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Multiple attribute decision making based on different types of linguistic information

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Abstract: Distance measures between exact linguistic variables and between uncertain linguistic variables are introduced respectively. Based on exact linguistic variables and uncertain linguistic variables, the concepts of positive linguistic ideal solution and negative linguistic ideal solution of attribute values are defined. To rank and select alternatives, based on the distance measures of two types of linguistic variables and the linguistic ideal solutions, a method for multiple attribute decision making with different types of linguistic information is proposed, by which all alternatives can be ranked. The method can carry out linguistic computation processes easily without loss of linguistic information, and thus makes the decision result reasonable and effective. Finally, the implementation process of the proposed method is illustrated and analyzed by a numerical example.

Key words: multiple attribute decision making; linguistic ideal solution; distance measure; linguistic variable

In multiple attribute decision making, a decision maker (DM) usually expresses his/her preference information over alternatives with linguistic variables. For example, when evaluating "comfort" or "design" of a car, linguistic labels like "good", "fair", "poor" are usually used; when evaluating a car's speed, linguistic labels like "fast", "very fast", "slow" can be used[1,2]. Several authors have paid attention to this research domain, and proposed some approaches to solving the multiple attribute decision making problem with linguistic information[2-4]. However, all these approaches are focused on the situations where all the linguistic information in a decision matrix takes the same form, either exact linguistic variables[2,3] or uncertain linguistic variables[4]. In this paper, we focus on the multiple attribute decision making problem with different types of linguistic information.

1 Basic Concepts

In the process of multiple attribute decision making, the decision maker may provide different types of linguistic information as a result of time-pressure, lack of knowledge or data, and his/her limited expertise related with the problem domain. In the following we will first review some basic concepts, which will be used throughout this contribution.

A multiple attribute decision making problem in a linguistic environment involves choices from a finite discrete set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, where the DM provides his/her preference information by using linguistic labels[5-9]. In Ref. [9], Xu defined a finite and totally ordered discrete linguistic label set $S = \{s_\alpha \mid \alpha = -t, \ldots, t\}$, whose cardinality value is an odd one, such as 7 and 9, and it requires that ① $s_\alpha < s_\beta$ iff $\alpha < \beta$; and ② There is a negation operator: $\mathrm{neg}(s_\alpha) = s_{-\alpha}$, especially, $\mathrm{neg}(s_0) = s_0$. For example, S can be assumed to be $S = \{s_{-4}$ = extremely poor, s_{-3} = very poor, s_{-2} = poor, s_{-1} = slightly poor, s_0 = fair, s_1 = slightly good, s_2 = good, s_3 = very good, s_4 = extremely good$\}$. In the process of information aggregation, however, some results may not exactly match any linguistic labels in S. To preserve all the information, Xu[9] extended the discrete linguistic label set S to a continuous linguistic label set $\bar{S} = \{s_\alpha \mid \alpha \in [-q, q]\}$, where q $(q > t)$ is a sufficiently large positive integer.

Definition 1[9] If $s_\alpha \in S$, then s_α is termed an original linguistic label, otherwise, s_α is termed a virtual linguistic label. In general, the DM uses the original linguistic labels to evaluate alternatives, and the virtual linguistic labels can only appear in the actual calculation.

Definition 2[9] For any two linguistic labels s_α, $s_\beta \in \bar{S}$, their operational laws are defined as follows: ① $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$; ② $\lambda s_\alpha = s_{\lambda\alpha}$, $\lambda \in [0, 1]$.

Definition 3[4] Let $\tilde{s} = [s_\alpha, s_\beta]$, where $s_\alpha, s_\beta \in \bar{S}$, s_α and s_β are the lower and upper limits, respectively, then $\tilde{s}$ is called an uncertain linguistic variable. Let $\tilde{S}$ be the set of all the uncertain linguistic variables.

Definition 4[4] For any three uncertain linguistic variables $\tilde{s} = [s_\alpha, s_\beta]$, $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}] \in \tilde{S}$, their operational laws are defined as follows: ① $\tilde{s}_1 \oplus$

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$\tilde{s}_2 = [s_{\alpha_1}, s_{\beta_1}] \oplus [s_{\alpha_2}, s_{\beta_2}] = [s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}]$; ② $\lambda\tilde{s} = [s_{\lambda\alpha}, s_{\lambda\beta}], \lambda \in [0,1]$.

Definition 5 Let $s_\alpha, s_\beta \in \bar{S}$, then we define the distance between s_α and s_β as follows:

$$d(s_\alpha, s_\beta) = |\alpha - \beta| \quad (1)$$

Definition 6 Let $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}], \tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}] \in \tilde{S}$, then we define the distance between $\tilde{s}_1$ and $\tilde{s}_2$ as follows:

$$d(\tilde{s}_1, \tilde{s}_2) = \frac{1}{2}[|\alpha_1 - \alpha_2| + |\beta_1 - \beta_2|] \quad (2)$$

2 An Ideal Solution Based Method

A multiple attribute decision making problem with different types of linguistic information is represented as follows:

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the set of alternatives, and $U = \{u_1, u_2, \ldots, u_m\}$ be the set of attributes. Let $\boldsymbol{w} = \{w_1, w_2, \ldots, w_m\}$ be the weight vector of attributes, where $w_i \geqslant 0, i = 1, 2, \ldots, m, \sum_{i=1}^{m} w_i = 1$. Suppose that $\boldsymbol{A} = (a_{ij})_{m\times n}$ is the linguistic decision matrix with different types of linguistic information, where a_{ij} is the attribute value, given by the DM, for the alternative $x_j \in X$ with respect to the attribute $u_i \in U$. Without loss of generality, let $a_{ij} \in S(i = 1, 2, \ldots, l; j = 1, 2, \ldots, n)$, and let $a_{ij} = [a_{ij}^L, a_{ij}^U] \in \tilde{S}\ (i = l+1, \ldots, m; j = 1, 2, \ldots, n)$. Suppose that $\boldsymbol{a}_j = \{a_{1j}, a_{2j}, \ldots, a_{mj}\}$ be the vector of the attribute values corresponding to the alternative $x_j (j = 1, 2, \ldots, n)$.

Definition 7 Let $\boldsymbol{A} = (a_{ij})_{m\times n}$ be the decision matrix with different types of linguistic information, then we call $\boldsymbol{I}^+ = (I_1^+, I_2^+, \ldots, I_m^+)$ the positive linguistic ideal solution of attribute values, where $I_i^+ = \max_j\{a_{ij}\}, i = 1, 2, \ldots, l$; $I_i^+ = [I_i^{+L}, I_i^{+U}] = [\max_j\{a_{ij}^L\}, \max_j\{a_{ij}^U\}], i = l+1, \ldots, m$.

Definition 8 Let $\boldsymbol{A} = (a_{ij})_{m\times n}$ be the decision matrix with different types of linguistic information, then we call $\boldsymbol{I}^- = (I_1^-, I_2^-, \ldots, I_m^-)$ the negative linguistic ideal solution of attribute values, where $I_i^- = \min_j\{a_{ij}\}, i = 1, 2, \ldots, l$; $I_i^- = [I_i^{-L}, I_i^{-U}] = [\min_j\{a_{ij}^L\}, \min_j\{a_{ij}^U\}], i = l+1, \ldots, m$.

In the following we propose a practical method based on distance measures and linguistic ideal solutions for the multiple attribute decision making problem with different types of linguistic information.

Step 1 Calculate the distance between the alternative x_j and positive linguistic ideal solution by Eqs. (1) and (2) and

$$d(\boldsymbol{a}_j, \boldsymbol{I}^+) = \sum_{i=1}^{m} w_i d(a_{ij}, I_i^+) \qquad j = 1, 2, \ldots, n \quad (3)$$

Step 2 Calculate the distance between the alternative x_j and negative linguistic ideal solution by Eqs. (1) and (2) and

$$d(\boldsymbol{a}_j, \boldsymbol{I}^-) = \sum_{i=1}^{m} w_i d(a_{ij}, I_i^-) \qquad j = 1, 2, \ldots, n \quad (4)$$

Step 3 Calculate the relative distance d_j corresponding to the alternative x_j:

$$d_j = \frac{d(\boldsymbol{a}_j, \boldsymbol{I}^+)}{d(\boldsymbol{a}_j, \boldsymbol{I}^+) + d(\boldsymbol{a}_j, \boldsymbol{I}^-)} \qquad j = 1, 2, \ldots, n \quad (5)$$

It is clear that the smaller the relative distance d_j is, the better the alternative x_j is.

Step 4 Rank all the alternatives and select the best one(s) in accordance with $d_j (j = 1, 2, \ldots, n)$.

From the above procedure, we know that the method first calculates the absolute distances between each alternative and two linguistic ideal solutions, and then based on these absolute distances, the method calculates the relative distances, which can be used to rank and select the alternatives. The prominent characteristic of the method is that it can carry out linguistic computation process easily without any loss of information, and thus makes the decision result reasonable and effective.

3 Illustrative Example

Let us consider a customer who intends to buy a car. Four types of cars $x_j (j = 1, 2, 3, 4)$ are available (adapted from Ref. [3]). The customer takes into account four attributes to decide which car to buy: ① u_1: economy, ② u_2: comfort, ③ u_3: design, and ④ u_4: safety. The DM evaluates these four types of cars $x_j (j = 1, 2, 3, 4)$ under the attributes $u_i (i = 1, 2, 3, 4)$ (whose weight vector is $\boldsymbol{w} = \{0.2, 0.4, 0.1, 0.3\}$) by using the linguistic label set $S = \{s_{-4}$ = extremely poor, s_{-3} = very poor, s_{-2} = poor, s_{-1} = slightly poor, s_0 = fair, s_1 = slightly good, s_2 = good, s_3 = very good, s_4 = extremely good$\}$, and gives a linguistic decision matrix as listed in Tab. 1.

Tab. 1 Linguistic decision matrix ***A***

u_i	x_1	x_2	x_3	x_4
u_1	s_{-2}	s_1	s_4	s_3
u_2	s_1	s_3	s_3	s_2
u_3	$[s_3, s_4]$	$[s_{-1}, s_1]$	$[s_{-2}, s_0]$	$[s_0, s_1]$
u_4	$[s_2, s_3]$	$[s_1, s_2]$	$[s_0, s_1]$	$[s_2, s_4]$

In the following, we utilize the proposed method to get the optimal car.

Step 1 From Tab. 1, we get the vectors of the attribute values corresponding to the alternative $x_j (j = 1, 2, 3, 4)$, and the vectors of the positive ideal solution and negative ideal solution as follows:

$$\boldsymbol{a}_1 = \{s_{-2}, s_1, [s_3, s_4], [s_2, s_3]\}$$

$$a_2 = \{s_1, s_3, [s_{-1}, s_1], [s_1, s_2]\}$$
$$a_3 = \{s_4, s_3, [s_{-2}, s_0], [s_0, s_1]\}$$
$$a_4 = \{s_3, s_2, [s_0, s_1], [s_2, s_4]\}$$
$$I^+ = \{s_4, s_3, [s_3, s_4], [s_2, s_4]\}$$
$$I^- = \{s_{-2}, s_1, [s_{-2}, s_0], [s_0, s_1]\}$$

Step 2 Calculate the distance between the alternative x_j and positive ideal solution by Eqs. (1) to (3):

$$d(a_1, I^+) = 2.15, \quad d(a_2, I^+) = 1.4$$
$$d(a_3, I^+) = 1.2, \quad d(a_4, I^+) = 0.9$$

Step 3 Calculate the distance between the alternative x_j and negative ideal solution by Eqs. (1), (2) and (4):

$$d(a_1, I^-) = 1.05, \quad d(a_2, I^-) = 1.8$$
$$d(a_3, I^-) = 2.0, \quad d(a_4, I^-) = 2.3$$

Step 4 Calculate the relative distance d_j corresponding to the alternative x_j by Eq. (5):

$$d_1 = 0.672, \quad d_2 = 0.437$$
$$d_3 = 0.375, \quad d_4 = 0.281$$

Step 5 Rank all the alternatives $x_j (j = 1, 2, 3, 4)$ in accordance with $d_j (j = 1, 2, 3, 4)$:

$$x_4 > x_3 > x_2 > x_1$$

and thus the best car is x_4.

4 Conclusion

In this paper, we have investigated the multiple attribute decision making problem with different types of linguistic information, in which the attribute values take the form of exact linguistic variables and uncertain linguistic variables. We have defined some distance measures, and introduced the concepts of positive and negative linguistic ideal solutions. Based on the distance measures and linguistic ideal solutions, we have proposed a practical method for ranking the decision alternatives, which can carry out linguistic computation processes easily without loss of information.

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不同类型语言信息下的多属性决策

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摘要:介绍了精确语言变量和不确定语言变量的距离测度,并且基于精确语言变量和不确定语言变量,给出了属性值的正语言理想解和负语言理想解的概念. 为了对决策方案进行排序和择优,基于2种语言变量的距离测度和语言理想解,提出了一种不同类型语言信息下的多属性决策方法. 该方法不仅易于对语言变量进行计算,而且在求解过程中不会丢失任何语言决策信息,从而保证了决策结果的合理性和有效性. 最后,利用算例对方法的运算过程进行了具体分析和说明.

关键词:多属性决策;语言理想解;距离测度;语言变量

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