

# Mathematical demonstration of correlation of optical prime code

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**Abstract:** Starting from the configuration of the optical prime code, a kind of key signature code for the optical code-division multiple access (OCDMA) system, based on the linear congruence theory in the finite Galois field, the correlation properties of the basic prime code, the extended prime code and the modified prime code are mathematically analyzed, the distribution of cross-correlation values is given and the overlap area of “1”s in the case of periodically circularly shifting is indicated. It is mathematically demonstrated that the maximum cross-correlation of the basic prime code is 2, and that of the extended prime code and the modified prime code is 1. The integrated correlation analysis process is proposed. The signal-interfere ratio (SIR) and the BER performance of the systems employing different signature codes are calculated, respectively, and the performances of OCDMA systems employing different signature codes mode are compared.

**Key words:** fiber-optic communication; optical code-division multiple access; optical prime code; correlation

The optical code-division multiple access (OCDMA) system has many advantages, such as information security, anti-jamming ability, permission of users to add/drop randomly, coding in the optical domain, so it is thought to be the most hopeful multiplexing technology in establishing all-optical networks. As a result, it has received much attention and research in recent years<sup>[1-2]</sup>. To be practically applied in the network, the OCDMA system needs a signature code with better performance, which directly depends on the correlation between the codewords. The optical prime code (OPC), a kind of key signature code, was firstly brought forward by Sharr et al.<sup>[3]</sup>, and from then on, the construction schemes of the extended prime code<sup>[4-5]</sup>, the modified prime code<sup>[6-7]</sup> and the 2<sup>n</sup> prime code<sup>[8-9]</sup> have been set up. Although one knows that the maximum cross-correlation of the basic prime code is 2, and that of the extended prime code and the modified prime code is 1, this has not been mathematically demonstrated. Considering this, this paper will start from the linear congruence theory in the finite Galois field, and mathematically analyze the correlation properties of the basic prime code, the extended prime code and the modified prime code to give an integrated analysis process of prime code correlation. And then, the performances of OCDMA systems employing different signature code modes are compared under the same transmitting speed and the same code-length.

## 1 Typical Configurations of Prime Code

### 1.1 Basic prime code (BPC)

According to the linear congruence theory in the Galois field, the basic prime code can simply be acquired. In the Galois field  $GF(p)$ , where  $p$  is a prime larger than 2, take  $S_i(j) = \{i \times j\}(\text{mod } p)$ ,  $i \in GF(p)$ ,  $j \in GF(p)$ , we get the prime sequence  $S_i = \{S_{i0}, S_{i1}, \dots, S_{ij}, \dots, S_{i(p-1)}\}$ <sup>[1]</sup>, which can be used to construct the basic prime code, whose code function  $C_i = \{C_{i0}, C_{i1}, \dots, C_{i(N-1)}\}$  ( $i=0, 1, \dots, p-1$ ),

$$\begin{cases} C_i(k) = 1 & \forall : k = S_{ij} + jp \\ C_i(k) = 0 & \forall : k \neq S_{ij} + jp \end{cases} \quad (1)$$

where  $j=0, 1, \dots, p-1$ .

The above expressions are employed to determine the locations of “1”s in the  $(0, 1)$  sequence of the prime code, the value of  $C_i(k)$  denotes the position of the  $k$ -th “1”. And then we acquire  $p$  basic prime codes whose length  $L = p^2$ , weight  $w = p$ , the maximum auto-correlation (AC in concision) side-lobe and crosscorrelation (CC in concision) are  $p-1$  and 2, respectively.

### 1.2 Extended prime code (EPC)

Because the maximum CC value of the BPC, 2, is not helpful to the performance of the system, Maric et al.<sup>[5]</sup> proposed a new prime code model by increasing the code-length, i. e., adding some “0”s in the  $(0, 1)$  sequence of the code, in order to improve the CC<sup>[3]</sup>. The code function they employed is

$$\begin{aligned} C_i(k) &= (ik)(\text{mod } p) + k(2p-1) \\ k &\in GF(p), i \in GF(p) \end{aligned} \quad (2)$$

Eq. (2) means that  $p-1$  “0”s are added to each sub-sequence of the BPC, which will make the “1”s

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more sparse in the whole  $(0, 1)$  sequence, and, therefore, the chance that two “1”s have the same location is cut down greatly, and CC is reduced. To code from Eq. (2), the length  $L = p(2p - 1)$ , weight  $w = p$ , the maximum AC sidelobe is also  $p - 1$ , but the maximum CC value is decreased from 2 to 1. Obviously, the improvement in the CC in the EPC bears a penalty of a decrease in bit rate because of the longer code-length.

### 1.3 Modified prime code

Another disadvantage of the BPC is that the maximum number of users it can support is just the same as the prime  $p$ . If we want to contain a large number of simultaneous users, the  $p$  must be large enough, which leads to the AC peak (proportioning to  $p^2$ ) being much higher than is needed. For better power efficiency, we can remove some “1”s in the  $(0, 1)$  sequence to lower the AC peak. The code function of MPC<sup>[4]</sup> is

$$C_i(j) = (ib_j)(\text{mod}p) + b_jp$$

$$j \in \{0, 1, \dots, w - 1\}, b_j \in \text{GF}(p), i \in \text{GF}(p) \quad (3)$$

Eq. (3) is equivalent to drawing out  $p - w$  “1”s in the  $(0, 1)$  sequence of the basic prime code.

## 2 Correlation Analysis for Prime Codes

To optical signature code with length  $L$ , weight  $w$ , the AC sidelobe between a code and its periodically cyclical shifting satisfies (4a), and the CC between different codes satisfies (4b), where  $\lambda_a$  and  $\lambda_c$  are the maximum AC sidelobe and the maximum CC value, respectively.

$$\sum_{i=0}^{L-1} x_i x_{i \oplus \tau} \leq \lambda_a \quad \forall x \in C, \tau \neq 0 \quad (4a)$$

$$\sum_{i=0}^{L-1} x_i y_{i \oplus \tau} \leq \lambda_c \quad \forall x, y \in C, x \neq y \quad (4b)$$

where  $\oplus$  denotes module  $L$  addition. If the code is denoted by vector, the above two constraints can be expressed as

$$\max\{C_i \cdot C_i^\tau \mid \tau \in \{1, 2, \dots, L - 1\}, i \in \text{GF}(p)\} \leq \lambda_a \quad (4c)$$

$$\max\{C_i \cdot C_j^\tau \mid \tau \in \{0, 1, \dots, L - 1\}, i \neq j\} \leq \lambda_c \quad (4d)$$

where  $C_i \cdot C_j^\tau$  means the scalar product of vectors  $C_i$  and  $C_j^\tau$ , and  $C_j^\tau$  denotes the  $\tau$ -cyclical shifting of code  $C_j$ .

In the asynchronous OCDMA system, the AC sidelobe comes from the interference between different chips of the same code, and CC comes from different codes (users). At the OCDMA transmitter and receiver, CC is the main source of the system BER. If the maximum CC value,  $\lambda_c$ , is used to calculate the system BER, the result will not be accurate, because the maximum CC value appears at only a few  $\tau$ -shiftings, not all the shiftings. Meanwhile, the apparent probability

of the maximum CC value is far less than that of other correlation values. For example, as for the basic prime code, the maximum CC is 2, the apparent probability of which is much less than that of CC, 1 or CC, 0. This will be identified in the following section.

### 2.1 Basic model for code correlation

From (1), we know that the  $(0, 1)$  sequence of the code can be evenly divided into  $p$  subsequences (sub in concision), each of which has just one “1”. To the  $i$ -th code,  $C_i$ , of the code family, the location scope of all  $p$  chips of the  $k$ -th ( $0 \leq k \leq p - 1$ ) sub are from  $kp$  to  $(k + 1)p - 1$ . Meanwhile, we know the location of the only “1” of this sub  $(ik)(\text{mod}p) + kp, k \in \text{GF}(p), i \in \text{GF}(p)$ , which means the “1” locates at the  $(ik)(\text{mod}p)$ -th in the  $k$ -th sub relatively. For another code,  $C_j (0 \leq j \leq p - 1, j \neq i)$ , chips between  $kp - \tau$  and  $(k + 1)p - 1 - \tau$  will move to the  $k$ -th sub after  $\tau$ -cyclical shifting, as shown in Fig. 1. Let  $\tau = ap + b (0 \leq b \leq p - 1)$ , the chips’ location is from  $(k - a - 1)p + (p - b)$  to  $(k - a)p + (p - b - 1)$  before shifting, shown as the shadowed area. If there are “1”s in the  $k$ -th sub, they must locate in the shadowed area before shifting. If the “1” originally locates in the  $(k - a - 1)$ -th sub, its relative position,  $(j(k - a - 1))(\text{mod}p)$  must satisfy

$$(j(k - a - 1))(\text{mod}p) \geq p - b \quad (5)$$

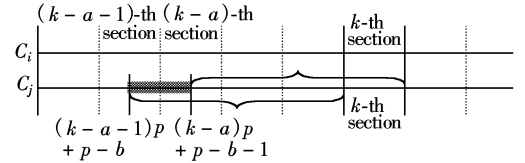


Fig. 1 Schematic of code periodically cyclical shifting

After  $\tau = ap + b$  cyclical shifting, its relative position in the  $k$ -th sub is

$$(j(k - a - 1))(\text{mod}p) + (k - a - 1)p + (ap + b) - kp = (j(k - a - 1))(\text{mod}p) + b - p \quad (6)$$

On the other hand, if the “1” comes from  $(k - a)$ -th sub, its relative position  $(j(k - a))(\text{mod}p)$  must satisfy

$$(j(k - a))(\text{mod}p) \leq p - b - 1 \quad (7)$$

After  $\tau = ap + b$  cyclical shifting, its relative position in the  $k$ -th sub is

$$(j(k - a))(\text{mod}p) + (k - a)p + (ap + b) - kp = (j(k - a))(\text{mod}p) + b \quad (8)$$

According to the definition of (1), the CC between codes  $C_j$  and  $C_i$  means that some “1”s of the two codes have the same position after cyclical shifting, i. e., the following equation must be satisfied

$$(j(k - a - 1))(\text{mod}p) + b - p = (ik)(\text{mod}p) \quad (9)$$

or

$$(j(k - a))(\text{mod}p) + b = (ik)(\text{mod}p) \quad (10)$$

Eqs. (9) and (10) are the linear congruence equa-

tions in the Galois field, whose solutions are

$$k = \frac{j(a+1) - b + \xi p}{j-i} \quad (11)$$

$$k = \frac{ja - b + \zeta p}{j-i} \quad (12)$$

where  $\xi, \zeta$  are integers. The values of  $k$  decided by (11) and (12) must satisfy the conditions of (5) and (7), respectively; otherwise, the solution is invalid. Obviously, if  $a, b$  make both (11) and (12) have no integer solution, the CC between codes  $C_j$  and  $C_i$  is zero at such a shifting  $\tau = ap + b$ . Similarly, if only one of the equations has an integer solution, CC is 1, and if both equations have integer solutions, CC is 2.

Tab. 1 gives the variation of the CC between codes  $C_3$  and  $C_5$  in GF(7) versus shifting  $\tau = ap + b$ . Tab. 2 shows the subsequences  $k$  in which the “1”s of the two codes have the same location.

**Tab. 1** CC of  $C_3$  and  $C_5$  vs. shifting  $\tau = ap + b$

$a$	$b$						
	0	1	2	3	4	5	6
0	1	1	2	1	0	1	1
1	1	1	0	2	2	1	0
2	1	2	1	0	0	1	2
3	1	0	1	2	2	0	1
4	1	1	1	0	1	2	1
5	1	1	2	2	0	0	1
6	1	1	0	0	2	2	1

**Tab. 2** Overlapping area of “1”s,  $k$  vs. shifting  $\tau = ap + b$

$a$	$b$						
	0	1	2	3	4	5	6
0	0	3	5, 6	2	—	0	3
1	6	2	—	0, 1	3, 4	6	—
2	5	0, 1	4	—	—	5	1, 2
3	4	—	3	5, 6	1, 2	—	0
4	3	6	2	—	0	3, 4	6
5	2	5	0, 1	3, 4	—	—	5
6	1	4	—	—	5, 6	1, 2	4

From Tabs. 1 and 2, we can see that CC varies with different shiftings  $\tau$ ; meanwhile, different CC values have different appearing probabilities.

## 2.2 Correlation property of basic prime code

$\lambda = 2$  means that there are just 2 “hit”s that happened between two codes at some cyclical shifting  $\tau$ . In this case, the distance between the two “1”s in one code equals the distance between two “1”s in another code, here “distance” means the position difference. From definition (1), the distance between two adjacent “1”s in code  $C_i$  is

$$d = ((i(k+1))(\bmod p) + (k+1)p) - ((ik)(\bmod p) + kp) \quad (13)$$

According to the characteristics of the finite congruence equation, we consider the distance in three cases:

- ① When  $i(k+1) < mp$ , we have  $d = p + i$ ;
- ② When  $ik < mp \leq i(k+1)$ , we get  $d = p + i - p = i$ ;
- ③ When  $ik \geq mp$ , we have  $d = p + i$ .

Thus, the distance of adjacent “1”s in  $C_i$  is either  $p + i$  or  $i$ . Considering the periodically cyclical shifting, the distance  $p + i$  appears  $p - i$  times, and  $i$ ,  $i$  times. As a result, we obtain the distances between any two “1”s in the code  $C_i$ .

$$d_{m,n}^i = m(p+i) + ni \quad 0 \leq m \leq p-i, 0 \leq n \leq i \quad (14)$$

where  $m$  and  $n$  are integers, cannot be zero, cannot be the maximum simultaneously. Otherwise, the distance will be zero or  $L$ . When the CC value between codes  $C_i$  and  $C_j$  is 2, there are  $m, n, m', n'$  satisfying  $d_{m,n}^i = d_{m',n'}^j$ , i. e.,

$$(m'(p+j) + n'j)(\bmod L) = (m(p+i) + ni)(\bmod L) \quad (15)$$

$$m' - m = \frac{(m+n)i - (m' + n')j}{p} = \frac{li - l'j}{p}$$

$$0 < l = m + n < p, 0 < l' = m' + n' < p \quad (16)$$

When  $i = 0$  or  $j = 0$ , both  $l'j$  and  $li$  are aliquant by  $p$ , therefore, no  $m, n, m', n'$  can satisfy (15). In such a case, CC is 1. When  $i \neq j \neq 0$ , because  $\min(l'j) = 1$ , we know  $\max(li) = (p-1)^2$ . From Eq. (16), we have

$$\max(m' - m) = \frac{(p-1)^2 - 1}{p} = p - 2 \quad (17)$$

In the above case, there must be  $m, n, m', n'$  satisfying (15), which means the CC value between  $C_i$  and  $C_j$  is 2 in some  $\tau$ -shiftings.

The maximum CC value of the basic prime code family is 2, i. e., in (4),  $\lambda_c = 2$ . However, the probability of  $\lambda = \lambda_c$  is far less than that of  $\lambda = 1$  or  $\lambda = 0$ . Tab. 3 shows the appearing times of  $\lambda = 2$  in the whole shifting process ( $0 \leq \tau \leq 48$ ) of the prime code family acquired in GF(7).

**Tab. 3** Appearing times of  $\lambda = 2$  between any two prime codes in GF(7)

$i$	$j$						$\sum_{j=1, j \neq i}^{p-1} C_i C_j$
	1	2	3	4	5	6	
1	—	10	10	6	12	12	50
2	10	—	6	12	6	12	46
3	10	6	—	10	12	6	44
4	6	12	10	—	6	10	44
5	12	6	12	6	—	10	46
6	12	12	6	10	10	—	50
$\sum_{i=1}^{p-1} \sum_{j=1, j \neq i}^{p-1} C_i C_j$							280

## 2.3 Correlation property of extended prime code

According to Eq. (2), we have the distance between adjacent “1”s in the  $(0, 1)$  sequence of the extended prime code,

$$d = ((i(k+1)) \pmod{p} + (k+1)(2p-1)) - (ik) \pmod{p} + k(2p-1) \quad (18)$$

There are three cases as follows:

- ① When  $i(k+1) < mp$ , we have  $d = 2p - 1 + i$ ;
- ② When  $ik < mp \leq i(k+1)$ , we get  $d = 2p - 1 + i - p = p - 1 + i$ ;
- ③ When  $ik \geq mp$ , we have  $d = 2p - 1 + i$ .

Thus, the distance of adjacent “1”s in  $C_i$  is either  $2p - 1 + i$  or  $p - 1 + i$ . Considering the periodically cyclical shifting, the distance  $2p - 1 + i$  appears  $p - i$  times, and  $p - 1 + i$ ,  $i$  times. As a result, we obtain the distances between any two “1”s in the code  $C_i$

$$d_{m,n}^i = m(2p - 1 + i) + n(p - 1 + i) \quad 0 \leq m \leq p - i, 0 \leq n \leq i \quad (19)$$

where  $m$  and  $n$  are integers, cannot be zero and cannot be the maximum simultaneously. Otherwise, the distance will be zero or  $L$ . When the CC value between codes  $C_i$  and  $C_j$  is 2, then there are  $m, n, m', n'$  satisfying  $d_{m,n}^i = d_{m',n'}^j$ , i. e. ,

$$(m'(2p - 1 + j) + n'(p - 1 + j)) \pmod{L} = (m(2p - 1 + i) + n(p - 1 + i)) \pmod{L} \quad (20)$$

$$(2m' + n') - (2m + n) = \frac{l(i-1) - l'(j-1)}{p} \quad 0 < l = m + n < p, 0 < l' = m' + n' < p \quad (21)$$

When  $i = 1$  or  $j = 1$ , both  $l'(j-1)$  and  $l(i-1)$  are aliquant by  $p$ , therefore, no  $m, n, m', n'$  can satisfy (20). In such a case, CC is 1. When  $i \neq j \neq 1$ ,  $\max(l(i-1) - l'(j-1)) = (p-1)(p-2) - 1 = p^2 - 3p + 1$ , i. e. ,

$$\max((2m' + n') - (2m + n)) = \frac{p^2 - 3p + 1}{p} \quad (22)$$

In the above case, there are no  $m, n, m', n'$  satisfying (22), which means the CC value between  $C_i$  and  $C_j$  is at most 1 in any  $\tau$ -shifting.

According to the analysis of sections 2.2 and 2.3, we know that the CC property of the extended prime code is better than that of the basic prime code, but the length of the extended code is longer than that of the basic prime code.

As for the modified prime code defined by (3),  $w < p, L = p^2$ , which can be acquired by taking out  $p - w$  “1”s from the basic prime code. The positions of the left “1”s have not been changed. Therefore, the CC property of the modified prime code is the same as that of the basic prime code. However, the AC property of the former is better than that of the latter.

### 3 Calculation of BER from Code Correlation

To analyze the system BER induced from multiple user interference, we should first analyze the distribution of the different CC values. That is to say that the influence of different CC values ( $\lambda_m \in \{0, 1, \dots, \lambda_c\}$ ) to system BER is different. Let  $q_m$  denote the appearing probabilities of  $\lambda_m$ , we obtain the average CC value

where  $\bar{\lambda}$  denotes the average probability that a “1” in a code “hits” a “1” in another code. To the codes with length  $L$  and weight  $w$ , each code has  $w$  “1”s uniformly. Thus, the probability of a certain “1” in the first code “hits” the “1”s in the second code is  $w/L$ . If the system employs the on-off key modulation mode, both data bits 0 and 1 are transmitted with equal probability,  $1/2$ . When a user transmits data 0, there will be no “hit” happening. Therefore, the practical “hit” probability is  $w/2L$ . Considering these, the “hit” probability that the  $w$  “1”s in the first code “hit”  $w$  “1”s in the second code is  $\bar{\lambda} = w^2/2L$ .

$$\bar{\lambda} = q_0 \cdot 0 + q_1 \cdot 1 + q_2 \cdot 2 + \dots q_{\lambda_c} \lambda_c = \sum_{m=0}^{\lambda_c} q_m \lambda_m \quad (23)$$

From the prime code, we can easily know  $\bar{\lambda}$ . The values of  $q_0, q_1$ , and  $q_2$  can be derived from Ref. [10]. Then we have the distribution variance of correlation values,  $\sigma^2 = 5/12 - 1/(6p) - 1/(3p^2)$  for the basic prime code and  $\sigma^2 = 3/16 + 1/(8(2p-1)) - 1/(16(2p-1)^2)$  for the extended prime code. Supposing that there are total  $K$  simultaneous users, a special user received the interference from other  $K-1$  users is  $(K-1)\sigma^2$ , the signal-interference ratios  $R_{\text{SIR,B}}$  of the basic prime code and  $R_{\text{SIR,E}}$  of the extended prime code, are respectively:

$$R_{\text{SIR,B}} = \frac{w^2}{2(K-1)\sigma^2} = \frac{p^2}{2(K-1)(5/12 - 1/(6p) - 1/(3p^2))} \quad (24)$$

$$R_{\text{SIR,E}} = \frac{p^2}{2(K-1)} \left( \frac{3}{16} + \frac{1}{8(2p-1)} - \frac{1}{16(2p-1)^2} \right) \quad (25)$$

In the DS-OCDMA system, in order to satisfy the BER requirement, the  $p$  used in code designing is much larger than 1 (normally 37 or 41). In this case, we have

$$R_{\text{SIR,B}} = \frac{12p^2}{10(K-1)} = \frac{1.2p^2}{K-1} \quad (26)$$

$$R_{\text{SIR,E}} = \frac{8p^2}{3(K-1)} = \frac{2.67p^2}{K-1} \quad (27)$$

Therefore, we obtain the system BER induced by cross-correlation between codes as

$$B_{\text{BER,B}} = \Phi \left( -\sqrt{\frac{R_{\text{SIR,B}}}{2}} \right) = 0.5 + 0.5 \operatorname{erf} \left( -\frac{\sqrt{R_{\text{SIR,B}}}}{2} \right) =$$

$$0.5 + 0.5 \operatorname{erf} \left( -0.548 \frac{p}{\sqrt{K-1}} \right) \quad (28)$$

$$B_{\text{BER}, E} = \Phi \left( -\sqrt{\frac{R_{\text{SIR}, E}}{2}} \right) = 0.5 + 0.5 \operatorname{erf} \left( -\frac{\sqrt{R_{\text{SIR}, E}}}{2} \right) =$$

$$0.5 + 0.5 \operatorname{erf} \left( -0.817 \frac{p}{\sqrt{K-1}} \right) \quad (29)$$

where the variation function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$$

Fig. 2 to Fig. 5 give  $R_{\text{SIR}, B}$ ,  $B_{\text{BER}, B}$ ,  $R_{\text{SIR}, E}$  and  $B_{\text{BER}, E}$  vs. the number of simultaneous users of the OCDMA system with the basic prime code or with the extended prime code, respectively. According to Fig. 2 to Fig. 5, we can see that when the prime number  $p$  increases, the system SIR increases soon and BER decreases fast, which indicates that the greater  $p$  is, the better the system performance. But the greater  $p$  will increase the code-length, which will depress the bit rate. So, for properly choosing  $p$ , we should make overall plans and take all factors into consideration.

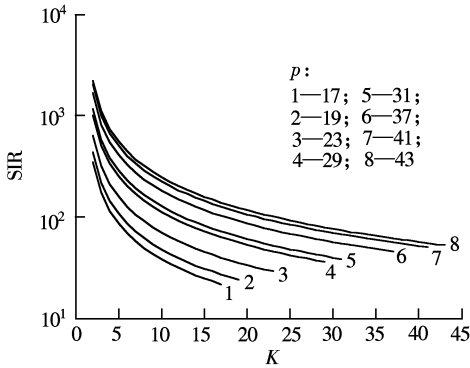


Fig. 2 SIR performance of prime code

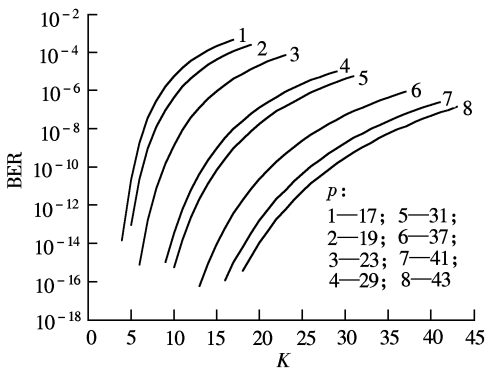


Fig. 3 BER performance of prime code

According to Figs. 2 to 5, as for the same prime code  $p$ , we can directly conclude that the SIR and the BER performance of the extended code  $(p(2p-1), p, p-1, 1)$  are far better than those of the basic prime code  $(p^2, p, p-1, 2)$ . However, the code-length of the former is about twice that of the latter, which indicates

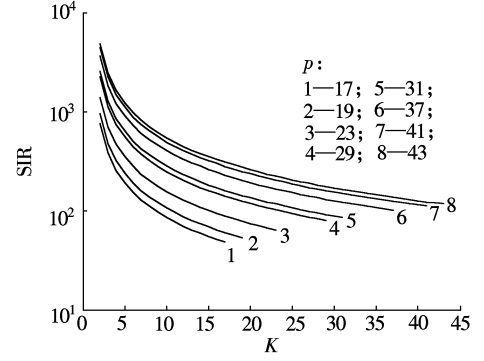


Fig. 4 SIR performance of extended prime code

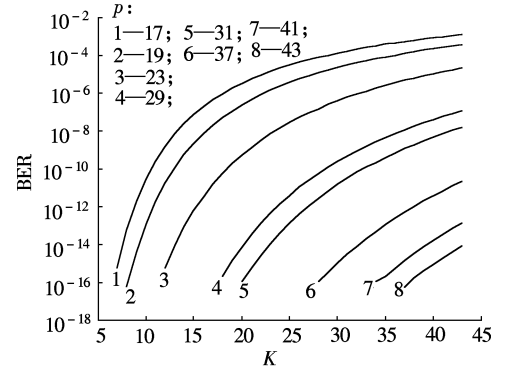


Fig. 5 BER performance of extended prime code

that the improvement in performance is under the penalty of a decrease in bit rate.

With the same code-length, the code weight of EPC is about 0.707 time of that of BPC. We can get the comparison of SIR and BER between the EPC and BPC, according to Eqs. (26) to (29). The comparison results are shown in Fig. 6 and Fig. 7.

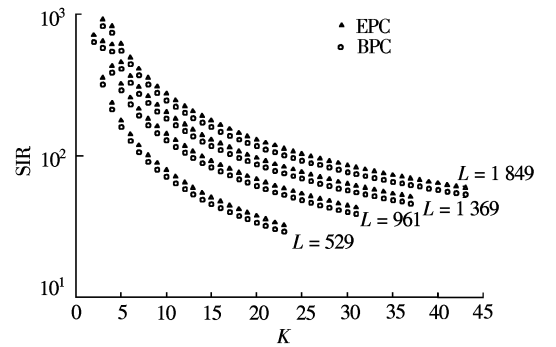


Fig. 6 SIR comparison between EPC and BPC

From Fig. 7, we find that the BER of the OCDMA system with the extended prime code is improved by one order compared to the BPC; therefore, we can say that the extended prime code has a better performance than the basic prime code.

Finally, it should be pointed out that one disadvantage of both the EPC and the BPC is that they have a high autocorrelation value, which leads to the quite

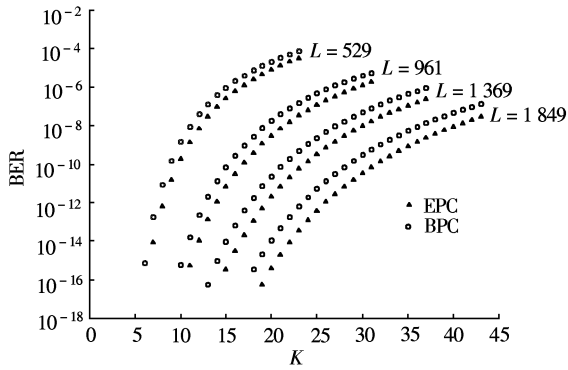


Fig. 7 BER comparison between EPC and BPC

low weight-autocorrelation ratio  $w/\lambda_a = p/(p-1)$ .

#### 4 Conclusion

The performance of the optical code family depends on the BER induced by multiple access interference, which is mainly based on the cross-correlation between signature codes. The optical prime code is one kind of basic signature code, and the correlation property of its code function directly determines the code performance. Starting from the linear congruence theory in the finite Galois field, this paper mathematically analyzes the correlation properties of the basic prime code, the extended prime code and the modified prime code, and then calculates the system SIR and BER induced by multiple access interference according to the distribution of the correlation values. The performance of OCDMA systems with the basic or extended prime codes are compared. The result indicates that the system performance with the EPC is better than that with the BPC under the same transmitting data rate.

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## 光素数地址码互相关特性数学分析

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**摘要:** 围绕 OCDMA 系统的重要地址码类型——光素数地址码的结构模型, 根据有限 Galois 域上的线性全等理论, 系统分析了基本素数码、扩展素数码、修正素数码的互相关特性, 指出码字中“1”位的重叠区域. 具体给出基本素数码最大互相关为 2、扩展素数码和修正素数码最大互相关为 1 的数学证明, 并给出完整的素数码相关性分析过程. 在此基础上, 计算基本素数码和扩展素数码的信号干扰比以及多址干扰误码率, 并比较基于不同素数码的 OCDMA 系统的误码性能, 给出定量比较结果.

**关键词:** 光纤通信; 光码分多址 (OCDMA); 光素数地址码 (OPC); 相关性

**中图分类号:** TN29