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Analysis of response time probability distribution of workflows

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Abstract: An evaluation approach for the response time probability distribution of workflows based on the fluid stochastic Petri net formalism is presented. Firstly, some problems about stochastic workflow net modeling are discussed. Then how to convert a stochastic workflow net model into a fluid stochastic Petri net model is described. The response time distribution can be obtained directly upon the transient state solution of the fluid stochastic Petri net model. In the proposed approach, there are not any restrictions on the structure of workflow models, and the processing times of workflow tasks can be modeled by using arbitrary probability distributions. Large workflow models can be efficiently tackled by recursively using a net reduction technique.

Key words: workflow; response time; stochastic workflow net; fluid stochastic Petri net

A workflow is a business process in which a set of logically related tasks are performed by different processing entities, be it with regard to either machines or humans. Individual cases are routed through the business process along the tasks that should be performed for them. Workflow management technology aims at the automated support and coordination of business processes to reduce costs and flow times, and to increase the quality of service and productivity.

In this paper we focus on the time between the start and the end of the processing of a single workflow instance, i. e. the response time. The response time is one of the most important performance indicators in business. Often, a low or stable response time is a desirable or even necessary characteristic of a business process. This makes it possible for an organization to be more responsive to customers' needs, thus providing a competitive edge. Therefore, it is important to evaluate the response time in business process redesign.

For workflow performance modeling stochastic durations should be integrated to model the processing times of tasks. That is because resources, especially human resources, do not deliver constant productivity. On the other hand, in order to reduce complexity of modeling, each selective routing is assigned a probability weight, instead of modeling the specific cause. Thus, stochastic approaches should be chosen to analyze the response time.

Simulation is a simple flexible technique suited for the performance evaluation of almost any type of business process[1]; however, simulation is an approximation method and may be time consuming. A time-efficient analytical approach that can deliver exact results would be preferable in most cases. In Refs. [2 – 6], the authors computed the average response time of a process using performance analysis techniques based on a general stochastic Petri net, a colored stochastic Petri net or a queuing theory. The hypothesis of exponential distributions in their models, however, gave more qualitative rather than quantitative results of real workflow systems. And, moreover, the average response time cannot indicate the stability of workflow performance. If the variation of the response time is large, the average is hardly suitable to give customers guarantees about delivery times.

To achieve a comprehensive and accurate performance evaluation we need to compute the response time probability distribution of workflows, and allow the workflow service characteristics to be modeled by using arbitrary probability distributions. Three approaches have been presented to obtain the response time distribution, and they are all based on variants of the stochastic timed Petri net. In Ref. [6], non-exponential distribution was approximated by phase type distribution, so the improved accuracy came at the price of increased state-space size. In Ref. [7], the discrete Fourier transform was used to analyze a class of discrete stochastic workflow nets. Another approach based on the Semi-Markovian stochastic Petri net was proposed in Ref. [8], in which the concurrent enabling of general distribution transitions was strictly prohibited. The above approaches, however, are feasible only for parti-

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cular kinds of workflows.

Petri nets are successfully applied in the modeling and analysis of workflow systems, thus we also apply a stochastic workflow net (SWN) to model workflows in this paper. We use a recently developed formalism called a fluid stochastic Petri net (FSPN) to compute the response time distribution. To reduce the computational complexity, we adapt the reduction method proposed in Ref. [8] to deal with large models. In Ref. [9], we present an evaluation approach for the average response time of workflows based on the steady state analysis of the FSPN models, but the existing numerical methods have problems regarding the analysis of the FSPN models of realistic workflows.

1 Stochastic Workflow Net

Definition 1 (place/transition net) A place/transition net is a tuple (P, T, F), where

- P is a finite set of places;
- T is a finite set of transitions such that $P \cap T = \Phi$;
- $F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs.

For any node $x \in P \cup T$, the preset of x is defined as ${}^{\bullet}x = \{y \mid yFx\}$, and the postset of x is defined as $x^{\bullet} = \{y \mid xFy\}$.

Definition 2 (workflow net[10]) Let $N = (P, T, F)$ be a place/transition net and $t^* \notin P \cup T$. Net N is a workflow net if and only if

- P contains a source place i: ${}^{\bullet}i = \Phi$, and a sink place o: $o^{\bullet} = \Phi$;
- The short-circuited net $N^* = (P, T \cup \{t^*\}, F \cup \{(t^* \times i), (o \times t^*)\})$ is strongly connected.

The SWN is a stochastically timed variant of the workflow net. In the SWN, the set of transitions is partitioned into three subsets: T_G, T_E, and T_I (i.e. $T = T_G \cup T_E \cup T_I$). T_G is a set of transitions with arbitrary firing time distributions. T_E is a set of exponentially distributed transitions and T_I is a set of immediate ones which have a constant zero firing time.

A timed transition $T_i \in T_E \cup T_G$ is depicted as a rectangular box, which stands for a task of the modeled workflow. The firing time distributions of the timed transitions model the stochastic processing times of the tasks. An immediate transition $t_i \in T_I$ only represents a routing relation, which does not have a counterpart in the workflow and is drawn as a thick bar. We denote the timed transitions with uppercase letters and the immediate ones with lowercase letters.

Fig. 1 shows an SWN model, which includes the six basic forms of junctions for workflow tasks defined by WfMC, namely, AND-join, AND-split, OR-join, OR-split, iteration and causality. The SWN models a workflow process composed of eight tasks: T_1, T_2, …, T_8. Transition t_9 is an immediate transition used to synchronize two parallel flows in this example, which does not represent a task.

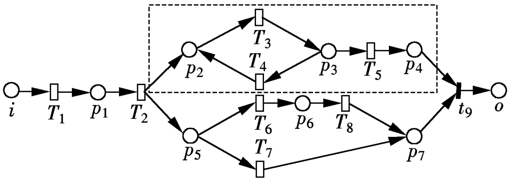

Fig. 1 An SWN model

An SWN model captures the control flow structure of the modeled workflow. The underlying stochastic process of the SWN model expresses the way that a workflow instance is handled. In the following sections we only consider the dynamic behavior of a single case in isolation within a workflow.

Each task has an infinite server semantics, that is to say, we assume that sufficient resources are always available when there is a task to be processed. In other words, the resource capacity is infinite; no waiting time can occur due to the lack of resources. In our approach, all processing times for one specific task—a timed transition—are independently sampled on the basis of the same probability distribution. For each transition $T_i \in T_G$, the enabling delay is represented with a general continuous distribution $F_i(\tau)$, whereas only an instantaneous firing rate λ_i is assigned to each transition $T_i \in T_E$ for the simplicity of expression. The instantaneous firing rate function $\lambda_i(\tau)$ of transition $T_i \in T_G$ with distribution $F_i(\tau)$ is given by

$$\lambda_i(\tau) = \lim_{\Delta\tau \to 0} \frac{\Pr\{X \leqslant \tau + \Delta\tau \mid X > \tau\}}{\Delta\tau} = \frac{\mathrm{d}F_i(\tau)}{\mathrm{d}\tau} \frac{1}{1 - F_i(\tau)} = \frac{f_i(\tau)}{\bar{F}_i(\tau)} \quad (1)$$

In Refs. [2 – 7] conflicts between timed transitions are resolved by race policy: when several timed transitions are enabled in a given marking, the transition with the shortest associated delay fires first (thus disabling the other conflicting transitions). That is not the case; the actual routing of the process control flow is determined by evaluation of the transition conditions of the process rather than by comparison of the processing times of the corresponding tasks, so we adopt a pre-selection policy. Not only conflicts between immediate transitions, but also conflicts between timed ones are resolved by a probabilistic choice (the firing probability of a transition is given by the fraction of its weight and the sum of the weights of all enabled transitions belonging to an effective conflict set). For SWN workflow models we only need to pay attention to all the conditional routings (i. e. OR-split junction).

For the analysis of the response time, it is necessary to specify the stochastic information that indicates the probabilities of transitions being fired at runtime and the processing times of tasks. The specification of transitional probabilities and distribution functions, which statistically describes workflow behavior at runtime, is based on data coming from specialists' experience and the workflow system log.

We also assume that the routing decisions and the task processing times are statistically independent. But in fact, they are not independent from each other, because the actual routings and processing times depend on the workflow attributes of any given case. A source of inaccuracy would be introduced to the model if the assumption of independence were made.

2 Fluid Stochastic Petri Net

The FSPN is a high-level modeling formalism that enables a simple description of a complex hybrid system with both continuous and discrete components[11]. The FSPN can also be thought of as a graphical language to represent (non-Markovian) stochastic processes with rewards[12]. Due to space limitation we only review some of the basic concepts and notations about the first-order FSPN that we use in this paper. A comprehensive description is given in Ref. [11].

A first-order FSPN is a tuple $(P, T, A, \lambda, W, \gamma, M_0)$, where the set of places $(P = P_C \cup P_D)$ is divided into the fluid (continuous) and the discrete places. The discrete places (the elements of P_D) are drawn as single-lined circles and hold an integer number of tokens as usual, whereas the continuous places (the elements of P_C) are drawn as two concentric circles and they hold a real-value amount of fluid. The set of transitions $T = T_E \cup T_I$ is composed of the exponentially distributed and the immediate transitions as defined in section 1.

The marking $M = (m, \boldsymbol{x})$ consists of a discrete part $m = (\#p_i, p_i \in P_D)$, where $\#p_i$ denotes the number of discrete tokens in discrete place p_i, and a continuous part, a vector representing the fluid level in each fluid place, $\boldsymbol{x} = (x_k, p_{ck} \in P_C)$. The initial marking is $M_0 = (m_0, \boldsymbol{x}_0)$. We use S to denote the partially discrete and partially continuous state space and S_D the discrete component of the state space.

The set of arcs $A = A_D \cup A_C \cup A_F$ is divided into three subsets: the discrete arcs (the elements of A_D), the continuous arcs (A_C) and the flush-out arcs (A_F). The discrete arcs are depicted as thin single-lined arrows, whereas the continuous arcs are drawn as double-lined arrows. The flush-out arcs, which are depicted as thick single-lined arrows, only connect continuous places to timed transitions, and describe the capability of a transition to flush out all the existing fluid from a continuous place when it fires.

The firing rate function $\lambda: T_E \times S \rightarrow \mathbf{R}^+$ is defined for exponential transitions. An exponential transition T_i, enabled in a discrete marking m_j with fluid level $\boldsymbol{x}$ in the continuous places, fires with rate $\lambda(T_i, m_j, \boldsymbol{x})$, allowing the firing rates to be dependent on the discrete and the continuous components of the marking.

The weight function $W: T_I \times S_D \rightarrow \mathbf{R}^+$ is defined for immediate transitions, and is used to solve conflicts. The function $\gamma: A_C \times S \rightarrow \mathbf{R}^+$ is called the flow rate function and describes the marking dependent flow of fluid across continuous arcs.

An FSPN model is shown in Fig. 2. There are two fluid places, p_{c3} and p_{c4}. Transition $T_i (i = 3, 4)$ and fluid place $p_{ci} (i = 3, 4)$ are connected by a continuous arc wit a constant unitary flow rate and a flush-out arc.

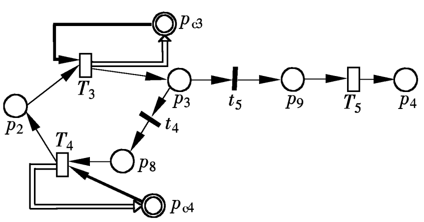

Fig. 2 An FSPN model

The stochastic marking process underlying the FSPN is a Markov process in continuous time with mixed discrete and continuous state space, $M(\tau) = \{(m(\tau), \boldsymbol{x}(\tau)), \tau \geqslant 0\}$, where $m(\tau)$ is the discrete marking at time τ, and $\boldsymbol{x}(\tau)$ is a random variable vector, representing the fluid levels in the fluid places at time τ. For the governing equations of the stochastic process, Ref. [11] should be referred to.

Up to now, the numerical methods developed for the solution of the FSPN are efficient only for those models with a very few fluid places.

3 Analysis

As discussed above, the existing techniques have problems regarding the analysis of large FSPN models coming from realistic workflows. We, therefore, need an appropriate net reduction method that preserves the external observable timing properties to facilitate the analysis of large workflow models. A reduction approach for the SWN models is presented in Ref. [8], which preserves soundness and response time distribution. Most of the SWN models are highly structured, thus the method can efficiently tackle large workflow models. By recursively using the approach we can decrease greatly the computation effort. In this section we describe how to transform one of the isolated subsystems (can be viewed as an SWN model) detected by using the reduction method into an FSPN model, and how to compute its response time distribution.

First, we have to separate conflict resolution from timing specification of transitions. The conflicts among timed transitions can be transferred to a barrier of conflicting immediate transitions, followed by the set of timed transitions.

A mapping approach to convert a non-Markovian SPN to an FSPN was described in Ref. [12]; however, the method could not be used in the case where the enabling degree of timed transitions might be more than one. As for SWN models, the enabling degree of any transition could only be zero or one when a single process instance in isolation within a workflow was considered. Therefore, the approach can be applied here to transform an SWN to an FSPN.

Because pre-selection policy is adapted to resolve the conflicts among timed transitions rather than race policy, there is no difference between preemptive resume (prs) and preemptive repeat different (prd) memory policies[12]. We will take all general distribution transitions as having a prs memory policy for simplicity.

For each general distributed transition $T_i \in T_G$ in the considered SWN model, we need to add a fluid place p_{ci}, which represents the memory of the transition. Transition T_i is connected to place p_{ci} with a fluid arc with an associated flow rate $\gamma((T_i, p_{ci}), M) = 1$. Transition T_i is also connected to fluid place p_{ci} with a flush-out arc so that fluid place p_{ci} looses all its fluid, as soon as T_i fires. Assuming that the probability distribution function of transition T_i is $F_i(\tau)$, and the probability density function (pdf) is $f_i(\tau)$, we can obtain the instantaneous firing rate function $\lambda_i(\tau)$ according to Eq. (1). The instantaneous firing rate of transition T_i in the resulting FSPN model is chosen to be dependent on the fluid level x_i of place p_{ci}, $\lambda(T_i, M) = \lambda(T_i, m_j, \boldsymbol{x}) = \lambda_i(x_i)$.

In particular, automatic tasks often have a deterministic processing time; the corresponding transitions in the SWN model would have a deterministic firing time. When the SWN model is converted into an FSPN model, the deterministic transitions would introduce vanishing markings dependent on fluid levels. To avoid this awkward situation, we propose that a deterministic distribution should be approximated using a normal distribution with kurtosis large enough.

Finally, the initial marking of the FSPN model is $M_0 = (m_0, \boldsymbol{x}_0) = ((\#i = 1), \boldsymbol{0})$, where i is the source place of the converted SWN model, that is, only the source place i has a token, and the fluid levels of all the fluid places are zero.

In the following, we give the equations for the stochastic marking process which describes the dynamic behavior of the FSPN model.

For any fluid place p_{ci} in the FSPN model, only one transition T_i may influence its fluid level. When transition T_i is enabled, the fluid in fluid place p_{ci} would increase at a constant unitary rate. Therefore, the actual rate of change of fluid level for fluid place p_{ci} in discrete marking m_j is

$$r_{ci}(m_j) = \begin{cases} 1 & \text{if } T_i \in E(m_j) \\ 0 & \text{otherwise} \end{cases}$$

where $E(m_j)$ denotes the set of enabled transitions in discrete marking m_j. All the possible actual flow rates for fluid place p_{ci} are collected into a diagonal matrix $\boldsymbol{R}(ci)$, and $r_{jj}(ci) = r_{ci}(m_j)$ is the element of $\boldsymbol{R}(ci)$.

Before giving out the infinitesimal generator matrix of the stochastic process, we should remove all the vanishing markings from the discrete state space by any standard general stochastic Petri net analysis technique. The matrix $Q(\boldsymbol{x}, \Phi)$ accounts for the transition rates among tangible states when no flush-out occurs, and $Q(\boldsymbol{x}, ci)$, with $p_{ci} \in P_C$, accounts for the transition rates among tangible states when flush-out of place p_{ci} occurs (There is no possibility in the FSPN model where a transition firing flushes out two or more fluid places simultaneously.). Formally speaking:

$$q_{jl}(\boldsymbol{x}, \Phi) = \sum_{\substack{T \in E(m_j) \mid m_j \to m_l \\ \text{no flush-out occurred}}} \lambda(T, m_j, \boldsymbol{x})$$

where $j \neq l$, $q_{jj}(\boldsymbol{x}, \Phi) = -\sum_{T \in E(m_j)} \lambda(T, m_j, \boldsymbol{x})$.

$$q_{jl}(\boldsymbol{x}, ci) = \begin{cases} \lambda(T_i, m_j, \boldsymbol{x}) & \text{if } T_i \in E(m_j) \wedge m_j \xrightarrow{T_i} m_l \\ 0 & \text{otherwise} \end{cases}$$

where $j \neq l$, $q_{jj}(\boldsymbol{x}, ci) = 0$.

Define the probability density function of the stochastic process $M(\tau) = \{(m(\tau), \boldsymbol{x}(\tau)), \tau \geqslant 0\}$ as $\pi(\tau, m_j, \boldsymbol{x}) = \partial \Pr\{m(\tau) = m_j, \boldsymbol{x}(\tau) \leqslant \boldsymbol{x}\} / \partial \boldsymbol{x}$ and let $\pi(\tau, \boldsymbol{x}) = [\pi(\tau, m_j, \boldsymbol{x}), m_j \in S_d]$ be a row vector of pdfs for all the discrete states. We denote Dirac's delta function as $\delta(x)$ and a projection operator $\theta(\boldsymbol{x}', i)$ as $\theta(\boldsymbol{x}', i) = (x_1, x_2, \ldots, x_{i-1}, x_i', x_{i+1}, \ldots, x_{|P_C|})$.

Theorem 1 For the FSPN model obtained by converting an SWN model using the fore mentioned approach, the probability density function vector $\pi(\tau, \boldsymbol{x})$ is governed by

$$\frac{\partial\pi(\tau,\boldsymbol{x})}{\partial\tau} + \sum_{p_{ci}\in P_C} \frac{\boldsymbol{R}(ci)\,\partial\pi(\tau,\boldsymbol{x})}{\partial x_i} = \pi(\tau,\boldsymbol{x})\,Q(\boldsymbol{x},\Phi) + \sum_{p_{ci}\in P_C} \delta(x_i)\int_0^{\infty} \pi(\tau,\theta(\boldsymbol{x}',i))\,Q(\theta(\boldsymbol{x}',i),ci)\,\mathrm{d}x_i'$$

The initial conditions are

$$\left.\begin{aligned} \pi(0,m_0,\boldsymbol{x}) &= \delta(\boldsymbol{x}) \\ \pi(0,m_j,\boldsymbol{x}) &= 0 \qquad \forall m_j \neq m_0 \end{aligned}\right\}$$

No boundary conditions are needed.

We can directly get the response time distribution we need by solving the above equations. Denote the final discrete state as m_e, the response time T_R distribution is

$F_\tau(T_R) = \Pr(T_R \leqslant \tau) = \Pr($The analyzed instance has been completed at the time $\tau) =$
$\Pr($The discrete state is m_e at the time $\tau) = \pi(\tau, m_e, 0)$

4 Example

In this section we use the example shown in Fig. 1 to illustrate the analysis technique presented in section 3. We only consider one of the reducible subsystems detected by the algorithm proposed in Ref. [8], which is composed of three places (p_2, p_3, p_4) and three transitions (T_3, T_4, T_5) and redrawn in Fig. 3.

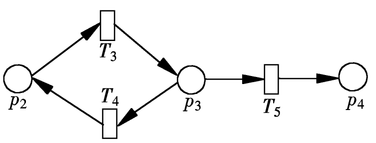

Fig. 3 A reducible submodel

In order to avoid the conflict between timed transitions T_4 and T_5, two immediate transitions t_4 and t_5 are added as in Fig. 4, which have the weights of timed transitions T_4 and T_5 ($W(T_4)$ and $W(T_5)$), respectively. Thus the firing probabilities of the two timed transitions remain the same.

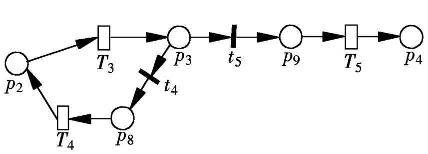

Fig. 4 The submodel being eliminated conflicts among timed transitions

By applying the mapping method to the SWN model shown in Fig. 4, in which only T_3 and T_4 are assumed to be general distributed transitions, we obtain the FSPN model drawn in Fig. 2.

Eliminating the intangible states leads to the tangible marking reachability graph shown in Fig. 5, in which a discrete marking is denoted as $m_j = (\#p_2, \#p_8, \#p_9, \#p_4)$.

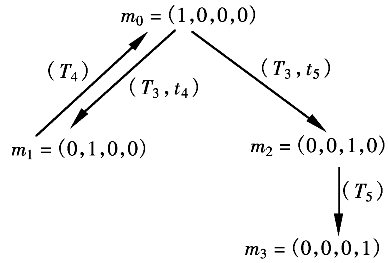

Fig. 5 The tangible state reachability gragh

The only one non-zero entry of diagonal matrix $\boldsymbol{R}(c3)$ is $r_{00}(c3) = 1$; the only one non-zero entry of diagonal matrix $\boldsymbol{R}(c4)$ is $r_{11}(c4) = 1$. We denote the fluid level vector by $\boldsymbol{x} = (x_3, x_4)$, then the three matrices $Q(\boldsymbol{x}, \Phi)$, $Q(\boldsymbol{x}, c3)$ and $Q(\boldsymbol{x}, c4)$ are given by

$$Q(\boldsymbol{x},\Phi) = \begin{bmatrix} -\lambda_3(x_3) & 0 & 0 & 0 \\ 0 & -\lambda_4(x_4) & 0 & 0 \\ 0 & 0 & -\lambda_5 & \lambda_5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q(\boldsymbol{x}, c3) = \begin{bmatrix} 0 & \lambda_3(x_3)\dfrac{W(T_4)}{W(T_4)+W(T_5)} & \lambda_3(x_3)\dfrac{W(T_5)}{W(T_4)+W(T_5)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q(\boldsymbol{x}, c4) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \lambda_4(x_4) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By solving the corresponding equations, we can obtain the response time distribution of this submodel, that is, $\pi(\tau, m_3, \boldsymbol{0})$. For the whole analysis of the example, the procedure proposed in Ref. [8] should be followed.

5 Conclusion

In this paper we have presented an approach for the evaluation of the response time probability distribution of workflows, which is based on the FSPN formalism. In our approach, the SWN model of the analyzed workflow is first constructed, and then the SWN model is converted into an FSPN model. The response time distribution can be achieved upon the numerical solution of the governing equations of the FSPN model. The computation effort of large workflow models can be decreased efficiently by recursively using a net reduction technique. A simple example illustrates the analysis procedure. Compared with other methods proposed up to now, there is not any restriction on the structure of workflow models and the processing times of workflow tasks can be modeled by using arbitrary probability distributions.

In addition, we recognize that the performance evaluation approach presented here is better suited for production workflows since they are more structured, predictable, and repetitive. In the case of ad hoc workflows, the stochastic information that indicates the dynamic behavior of workflows can hardly be specified.

References

[1] Miller J A, Sheth A P, Kochut K J, et al. Simulation modeling within workflow technology[C]//*Proceedings of the* 1995 *Winter Simulation Conference*. Arlington, 1995: 612 − 619.

[2] Lin Chuang, Tian Liqin, Wei Yaya. Performance equivalent analysis of workflow systems [J]. *Journal of Software*, 2002, **13**(8): 1472 − 1480. (in Chinese)

[3] Dehnert J, Freiheit J, Zimmermann A. Modeling and performance evaluation of workflow systems[C]//*Proceedings of* 4*th World Multiconference on Systemics*, *Cybernetics and Informatics*. Orlando, 2000: 632 − 637.

[4] Schomig A K, Rau H. A Petri net approach for the performance analysis of business processes [R]. Würzburg, Germany: University of Würzburg, 1995.

[5] Ferscha A. Qualitative and quantitative analysis of business workflows using generalized stochastic Petri net [C]// Chroust G, Benczur A, eds. *Proceedings of CON'*94: *Workflow Management—Challenges*, *Paradigms and Products*. Linz, Austria, 1994: 222 − 234.

[6] Zerguini L. Approximate computation of response time distribution in workflows[C]//*Proceedings of High Performance Computation Symposium*. San Diego, 2002: 280 − 287.

[7] van der Aalst W M P, van Hee K M, Reijers H A. Analysis of discrete-time stochastic Petri nets [J]. *Statistica Neerlandica*, 2000, **54**(2): 237 − 255.

[8] Zerguini L. Analysis of large workflow models [C]//*Proceedings of* 2002 *IEEE International Conference on Systems*, *Man and Cybernetics*. Hammamet, Tunisia, 2002: 188 − 194.

[9] Lu Guangsong, Ge Yunjian, Qian Pengan. Computation of the response time of a workflow based on FSPN [J]. *Journal of University of Science and Technology of China*, 2005, **35**(2): 232 − 239.

[10] van der Aalst W M P. Putting Petri nets to work in the workflow arena[C]//van der Aalst W M P, Colom J M, et al, eds. *LINCOM Studies in Computer Science*. München, Germany, 2003: 1 − 20.

[11] Gribaudo M, Sereno M, Horváth A, et al. Fluid stochastic Petri nets augmented with flush-out arcs: modeling and analysis [J]. *Discrete Event Dynamic Systems*, 2001, **11** (1): 97 − 117.

[12] Gribaudo M, Sereno M, Bobbio A. Fluid stochastic Petri nets: an extended formalism to include non-Markovian models[C]//Buchholz P, ed. *Proceedings of* 8*th IEEE International Workshop on Petri Nets and Performance Models*. Zaragoza, Spain, 1999: 74 − 82.

工作流响应时间概率分布分析

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摘要:提出了一种基于流体随机 Petri 网的工作流响应时间概率分布计算方法. 首先讨论了利用随机工作流网建模的一些相关问题,然后描述了如何将随机工作流网模型转化为流体随机 Petri 网模型,最后给出了该种流体随机 Petri 网模型的动态方程,说明工作流响应时间的概率分布可直接由流体随机 Petri 网模型的暂态解得到. 该方法对工作流模型的结构没有提出任何限制,且工作流任务的处理时间可取任意概率分布,通过递归地使用网化简技术可有效地处理大型工作流模型.

关键词:工作流;响应时间;随机工作流网;流体随机 Petri 网
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