspension. However, there is an obvious difference in the correlation dimension D_2 of suspension. The D_2 of the off-road jeep suspension is larger than that of the car suspension. This strong correspondence between the type of vehicle front suspension and correlation dimension D_2 can be used to identify the type of front suspension.

3) There is not any correspondence between the type of vehicle front suspension and the minimum embedding dimension $M_{\min}$.

4.3 Test result and analysis for rear suspension of different types of vehicles

Reviewing the data and results of the tests, it is found that the system parameters are almost coherent, and the chaos parameters are the same, calculating from the vibration curve of left-right rear suspension of the test vehicle. That means there is a symmetry between the left rear-wheel and the right one of the test vehicle. The system parameters and chaos parameters are obtained and expressed in Tab. 3.

From Tab. 3, what can be learned are listed below:

1) The first inherent frequency f_0 of rear suspensions of different types of vehicles is about 2 Hz which is larger than the one of front suspensions. The anti-vibration domain begins from about 2.8 Hz in which the most sensitive frequency band to the human body from 4 Hz to 8 Hz is still involved.

Tab. 3 Vibration parameters of rear suspension of different types of vehicles (original condition)

Test vehicles	Vehicle types	Rear suspension types	System parameters		Chaos parameters	
			f_0/Hz	ξ	$M_{\min}$	D_2
BJ212	Off-road jeep	Leaf spring rigid axle suspension	2.034	0.157 6	61.82	0.568 1
BJ2020S			1.860	0.185 4	136.5	0.551 6
SUZUKI 2.5	Car	Coil spring independent suspension	2.049	0.475 3	46.02	0.364 4
HONDA 3.0			1.993	0.185 5	33.97	0.316 0

2) Similar to the situation of the front suspension, there is a strong correspondence between the type of vehicle rear suspension and correlation dimension D_2. This feature can be used to identify different types of rear suspension. Meanwhile, the correlation dimension D_2 of rear suspension of the off-road jeep is larger than the one of the car.

3) There is not any correspondence between the type of test vehicle rear suspension and the minimum embedding dimension $M_{\min}$.

5 Conclusions

1) The anti-vibration domain begins from about $\sqrt{2}f_0$ while the first inherent frequency of linear anti-vibration device is f_0. The anti-vibration performance of vehicle front suspension should be estimated in the frequency band from 2 Hz to 8 Hz.

2) The minimum embedding dimension $M_{\min}$ can be used to estimate the change in the anti-vibration performance of the front suspension of the off-road jeep. If the $M_{\min}$ is smaller, the anti-vibration performance is worse, and the corresponding stiffness and damp of front suspension of the off-road jeep are smaller.

3) Correlation dimension D_2 can be used to identify different suspension types such as those of the off-road jeep and the car. The D_2 of the off-road jeep is larger than the one of the car.

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汽车悬架隔振性能与混沌特征关系的实验分析

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摘要:根据汽车隔振基本原理,提出了悬架振动的混沌描述问题.采用汽车制动-悬架隔振效率实验台获取了实验汽车前、后悬架的振动曲线,计算了系统参数如一阶固有频率和阻尼比,并计算了混沌参数如最小嵌入相空间维数和关联维,获得了汽车悬架的隔振性能、混沌参数与系统参数三者之间的对应关系.研究结果表明:对于吉普车型,可采用最小嵌入相空间维数 $M_{\min}$ 评价前悬架隔振性能的变化, $M_{\min}$ 值越小,隔振性能趋差,对应于前悬架的刚度和阻尼值越小;对于不同车型,可采用关联维 D_2 区分吉普车型或轿车型的悬架,吉普车型的 D_2 值高于轿车型.

关键词:汽车;悬架;隔振;混沌

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