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Dynamic finite element model updating using meta-model and genetic algorithm

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Abstract: Current dynamic finite element model updating methods are not efficient or restricted to the problem of local optima. To circumvent these, a novel updating method which integrates the meta-model and the genetic algorithm is proposed. Experimental design technique is used to determine the best sampling points for the estimation of polynomial coefficients given the order and the number of independent variables. Finite element analyses are performed to generate the sampling data. Regression analysis is then used to estimate the response surface model to approximate the functional relationship between response features and design parameters on the entire design space. In the fitness evaluation of the genetic algorithm, the response surface model is used to substitute the finite element model to output features with given design parameters for the computation of fitness for the individual. Finally, the global optima that corresponds to the updated design parameter is acquired after several generations of evolution. In the application example, finite element analysis and modal testing are performed on a real chassis model. The finite element model is updated using the proposed method. After updating, root-mean-square error of modal frequencies is smaller than 2%. Furthermore, prediction ability of the updated model is validated using the testing results of the modified structure. The root-mean-square error of the prediction errors is smaller than 2%.

Key words: finite element model; model updating; response surface model; genetic algorithm

In modern engineering, the precise finite element model (FEM) plays a key role in dynamic design. But there are always errors in the finite element modeling of a structure due to various assumptions and uncertainties. The FEM must be updated to minimize the deviation between finite element analysis (FEA) results and experimental modal analysis (EMA) results. In the past thirty years, a variety of updating methods have been proposed[1]. These methods can be divided into two categories depending on how updating objects are defined: matrix updating and design parameter updating. For the former, elements of mass matrix and stiffness matrix are taken as updating objects. For the latter, design parameters such as the Young's modulus, density and cross section area are taken as updating objects. It is obvious that, for the latter, the updated value can be easily interpreted by the engineers because of its obvious physical significance.

Design parameter updating used to be concluded as constrained optimization problems. Generally the problems are solved according to sensitivities of modal parameters with respect to design parameters. However, it can possibly lead to local optima solutions. To determine the global optima, the genetic algorithm (GA) has been adopted in model updating[2,3]. The genetic algorithm[4] is a kind of global optimization method. The basic idea of the GA is to find the best candidate solution (named "individual" in the GA), which leads to the maximum objective function (named "greatest fitness value"), from a lot of individuals in single step (named "generation" in the GA) of optimization iteration (named "evolution" in the GA). The sensitivity analysis is no longer necessary because only the fitness value of individual is required. Therefore, it is able to deal with such problems that cannot be solved according to sensitivity information.

As to FEM updating, individual is a group of feasible solutions for design parameters. Its fitness value is the function of deviation between EMA results and FEA results. To evaluate the fitness of each individual, FEA must be performed using the design parameters corresponding to that individual. Generally, there are hundreds of individuals in a generation of the GA. This implies that hundreds of FEA must be performed in a single generation. This is really computational-in-

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tensive for structure with great dimensions of freedoms. Hence, current updating method[2,3] based on the GA is not applicable for real structures.

In recent years, meta-models have been accepted in engineering[5-7]. Fast meta-models are mathematical models constructed according to sampling data. They are utilized as the surrogate models for FEMs to describe the functional relationship between physical parameters and structural features such as modal frequencies. The response surface model (RSM)[8] is one of the major meta-models. Polynomials are used to approximate the map function between physical parameters and structural features by using regression analysis.

1 Updating Using RSM and GA

The FEM updating using the RSM and the GA is a two-stage method. The first stage is the construction of the RSM using the experimental design technique and the regression analysis (RA) method. The second stage is to perform the global search using the GA.

In the second stage, the RSM is used to define the fitness function and to substitute FEM in the evolution to compute the fitness values. Fig. 1 shows the flowchart of the proposed method. The details of the two stages are introduced in the following sections.

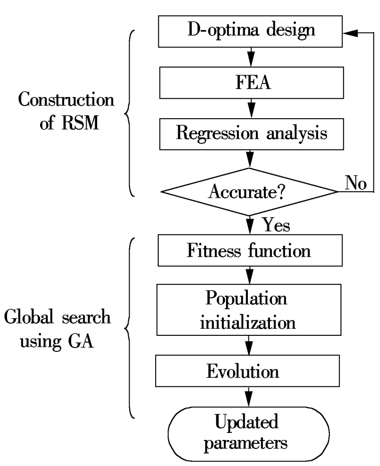

Fig. 1 Flowchart of updating based on RSM and GA

1.1 Construction of response surface model

Supposing that the following polynomial model describes the functional relationship of modal frequency with respect to design parameters,

$$f = \beta_0 + \sum_{j=1}^{m} \beta_j p_j \tag{1}$$

where f is the modal frequency, $p_j \in [-1, 1]$ is the normalized design parameter, and β_0 and β_j are unknown coefficients.

Supposing that there are n groups of samples

$$\left.\begin{aligned} f_1 &= \beta_0 + \beta_1 p_{11} + \dots + \beta_m p_{1m} \\ f_2 &= \beta_0 + \beta_1 p_{21} + \dots + \beta_m p_{2m} \\ &\vdots \\ f_n &= \beta_0 + \beta_1 p_{n1} + \dots + \beta_m p_{nm} \end{aligned}\right\} \tag{2}$$

where m is the number of parameters to be updated.

Rewrite Eq. (2) into matrix form,

$$\boldsymbol{F} = \boldsymbol{X\beta} \tag{3}$$

where

$$\boldsymbol{F} = \{f_1, f_2, \dots, f_n\}^{\mathrm{T}}$$

$$\boldsymbol{X} = \begin{bmatrix} 1 & p_{11} & p_{12} & \cdots & p_{1m} \\ 1 & p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & p_{n1} & p_{n2} & \cdots & p_{nm} \end{bmatrix} \tag{4}$$

$$\boldsymbol{\beta} = \{\beta_0, \beta_1, \dots, \beta_m\}^{\mathrm{T}}$$

The unknown coefficient vector $\boldsymbol{\beta}$ can be estimated by solving Eq. (3),

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{F} \tag{5}$$

The accuracy of the polynomial model (1) must be verified and the following indicator is used,

$$E = \frac{1}{n}\sum_{j=1}^{n}(f_j - \hat{f}_j)^2 \tag{6}$$

where n is the number of samples, f_j is the FEA result, and $\hat{f}_j$ is the results output by the RSM.

The procedure can be concluded as follows:

① Determine sampling points using D-optima design. D-optima[9] is a kind of experimental design technique which determines the best sampling points for the estimation of a polynomial's unknown coefficients given the order and the number of independent variables of the polynomial model. In this study, D-optima is used to determine matrix $\boldsymbol{X}$ in Eqs. (3) and (4).

② Compute the sampling data using FEA. In this step, FEA is performed n times. The row of matrix $\boldsymbol{X}$ is taken in turn to determine the value of design parameters as the input of FEA in each time. The result of this step is vector $\boldsymbol{F}$ in Eqs. (3) and (4).

③ Estimate the coefficients based on sampling data.

④ Verification of the RSM. If the RSM is not accurate, go back to step ①. Augmentation sampling points can be determined by D-optima.

1.2 Global search using genetic algorithm

After the RSM is constructed and verified, it is adopted in the global search using the GA. Major steps of the application of the genetic algorithm include the construction of fitness function, the initialization of population and the evolution. Besides, the definition of variable bounds, the coding and decoding method and the design of operators must be considered.

Among all the problems, construction of fitness function is the most important because the fitness is the key information for the evolution. In this study, the purpose of the FEM updating is to find a group of design parameters $\bar{\boldsymbol{P}} = (\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_m)$, which minimize the deviation between experimental modal frequency and analytical modal frequency. In this paper, the deviation is described by the square of relative error of modal frequency. Considering that the GA seeks the individual with the greatest fitness value, the reciprocal of the error is adopted as the fitness function,

$$f_{\text{fitness}} = \sqrt{\frac{1}{\left(\dfrac{f_a - f_e}{f_e}\right)^2}} \tag{7}$$

where f_e stands for the experimental modal frequency, and f_a stands for the analytical modal frequency. Instead of using FEA in the current method[2,3], the RSM is adopted to compute the analytical modal frequency.

The initialization of population is generally performed by randomly selecting hundreds of groups of values for the design parameters. Each group is called "individual". The RSM is used to compute analytical modal frequency taken every individual as the input of the polynomial model. Then the analytical modal frequency is adopted to compute the fitness for the individual according to Eq. (7).

The evolution starts from the initial population and finishes after several generations. In every generation, the selection, the crossover and the mutation are performed. To accelerate the convergence, the elitist model is employed. The best individual of the current generation is copied to the next generation without being operated.

The design parameters are normalized in the construction of the RSM, the bounds are [−1, 1]. Float representation is adopted in coding and decoding.

2 Application Example

A chassis model of NJ6550 is adopted as an application example. FEA and modal testing are performed. The FEM is updated using the proposed method.

2.1 Finite element analysis

ANSYS is employed in the FEA. Fig. 2 shows the FEM of the chassis. Spring elements are used to model the joints. Altogether, the FEM consists of 100 beam elements and eight spring elements.

Because only the low order modes greatly affect the dynamic performance of the chassis, only the first twelve modes are considered.

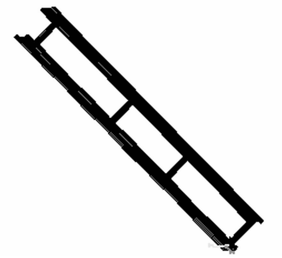

Fig. 2 FEM of the chassis model

2.2 Experimental modal analysis

2.2.1 Modal testing

The chassis model is suspended by elastic band. The suspension frequency is 1 Hz. Due to the limited number of the available sensors, two arrangements are made for the measurement of the vertical and the horizontal dynamic characteristics, respectively.

For each arrangement, multiple reference hammer testing is performed. Three acceleration sensors are located on point 1, point 10 and point 30. Impulse excitation is put on all the thirty points in turn. Fig. 3 shows the location of points.

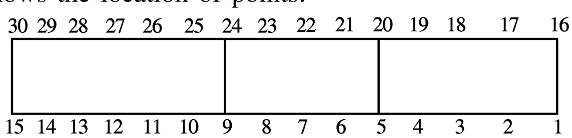

Fig. 3 Location of points

Fig. 4 shows the reciprocal of frequency response functions (FRF) between point 1 and point 30 in the vertical direction testing.

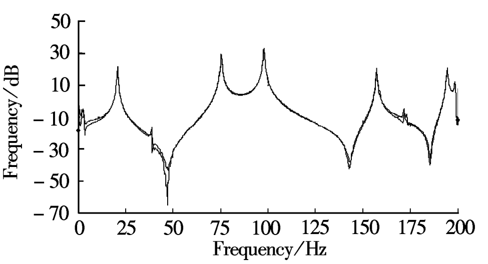

Fig. 4 Typical FRF

2.2.2 Modal identification

The rational fraction orthogonal polynomial (RFOP) method is adopted to identify modal parameters. Fig. 5 displays modal frequencies and mode shapes of the first six modes. Figs. 5(a), (c) and (e) are vertical modes and correspond to the first three significant peaks in Fig. 4.

2.3 FEM updating

The FEM is updated using the proposed method. Third order polynomial models are fitted for each modal frequency with respect to its sensitive design parameters.

The design parameters are the Young's modulus E of beams, the material density of beams and the spring stiffness. Details are listed in the first column of

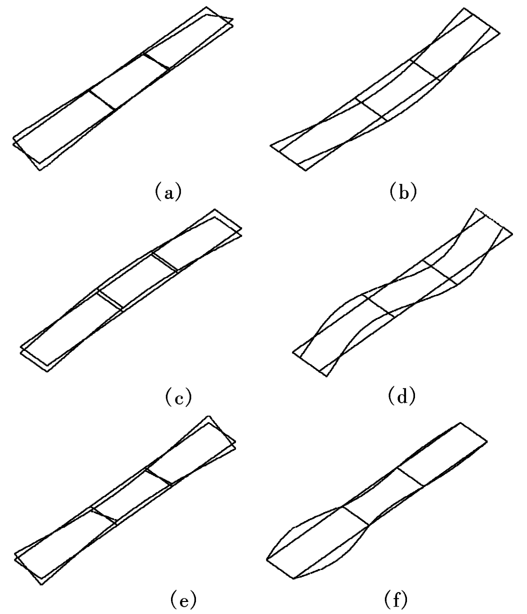

Fig. 5 Identified modal frequencies and mode shapes. (a) 20.78 Hz; (b) 38.81 Hz; (c) 75.40 Hz; (d) 89.61 Hz; (e) 97.91 Hz; (e) 126.44 Hz

Tab. 1. Fig. 6 is the visualization of the response surface of the second modal frequencies.

Global search based on the genetic algorithm is then performed. The initial population has 256 individuals. Crossover probability and mutation probability are 0.9 and 0.01, respectively. Nine experimental modal frequencies are used in the updating. Tab. 1 lists the initial value and updated value of the parameters.

Tab. 1 Updating design parameters

Item	Initial value	Updated value
E (long beam)/GPa	210	204
E (short beam)/GPa	210	200
Density of beam/(kg · m^{-3})	7 850	7 813
Length (short beam)/m	0.380 0	0.380 5
Spring stiffness (x)/(MN · m^{-1})	20.0	20.8
Spring stiffness (y)/(MN · m^{-1})	100	97
Spring stiffness (z)/(MN · m^{-1})	2.00	1.46

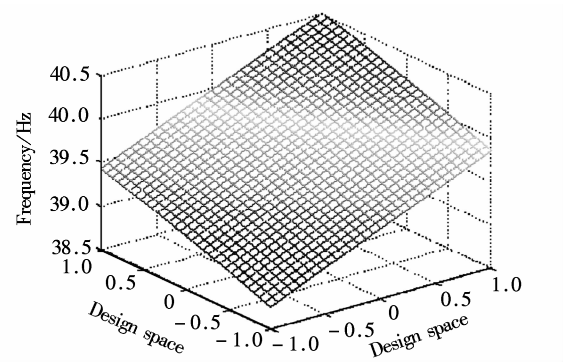

Fig. 6 Visualization of response surface model

Tab. 2 lists the initial analytical results, the results after updating and their corresponding errors. The root-mean-square error decreases from 4.96% to 1.60%. The maximum error decreases from 10.08% to 3.66%.

Tab. 2 Modal frequency error

Mode number	Test result/Hz	Initial analytical result/Hz	Initial error/%	Analytical result after updating/Hz	Error after updating/%
1	20.78	21.69	4.38	20.43	−1.68
2	38.81	38.29	−1.34	38.11	−1.80
3	75.40	77.86	3.26	75.83	0.57
4	89.61	90.29	0.76	88.95	−0.74
5	97.91	102.21	4.39	98.70	0.81
6	126.44	134.50	6.37	129.59	2.49
7	157.33	162.26	3.13	158.09	0.48
8	159.11	162.73	2.28	158.94	−0.11
9	171.69	172.97	0.75	170.19	−0.87
10	173.38	190.87	10.08	175.43	1.18
11	194.52	207.40	6.62	201.65	3.66
12	198.58	211.40	6.46	200.59	1.01

2.4 FEM validation

To validate the qualification of the updated model, the predication ability must be taken into account[10]. Therefore, the structure is modified by adding a beam between point 3 and point 18. Also, the modified structure is tested and modal parameters are identified to be used in the validation. The modal frequencies of the modified structure are listed in the second column of Tab. 3.

Tab. 3 Prediction error of modal frequencies

Mode number	Test result/Hz	Predicted result/Hz	Prediction error/%
1	22.12	21.90	−0.99
2	40.89	39.89	−2.44
3	75.51	75.91	0.53
4	95.55	95.07	−0.50
5	99.81	100.92	1.11
6	127.85	130.76	2.28
7	157.12	158.15	0.66
8	158.92	159.75	0.52
9	167.90	167.68	−0.13

Beam elements are added to the updated FEM and the FEA is performed to predict the modal frequencies. The analytical results are then compared with the identified results. Tab. 3 lists the prediction and the experimental results. The root-mean-square and the maximum of the prediction error are 1.28% and −2.44%, respectively.

3 Conclusion

A finite element model updating method based on the response surface model and the genetic algorithm is proposed. The proposed method has two advantages. First, in the implementation of the GA, the FEM is replaced by the RSM in the fitness evaluation. As a result, the computational-intensive FEA is no longer necessary. Secondly, the RSM is able to demonstrate the functional relationship with visualization, which can be employed to judge whether the local optima exists.

A real chassis model is taken as an application example. FEA and modal testing are performed. The proposed method is used to update the FEM. The root-mean-square error of modal frequencies decreases from 4.96% to 1.60%. Also updating qualification is validated by assessing the prediction ability. The root-mean-square error of prediction is 1.28%.

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一种利用等效模型与遗传算法的动态有限元模型修正方法

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摘要:为了解决现有动态有限元模型修正方法计算效率不高或者可能获得局部最优解的问题,提出了一种利用等效模型和遗传算法的动态有限元模型修正新方法. 首先,在设计参数的取值范围内,根据预设的多项式模型的阶次以及自变量的个数,利用试验设计方法获得拟合响应面模型所需要的最优样本点;通过有限元分析获得样本数据,并利用回归分析获得响应面模型,从而以响应面模型逼近结构特征与设计参数之间的函数关系. 然后,在遗传算法的适应度评估环节,利用响应面模型替代有限元模型计算对应于一组设计参数的结构特征,并计算遗传个体的适应度,最终通过进化获得最优解,即为修正后的设计参数. 以汽车车架模型为例,对其进行有限元分析与模态试验,并利用所提出的方法进行模型修正. 修正后,模态频率误差的均方值小于2%. 用修改后结构的动态特性的测试结果,对修正后有限元模型的预测能力进行检验,模态频率预测误差的均方值小于2%.

关键词:有限元模型;模型修正;响应面模型;遗传算法

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