Performance analysis on captive silicon micro-accelerometer system

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Abstract: The operational principle and the lumped parameters model of capacitive micro-accelerometer are introduced. The equivalent stiffness of different directions of the accelerometer is given. From the point of view of energy and mechanics, expressions of some key parameters, such as the damping, sensitivity, resolution of the accelerometer, are derived. The accelerometer noise behavior of mechanical-thermal noise in the open-loop system, along with the dynamic range of the open-loop system and closed-loop system is analyzed. The result is that the noise of the capacitive micro accelerometer is dominated by the magnitude of mechanical-thermal noise. At the same time, the magnitude of mechanical-thermal noise depends on the temperature and magnitude of mechanical damp. The result of the measurement from the implemented closed-loop micro-accelerometer system shows that the resolution is the level of mg, and the measurement range is from $-50g$ to $50g$.

Key words: micro-accelerometer; sensitivity; spectral density; mechanical-thermal noise; damping

The micro mechanical accelerometer is a miniature inertial instrument, which is based on the technology of micro electromechanical system (MEMS). The capacitive accelerometer is one type of micro silicon accelerometers and has the virtues of high precision, fine noise character, small temperature sensitivity, low power consumption and simple structure. It has turned out to be the main stream of the micro silicon accelerometer development.

In this paper, the operational principle of the accelerometer is discussed in detail. Some key parameters of the accelerometer have been derived. The noise character and the dynamic measurement range of the open-loop and the closed-loop accelerometer system are also analyzed. The practical measurement result is given.

1 Operational Principle

Fig. 1 is a simplified figure of the capacitive accelerometer. The sensing part of the accelerometer is made up of the proof mass and the flexural beam. The proof mass is connected to the mono-crystalline silicon substrate and is suspended on top of the substrate by the flexible beam. The two poly-silicon plates, which are used as the sensing electrode, are connected to the substrate by the supporting rib and are suspended equi-distantly from the proof mass. Differential capacitors are formed between the plates and the proof mass. When acceleration is exerted about the z-axis,

because of the inertial force, the proof mass will produce a mini displacement Δd about the z-axis. Capacitance between the proof mass and the up-down plates will change. When difference between the capacitance of the capacitors is measured by the measure circuit, the acceleration will be computed[1].
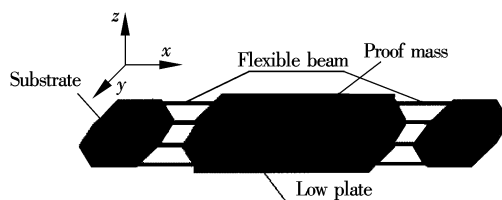

Fig. 1   Structure of the capacitive silicon micro-accelerometer

Fig. 2 is a lumped model of a capacitive accelerometer system. The system is made up of proof mass, suspension, damper and circuit in converting the acceleration to an electrical signal.
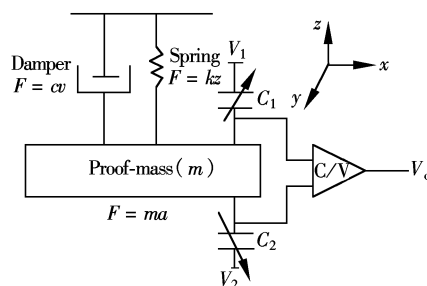

Fig. 2   Lumped model of an accelerometer system

2 Key Parameters

2.1 Equivalent stiffness of the suspension

Based on the theory of mechanics, the stiffness of the single end fixed beam without residual stress can be obtained by[2]

$$k_z = \frac{Ew^3 h}{l_b^3}, \qquad k_x = \frac{Ewh^3}{l_b^3} \qquad (1)$$

where E is the Young's modulus, l_b is the length of the beam, w is the width of the beam, h is the thickness of the beam, k_z is the stiffness about the z-axis, and k_x is the stiffness about the x-axis.

When the proof mass vibrates with small angle about the y-axis, the equivalent stiffness k_y about the y-axis, which is derived by the vibrating differential equation, can be written as

$$k_y = 3 \frac{Ewh^3 W^2 + w^3 h H^2}{l_b^3 (W^2 + H^2)} \qquad (2)$$

where W and H are the width and thickness of the proof mass. Thus, the equivalent stiffness of the accelerometer is only the function of the structure parameters of the suspension and proof mass.

2. 2　Equivalent mass

Based on the energy view, the equivalent mass of the system can also be computed. If the system is treated as a lumped mass system, the kinetic energy of the simplified system is equal to the kinetic energy of the true system. The expression is as follows:

$$\frac{1}{2} m_e v_{max}^2 = \sum_{i=1}^{N} \frac{1}{2} \int_{-L/2}^{L/2} \frac{m_i v_i^2(\xi)}{L_i} d\xi \qquad (3)$$

where m_e is the equivalent mass, v_{max} is the velocity of the centroid of the system, m_i is the distribution mass of the system, and v_i is the velocity of the distribution mass of the system.

2. 3　Damping

The performance of the accelerometer is affected not only by the stiffness of the suspension, but also by the gas damp force. Fig. 3 is a schematic cross-section of a typical capacitive accelerometer. When the proof mass is at the center, d_0 is the distance between the bottom of upper plate of the capacitor and the top of the proof mass. Compared with d_0, when the displacement of the proof mass is small, the damping force for a rectangular plate with no holes is approximately given as[3]

$$f_c = \frac{f(W_p/L)\mu W_p^3 L \partial d}{d_0^3 \partial t} \qquad 0 < \frac{W_p}{L} < 1 \qquad (4)$$

where $f\left(\dfrac{W_p}{L}\right) = 1 + \dfrac{W_p}{L}\left(0.2\dfrac{W_p}{L} - 0.785\right)$, W_p is the width of the plate, L is the length of the plate, μ is the viscosity of the fluid, and d is the displacement of the proof mass.

If one of the plates is perforated, the damping force is approximately given as[3]

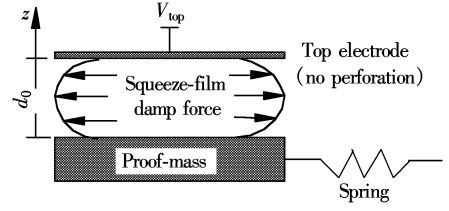$$f_{c\text{-perf}} = \frac{12\mu B(A) S^2 \partial d}{\pi N d_0 \partial t} \qquad (5)$$

Fig. 3　Model of the parallel-plate capacitive accelerometer

where $B(A) = \dfrac{1}{4}\ln\dfrac{1}{A} - \dfrac{3}{8} + \dfrac{1}{2}A - \dfrac{1}{8}A^2$, S is the area of the plate, N is the total number of the holes in the perforated plate, and A is the fraction of open area in the plate.

When the displacement of the proof mass is significant with respect to the air-gap, the damping force becomes a function of the displacement of the proof mass. When the plate is not perforated, the damping force is described as[3]

$$f_c = \frac{1}{(1-\gamma)^{3/2}} \frac{f(W/L)\mu W^3 L \partial d}{d_0^3 \partial t} \qquad (6)$$

where γ is the ratio of the mass displacement to the air-gap.

When the plate is perforated, the damping force becomes[3]

$$f_{c\text{-perf}} = k\mu S^2 \left[\frac{1}{(d_0 + d)^3} + \frac{1}{(d_0 - d)^3}\right]\frac{\partial d}{\partial t} \qquad (7)$$

where $k = 12B(A)/(\pi N)$.

Thus, as the moving amplitude of the proof mass increases, the damping force will increase faster.

2. 4　Sensitivity

As shown in Fig. 2, the motion of the proof mass causes the change in the capacitance of the capacitor and the change of the capacitance will be used to measure the acceleration. The overall system sensitivity can be written as

$$S_0 = \frac{dC}{da} = \frac{dC}{dz}\frac{dz}{da} \qquad (8)$$

where $\dfrac{dz}{da}$ is defined as $\dfrac{dz}{da} = \dfrac{m}{k} = \dfrac{1}{\omega_n^2}$.

When the edge dimension of the capacitor is much greater than that of the air gap, the capacitance of each capacitor can be approximately written as

$$C = \frac{\varepsilon A_e}{d_c} \qquad (9)$$

where ε is the permittivity, A_e is the electrode area, and d_c is the initial air-gap. Differential equation of Eq. (9) is

$$\frac{dC}{dz} = -\frac{\varepsilon A_e}{d_c^2} \qquad (10)$$

Thus, the sensitivity of the accelerometer is

$$S_0 = \left| \frac{\mathrm{d}C}{\mathrm{d}a} \right| = \frac{\mathrm{d}C}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}a} = \frac{\varepsilon A m}{k_z d_c^2} = \frac{\varepsilon A_e}{d_c^2 \omega_n^2} \qquad (11)$$

If the displacement of the proof mass is very small, Eq. (9) can be appropriated as

$$S_0 = \frac{\mathrm{d}C}{\mathrm{d}a} \approx \frac{2\varepsilon A_e \Delta z_{\mathrm{deft}}}{d_0^3 \, \omega_n^2} \qquad (12)$$

where Δz_{deft} is the deflection of the proof mass from its equilibrium position.

When A_e becomes comparable to or greater than the air-gap, the effects of the fringe fields must be included. A more accurate expression is given by[4]

$$C = \frac{\varepsilon A_e}{d_c} + \frac{\varepsilon w_e}{\pi}\ln\left(\frac{w_e \pi}{d_c}\right) \qquad (13)$$

where w_e is the edge dimension. Decreasing k_z and d_0, together with increasing m and A, can improve the sensitivity of the accelerometer.

2.5 Resolution

Resolution is the minimum acceleration that can be detected reliably by the accelerometer system. The minimum acceleration is determined by the mechanical noise, thermal noise and the noise from the detected circuit. An equivalent accelerometer spectral density, caused by the mechanical noise and the thermal noise, can be calculated by[5]

$$D = \sqrt{\frac{4 k_B T \omega_n}{Q m}} \qquad (14)$$

where T is the absolute temperature, $Q = \sqrt{km}/c$ is the quality factor, and k_B is the Boltzman constant.

From Eq. (14), we can see that in order to achieve a low noise floor, we need a large mass m and high Q. Reducing the damping can increase the quality factor. Increasing the proof mass and perforating the plate can also decrease the noise floor of the system.

3　Noise

3.1　Noise of the open-loop accelerometer

The thermal kinematics equation of the accelerometer is described as[6]

$$m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + c\frac{\mathrm{d}z}{\mathrm{d}t} + k_z z = F_n \qquad (15)$$

where z is the displacement of the proof mass, m is the mass of the proof mass, F_n is the driving force, and c is the damping. Based on the thermo-kinetics, the motional mass of the micro mechanical thermal sensor in

the molecular level will produce the Brownian motion. Thus, the mean square displacement of the model of mass-spring from the thermal motion is given by[7]

$$\Delta z_{\mathrm{mns}} = \sqrt{\frac{k_B T}{k_z}} \qquad (16)$$

So the displacement depends not only on the damping of the fluid, but also on the stiffness of the suspension. As we know, the minimum acceleration $a_{\min}$, which the accelerometer can detect will be larger than the mean square of the noise. Thus

$$a_{\min} = \frac{k_z}{m}\Delta z_{\mathrm{mns}} = \frac{\sqrt{k_B T k_z}}{m} \qquad (17)$$

Decreasing k_z or increasing m can decrease the sensitivity threshold of the accelerometer. But the increase of the sensitivity will cause the increase of the proof mass displacement.

The measurable maximum acceleration of the open-loop accelerometer depends on the sensor's dimension. The maximum displacement of the proof mass must be smaller than $\Delta z_{\max}$, where $\Delta z_{\max}$ depends on d_0 and h_0. The corresponding maximum acceleration can be written as

$$a_{\max} = \frac{k_z \Delta z_{\max}}{m} \qquad (18)$$

Thus, the dynamic range, which the open-loop accelerometer can measure is given as

$$R_o = \frac{a_{\max}}{a_{\min}} = \Delta z_{\max}\sqrt{\frac{K_z}{k_B T}} \qquad (19)$$

3.2　Noise of the servo accelerometer

The block diagram of the servo accelerometer is shown in Fig. 4. The transfer function of the input force F_{in} to the output voltage V_{fb} is written as

$$H_F = \frac{V_{\mathrm{fb}}}{F_{\mathrm{in}}} = \frac{K K_{\mathrm{pos}}}{f_z \tau s^2 + (f_z + K_z \tau) s + K_z + K_{\mathrm{pos}} K K_{\mathrm{fb}}} \qquad (20)$$

where K is the gain, K_{pos} is the gain of the displacement detector, τ is the time constant, and $\tau_2 = f_z/k_z$. The output noise density of the system is given as[7]

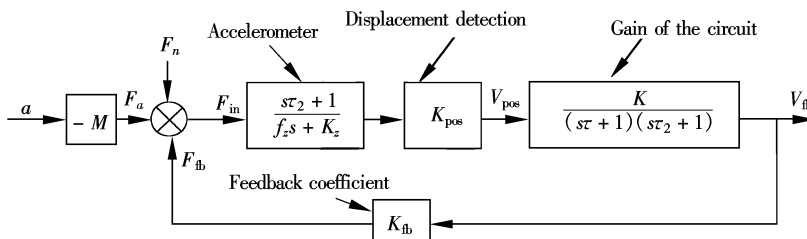$$\overline{V_{\mathrm{fb}}^2} = |H_F|^2 \frac{4 k_B T f_z}{1 + (\omega \tau_2)^2} \qquad (21)$$

Fig. 4　Block diagram of the servo accelerometer

The measurable minimum acceleration is calculated by

$$a_{\min} = \frac{K_z + K_{pos}KK_{fb}}{mKK_{pos}}V_{fbms} \approx \frac{\sqrt{k_B TK_{pos}KK_{fb}}}{m} \quad (22)$$

The measurable maximum acceleration in the closed-loop accelerometer is determined by the maximum feedback voltage $V_{fbmax} = V_0$, where V_0 is the bias voltage imposed on the plates. The mean square acceleration is decided by

$$\overline{a_n^2} = \frac{\overline{F_n^2}}{m^2} = \frac{4k_B Tf_z}{m^2} \quad (23)$$

The dynamic range of the servo accelerometer system is described as

$$R_c \approx V_0 \sqrt{\frac{K_{fb}}{k_B TK_{pos}K}} \quad (24)$$

Whether it be the open-loop accelerometer system or the closed-loop accelerometer system, their equivalent noise spectral density is the same and can be written as[7]

$$\overline{a_n^2} = \frac{\overline{F_n^2}}{m^2} = \frac{4k_B Tf_z}{m^2} \quad (25)$$

Based on the preceding theory, a closed-loop accelerometer system has been designed. The measurement result of the system is that the sensitivity is the level of mg and the measurable range is from $-50g$ to $50g$. The accelerometer system has been used in the automotive brake system.

4 Conclusion

In this paper, the factors, which affect the micro silicon capacitive accelerometer performance are analyzed in detail. It can supply the design of the accelero-meter with some theoretical reference.

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电容式硅微加速度计系统的性能分析

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摘要:介绍了微加速度计的工作原理,得出了不同方向上的加速度计的等效刚度.基于加速度计的集总模型分析,从能量和运动角度推导得出了加速度计的等效质量、精度和灵敏度的具体表达形式.从力学观点,总结了加速度计阻尼的影响因素.对开环和闭环加速度计系统的机械−热噪声的特性和它们各自检测的动态范围进行研究后发现,电容式微加速度计的噪声主要由机械热噪声决定,并取决于加速度计系统的温度和阻尼.对闭环加速度计系统的测量结果表明,系统的精度可以达到 mg 数量级,量程为 ±50g 的范围.
关键词:微加速度计;灵敏度;谱密度;机械热噪声;阻尼
中图分类号:TH824.4