Journal of Southeast University (English Edition) Vol. 22, No. 2, pp. 204 – 207 June 2006 ISSN 1003—7985

Multi-objective integrated optimization based on evolutionary strategy with a dynamic weighting schedule

Fu Wujun Zhu Changming Ye Qingtai

(School of Mechanical Engineering, Shanghai Jiaotong University, Shanghai 200030, China)

Abstract: The evolutionary strategy with a dynamic weighting schedule is proposed to find all the compromised solutions of the multi-objective integrated structure and control optimization problem, where the optimal system performance and control cost are defined by H_2 or H_∞ norms. During this optimization process, the weights are varying with the increasing generation instead of fixed values. The proposed strategy together with the linear matrix inequality (LMI) or the Riccati controller design method can find a series of uniformly distributed non-dominated solutions in a single run. Therefore, this method can greatly reduce the computation intensity of the integrated optimization problem compared with the weight-based single objective genetic algorithm. Active automotive suspension is adopted as an example to illustrate the effectiveness of the proposed method.

Key words: integrated design; multi-objective optimization; evolutionary strategy; dynamic weighting schedule; suspension system

The integrated structure/control optimization has been acknowledged as an advanced and systematic methodology to achieve optimal system performance and minimum control cost of the closed-loop system in the vibration suspension. There exist many approaches[1-2] used to solve this complex multi-objective optimization problem. And the weight-based single objective genetic algorithm (SOGA) is most popularly adopted by many researchers[3-5] because of its robust, multipoint and parallel searching characteristics. However, for a multi-objective optimization problem, there often exist a series of compromised solutions, usually called Pareto optimal sets. If all the Pareto solutions can be obtained, the designer can have a full view of the optimization problem. Because the SOGA can only get one solution for a single run, it should be implemented many times in order to get all the Pareto solutions, which greatly increase the computation intensities (time). What is more, the objective weights cannot be clearly defined in most cases. These are required for most controller design methods such as linear matrix inequality (LMI) and the Riccati method, so the weight-based approaches are very difficult to implement for realistic problems.

In this paper, the evolutionary strategy (ES) with a dynamic weighting schedule is proposed to solve the integrated optimization problem. This method can not only obtain the whole Pareto sets in a single run, but it can also avoid the selection of the objective weights precisely beforehand.

Received 2005-11-29.

Biographies: Fu Wujun (1973—), male, graduate; Zhu Changming (corresponding author), male, professor, zhuchangming@sjtu.edu.cn.

1 Modeling

Considering a linear time-invariant continuous time system,

$$\left.\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}(\boldsymbol{p})\boldsymbol{x} + \boldsymbol{B}_1(\boldsymbol{p})\boldsymbol{\omega} + \boldsymbol{B}_2(\boldsymbol{p})\boldsymbol{u} \\ \boldsymbol{z} &= \boldsymbol{C}_z(\boldsymbol{p})\boldsymbol{x} + \boldsymbol{D}_{z1}(\boldsymbol{p})\boldsymbol{\omega} + \boldsymbol{D}_{z2}(\boldsymbol{p})\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}_y(\boldsymbol{p})\boldsymbol{x} + \boldsymbol{D}_{y1}(\boldsymbol{p})\boldsymbol{\omega} + \boldsymbol{D}_{y2}(\boldsymbol{p})\boldsymbol{u} \end{aligned}\right\} \tag{1}$$

where $\boldsymbol{x}$ is the system state vector, $\boldsymbol{z}$ is the performance output, $\boldsymbol{y}$ is the sensor output, $\boldsymbol{\omega}$ is the external disturbance, $\boldsymbol{u}$ is the control input, and $\boldsymbol{p}$ is the set of the structural parameters such as mass, stiffness, and damping coefficients.

In most integrated designs, the objectives can be classified into the system performance J_p and the control cost J_c (or the robust stability)[3,6], which can be usually defined by H_2 or H_∞ norms. So the weighted performance J is expressed as

$$J = w_1 J_\mathrm{p} + w_2 J_\mathrm{c} \tag{2}$$

where w_1 and w_2 are the corresponding weights for J_p and J_c, respectively.

The integrated structure/control design can be formulated as

$$\begin{aligned} &\min: J_\mathrm{p} \text{ and } J_\mathrm{c} \\ &\text{Subject to } \dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{p})\boldsymbol{x} + \boldsymbol{B}_1(\boldsymbol{p})\boldsymbol{\omega} + \boldsymbol{B}_2(\boldsymbol{p})\boldsymbol{u} \\ &\qquad \boldsymbol{u} = -\boldsymbol{K}\boldsymbol{y},\ \boldsymbol{p} \in \{\boldsymbol{p}_\mathrm{l}, \boldsymbol{p}_\mathrm{u}\} \\ &\text{Design variables: } \boldsymbol{p} \text{ and } \boldsymbol{K} \end{aligned} \tag{3}$$

where $\boldsymbol{K}$ is the active controller designed by the LMI or the Riccati method to optimize the weighted performance J; $\boldsymbol{p}_l$ and $\boldsymbol{p}_u$ are the lower and upper limits of the structural parameters, respectively.

2 Integrated Optimization based on Dynamic Weighting Schedule

2.1 Application of dynamic weighting schedule

When the LMI and the Riccati method are adopted to design the active controller in order to optimize the weighted performance J defined by Eq. (2), the weights w_1 and w_2 must be changed during the integrated optimization process in order to get as many as possible of the Pareto optimal solutions.

The dynamic weighting schedule[7] shown in Fig. 1 can be used in this optimization process, which is defined as

$$\left.\begin{aligned} w_1(t) &= |\sin(2\pi t/F)| \\ w_2(t) &= 1.0 - w_1(t) \end{aligned}\right\} \tag{4}$$

where F denotes the varying frequency and t is the current generation of the evolutionary strategy. Obviously, the weights w_1 and w_2 are varying gradually with the generation $F = 150$ and the weight w_1 has two loops 1-0-1 in the whole generation from 0 to 150.

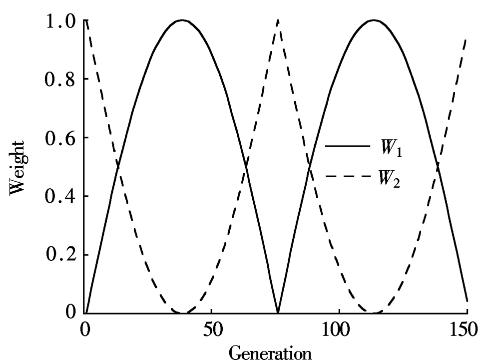

Fig. 1 Dynamic weighting schedule based on generation

2.2 Integrated optimization based on elitist evolutionary dynamic weighting aggregation

Jin et al. [7] first suggested the dynamic weighting aggregation (DWA) for multi-objective problem optimization; however the results in the following section show that the DWA cannot find uniformly distributed solutions to the integrated optimization problem. The elitist evolutionary dynamic weighting aggregation (EEDWA) is proposed in this part based on the standard $(u + \lambda)$-ES[8]. The integrated structure/control design is realized by combining EEDWA and LMI or the Riccati controller design method. The detailed steps are described as follows:

Step 1 Code the structural parameters. Make the dynamic weighting schedule for every generation and randomly generate the μ individuals to form the parent population.

Step 2 Generate the new λ individuals (offspring) by recombination and mutation operations based on the μ individuals in the parent population. So, the whole population is composed of the $\lambda + \mu$ individuals.

Step 3 Set the weights w_1 and w_2 for all the $\lambda + \mu$ individuals according to the current generation number and the dynamic weighting schedule. Optimize its controller $\boldsymbol{K}$ via the LMI or the Riccati method and obtain the system performance and the control cost of the closed-loop system as the fitness of the ES.

Step 4 Non-dominated sort all the individuals according to their fitness and save all the found Pareto solutions in the $\lambda + \mu$ individuals to the special archive[7,9].

Step 5 If the terminated condition is satisfied, output the solutions in the special archive. Or continue the next step.

Step 6 Select the best μ individuals from the current $\lambda + \mu$ individuals to form the new parent population according to their non-dominated levels, and then go to step 2.

3 Case Studies

Active automotive suspensions to achieve riding comfort and stability have been researched for decades. The suspension model provided in Ref. [6] is used to illustrate the integrated structure/control design method based on EEDWA. The quarter car model is shown in Fig. 2, where m_s and m_{us} are the sprung and unsprung masses, respectively; k_{us} is the tire stiffness; k_s and c_s are the suspension stiffness and damping coefficients; x_s, x_{us} and x_g are the vertical displacements of the sprung mass, the unsprung mass and the road, respectively; and u is the active control force.

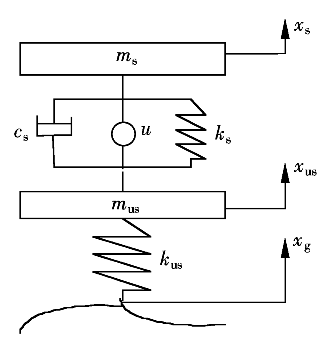

Fig. 2 Quarter car model

If the state $\boldsymbol{x} = \{x_{us} - x_g,\ \dot{x}_{us},\ x_s - x_{us},\ \dot{x}_s,\ \dot{x}_g\}^T$, the state space representation of the suspension model can be expressed as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(k_s, c_s)\boldsymbol{x} + \boldsymbol{B}_1(k_s, c_s)\omega + \boldsymbol{B}_2(k_s, c_s)u \quad (5)$$

$$\boldsymbol{A}(k_s, c_s) = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ -\dfrac{k_{us}}{m_{us}} & -\dfrac{c_s}{m_{us}} & \dfrac{k_s}{m_{us}} & \dfrac{c_s}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & \dfrac{c_s}{m_s} & -\dfrac{k_s}{m_s} & -\dfrac{c_s}{m_s} & 0 \\ 0 & 0 & 0 & 0 & -w_f \end{bmatrix}$$

$$\boldsymbol{B}_1(k_s, c_s) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ w_f \end{bmatrix}, \quad \boldsymbol{B}_2(k_s, c_s) = \begin{bmatrix} 0 \\ -\dfrac{1}{m_{us}} \\ 0 \\ \dfrac{1}{m_s} \\ 0 \end{bmatrix}$$

where ω denotes a zero-mean, white and Gaussian disturbance; and w_f is the cut-off frequency used to filter the disturbance ω to simulate the real ground noise.

The system performance can be defined as the following H_2 norms:

$$J_p = w_1 \left\| \begin{matrix} r_1 \ddot{x}_s \\ r_2(x_{us} - x_g) \\ r_3(x_s - x_{us}) \end{matrix} \right\|_2, \quad J_c = w_2 \| r_4 u \|_2 \quad (6)$$

where r_1, r_2 and r_3 are the coefficients to normalize the acceleration, the tire deflection and the suspension stroke, respectively; and r_4 is the coefficient to normalize the control force.

The system parameters shown in Tab. 1 are used to design the full state feedback controller. The normalization coefficients r_1, r_2, r_3, r_4 for the active suspension system performance are 1, 300, 100, $3.344\,7 \times 10^{-3}$. The numbers of parent and offspring populations are 15 and 100, respectively; and the maximum generation is 150.

Tab. 1 Parameters for the quarter car model

Items	Value
Sprung mass m_s/kg	480
Unsprung mass m_{us}/kg	48
Tire stiffness k_{us}/(kN·m^{-1})	190
Suspension stiffness k_s/(kN·m^{-1})	[0, 160]
Suspension damping c_s/(kN·s^2·m^{-1})	[0, 16]

The optimization results are shown in Figs. 3 and 4. Fig. 3 shows the relationship between the objectives and the design variables. The results in Fig. 3 reveal that the suspension stiffness and damping coefficients in the integrated design are varying with the control cost instead of fixed values as in the sequential counterpart. Fig. 4 shows the Pareto front between the system performance and the control cost found by the proposed method, the SOGA and the DWA. For the sake of comparison, the results obtained by the conventional sequential design method (design the structure first, then optimize the controller sequentially) is also shown in Fig. 4. The results show that the integrated design methods outperform the sequential counterpart. The EEDWA finds 284 uniformly distributed compromised solutions in a single run; the SOGA obtains only 10 solutions by running for 10 times, although the SOGA has the same numbers of the population and the maximum generation as the EEDWA. As stated in section 2, the DWA cannot find the uniformly distributed solutions. Pareto dominance plays an important role for the EEDWA searching a series of uniformly distributed Pareto solutions, which is not only used in archiving (step 2), but also used in the selection of the offspring (step 6). These results reveal that the EEDWA is effective in searching the Pareto fronts of the integrated optimization problem.

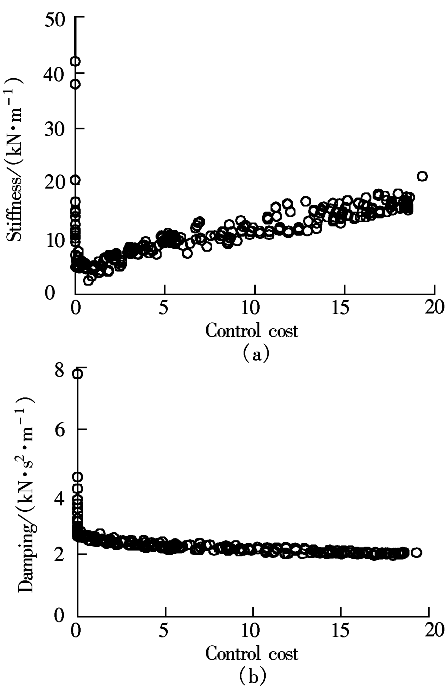

Fig. 3 Relationship between control cost vs. stiffness and damping

The Pareto fronts can help the decision maker to have a full view of the integrated design problem. For example, it is observed from Fig. 4 that if the required control system performance index is larger than about 60, the active controller is not necessary to use, because the passive control system can satisfy this requirement. The system performance cannot be smaller than 54. 4, even if the active controller is applied. And it is easy for the decision maker to choose the final so-

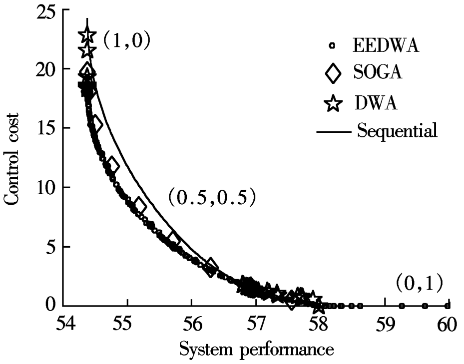

Fig. 4 Pareto front between system performance and control cost

lution from the Pareto fronts because each solution is naturally associated with a couple of weights, as shown in Fig. 4, which defines the relative importance of the optimization objectives, when the EEDWA is used to optimize the problem.

4 Conclusion

This paper describes a new method to solve the integrated structure/control design problem for achieving optimal system performance and minimum control cost. The $(u+\lambda)$-ES with a dynamic weighting schedule is proposed to solve the multi-objective optimization problem. The proposed method can not only find all the uniformly distributed Pareto optimal solutions in a single run but can avoid selecting the weights precisely beforehand, which greatly reduces the computation intensity compared to the SOGA. What is more, the designer can benefit from this method by having a full view of the integrated design problem.

Acknowledgement The authors would like to express their appreciations to Dr. Jin Yaochu for his helpful discussion on this paper.

References

[1] Smith M J, Grigoriadis K M, Skelton R E. Optimal mix of passive and active control in structures [J]. *Journal of Guidance, Control and Dynamics*, 1992, **15**(4): 912 – 919.

[2] Yang Y, Chen Y. Multiobjective optimization of hard disk suspension assemblies: Part Ⅱ—Integrated structure and control design [A]. In: *Proceedings of the American Control Conference* [C]. Seattle, 1995. 2830 – 2834.

[3] Chen Wuwei, Wang Qirui, Zhu Wanling. Integrated structure-control design for a suspension system based on genetic algorithm and H_∞ control [J]. *Journal of Vibration and Engineering*, 2003, **16**(2): 143 – 148. (in Chinese)

[4] Liu X J, Begg D W, Fishwick R J. Genetic approach to optimal topology/controller design of adaptive structures [J]. *International Journal for Numerical Methods in Engineering*, 1998, **41**(5): 815 – 830.

[5] Tsujioka K, Kajiwara I, Nagamatsu A. Integrated optimum design of structure and H_∞ control system [J]. *AIAA Journal*, 1996, **34**(1): 159 – 165.

[6] Fathy H K, Papalambros P Y, Ulsoy A G, et al. Nested plant/control optimization with application to combined passive/active automotive suspensions [A]. In: *Proceedings of the American Control Conference* [C]. Denver, 2003. 3375 – 3380.

[7] Jin Yaochu, Okabe T, Sendhoff B. Adapting weighting aggregation for multiobjective evolution strategies [A]. In: *Proceedings of the First International Conference on Evolutionary Multi-Criterion Optimization* [C]. Zurich, 2001. 96 – 110.

[8] Wang Xiaoping, Cao Lining. *Genetic algorithm—theory, application and software implementation* [M]. Xi'an: Xi'an Jiaotong University Press, 1998. (in Chinese)

[9] Knowles J, Corne D. Approximating the nondominated front using the Pareto archived evolutionary [J]. *Evolutionary Computation*, 2000, **8**(2): 149 – 172.

基于动态加权计划和进化算法的多目标集成优化设计

傅武军　朱昌明　叶庆泰

(上海交通大学机械与动力工程学院,上海 200030)

摘要:为了求解结构和控制器多目标集成优化设计问题,获得用 H_2 或 H_∞ 范数定义的最优系统性能和控制代价的所有非劣解,采用了基于动态加权计划的进化算法. 该算法采用的权值随着进化代数的变化而变化,而不是固定值,通过结合线性矩阵不等式(LMI)或 Riccati 控制器设计方法,一次运行就能够得到均匀分布的非劣解. 与加权的单目标遗传算法相比,该方法可以大大减少求解集成优化设计问题的计算强度. 通过汽车悬架的集成设计表明了该方法的有效性.

关键词:集成设计;多目标优化;进化策略;动态加权计划;悬架系统

中图分类号:U463. 33