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Robust edge detection based on stationary wavelet transform

Zhang Guobao Liu Quan

(Research Institute of Automation, Southeast University, Nanjing 210096, China)

Abstract: By combining multiscale stationary wavelet analysis with fuzzy c-means, a robust edge detection algorithm is presented. Based on the translation invariance built in multiscale stationary wavelet transform, components in different transformed sub-images corresponding to a pixel are employed to form a feature vector of the pixel. All the feature vectors are classified with unsupervised fuzzy c-means to segment the image, and then the edge pixels are checked out by the Canny detector. A series of images contaminated with different intensive Gaussian noises are used to test the novel algorithm. Experiments show that fairly precise edges can be checked out robustly from those images with fairly intensive noise by the proposed algorithm.

Key words: edge detection; stationary wavelet; multiscale analysis; fuzzy c-means

Edge detection is still an important and difficult problem in image processing. Wavelet analysis has favorable localized character in both spatial domain and frequency domain, and appears to be an ideal tool to detect image edges. A typical algorithm is proposed by Mallat and Zhong[1] in which a dyadic wavelet transform has been constructed to detect edges. The algorithm is a combination of the wavelet transform and the gradient edge detection method. Experiments have shown that the wavelet-based multi-scale or multi-resolution edge detection algorithm possesses some advantages over those traditional edge detectors.

Earlier wavelet-based edge detection algorithms are based on downsampling wavelet transform. An essential property in image processing, known as the translation invariance, is lost[2]. It is almost impossible to fuse all the useful information embedded in different sub-images into an edge detection algorithm to find precise and robust image edges.

Stationary wavelet transform without downsampling has been proposed recently. It has to do with the property of translation invariance. In this paper, a novel edge detection algorithm based on stationary wavelet transform is discussed to improve precision and robusticity of the wavelet-based edge detection algorithm. According to the novel algorithm, an image is first decomposed with multiscale stationary wavelets, data in each decomposed image corresponding to the same pixel is used to form a feature vector. All the feature vectors are then segmented into clusters by fuzzy c-means. Finally, the Canny detector[3] is utilized to find the edges from the segmented image.

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Biography: Zhang Guobao(1965—), male, doctor, associate professor, guobaozh@ seu. edu. cn.

1 Feature Vector Based on Stationary Wavelet Transform

Traditionally two-dimensional discrete wavelet transform (DWT) is implemented by using digital filters and downsamplers with two separable one-dimensional scaling and wavelet functions[1]. An image $f(x, y)$ is first transformed along its rows/columns, and then along the columns/rows. It results in four quarter-size sub-band images labeled LL, LH, HL, and HH, respectively; where H indicates highpass filter and L indicates lowpass filter; HL indicates highpass filter followed by lowpass filter; and HH indicates highpass filter followed by highpass filter. The same process of the next scale wavelet transform is repeated on the LL image, while the other three sub-images remain invariable.

If an image is decomposed as before, the size of the transformed sub-images becomes quarter-size because of downsampling. It is very clear that such a DWT cannot maintain the position translation invariance which is a key to detecting precise edges by using multiscale methods. A solution to this problem is stationary wavelet transform, without downsampling, producing sub-band images of the same size as the original one[4]. Fig. 1 shows the process of two-dimensional stationary wavelet transforms in block diagram form. If an image is decomposed with stationary wavelet trans-

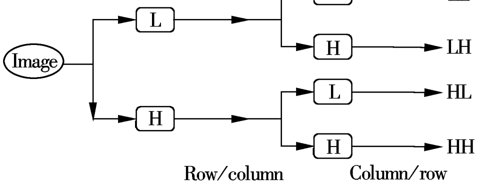

Fig. 1 Two-dimensional stationary wavelet transforms

form, it maintains the position translation invariance.

While decomposing an image, the number of the scales n is an important parameter. Many experiments show that it is enough for edge detection to set n smaller than 3. In this paper, $n=2$, so there are nine component images (including the original image). Let $x_1, x_2, \dots, x_8, x_9$ denote nine values corresponding to a pixel existing in each image respectively. The nine elements can be expressed in the form of a nine-dimensional vector $\boldsymbol{X}$ where $\boldsymbol{X} = \{x_1, x_2, \dots, x_9\}^{\mathrm{T}}$. The vector is taken as the feature vector of the pixel. If the images are of size $M \times N$, there will be a total of $M \times N$ nine-dimensional vectors. Fig. 2 shows the formation of a nine-dimensional vector $\boldsymbol{X} = \{x_1, x_2, \dots, x_9\}^{\mathrm{T}}$ in the nine sub-images.

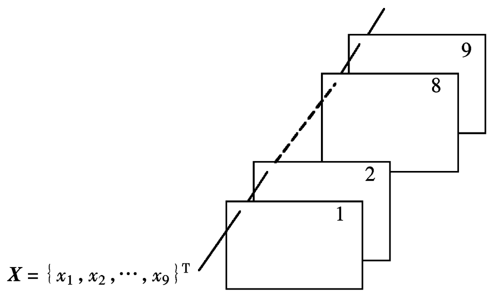

Fig. 2 Formation of a feature vector

2 c-Means Edge Detection Algorithm

Fuzzy clustering has been proved to be very effective in dealing with information contaminated with noises. According to the fuzzy clustering framework, each cluster is a fuzzy set and each pixel in the image has a membership value associated to each cluster, ranging from 0 to 1, measuring how much the pixel belongs to that particular cluster[5].

Let $\boldsymbol{X} = \{x_i,\ i = 1, 2, \dots, n\}$ is a set which contains n pixel, c is the cluster number, $m_j (j = 1, 2, \dots, c)$ is the center of each cluster, and $\mu_j(x_i)$ is the membership of a pixel x_i to the cluster j. The fuzzy c-means clustering algorithm tries to seek the minimum of a heuristic global cost function:

$$J = \sum_{j=1}^{c} \sum_{i=1}^{n} [\mu_j(x_i)]^b \| x_i - m_j \|^2 \qquad (1)$$

where b is a free parameter chosen to adjust the "banding" of different clusters. If $b > 1$, the criterion allows each pattern to belong to multiple clusters.

The probabilities of cluster membership for each point are normalized as

$$\sum_{j=1}^{c} \mu_j(x_i) = 1 \qquad (2)$$

Then the minimum of J in Eq. (2) is determined by

$$\frac{\partial J}{\partial m_j} = 0, \quad \frac{\partial J}{\partial \mu_j(x_i)} = 0 \qquad (3)$$

Its solution is

$$m_j = \frac{\sum_{i=1}^{n} [\mu_j(x_i)]^b x_i}{\sum_{i=1}^{n} [\mu_j(x_i)]^b} \qquad j = 1, 2, \dots, c \qquad (4)$$

and

$$\mu_j(x_i) = \frac{(1/\| x_i - m_j \|^2)^{\frac{1}{b-1}}}{\sum_{r=1}^{c} (1/\| x_i - m_r \|^2)^{\frac{1}{b-1}}}$$

$$i = 1, 2, \dots, n; j = 1, 2, \dots, c \qquad (5)$$

It seems impossible to get analytic solutions for Eqs. (4) and (5) to minimize J. Cluster means and point probabilities μ_j are estimated iteratively with a bearable cut-off error ε according to the following procedures:

① To initialize the cluster number c, the parameter $b=2$, the cut-off error $\varepsilon > 10^{-5}$, the cluster centrals $m_j^{(0)}$, and the iterative number $k=0$.

② To calculate the membership by

If $\forall\, i, r, d_{ir}^{(k)} > 0$, then

$$\mu_j^{(k)}(x_i) = \frac{(1/d_{ij}^{(k)})^{\frac{1}{b-1}}}{\sum_{r=1}^{c} (1/d_{ir}^{(k)})^{\frac{1}{b-1}}}$$

If $\exists\, i, r, d_{ir}^{(k)} = 0$, then $\mu_r^{(k)}(x_i) = 1$, $(j \neq r)$, otherwise $\mu_j^{(k)}(x_i) = 0$, where $d_{ij}^{(k)} = \| x_i - m_j^{(k)} \|^2$.

③ To update the centers $m_j^{(k)}$ by

$$m_j^{(k)} = \frac{\sum_{i=1}^{n} [\mu_j^{(k)}(x_i)]^b x_i}{\sum_{i=1}^{n} [\mu_j^{(k)}(x_i)]^b}$$

④ To test the convergence of membership value. If $\| m_j^{(k)} - m_j^{(k+1)} \| < \varepsilon$, then it stops, otherwise let $k = k+1$ and repeat from step ②.

The cluster numbers can simply be determined from the fluctuation of a histogram, or as a variable parameter updated if the algorithm does not reach convergence after a limited iterative loop, for example, $k = 20$.

If the algorithm reaches convergence, the cluster centers and the memberships have been found, and the image has been segmented into c clusters. Then the Canny detector is employed to detect edges in the segmented images. Fig. 3 gives the block diagram of this novel edge detection algorithm.

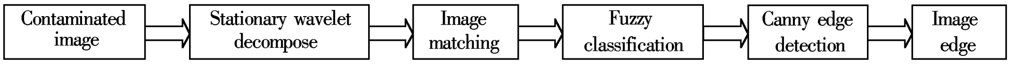

Fig. 3 Overall scheme of the proposed algorithm

3 Experiments

The daubechie-4 (db-4) wavelet is selected here because it has the least compact support with the given vanishing moment. It has been demonstrated that db-4 possesses a strong capability of resisting noise at image processing. Some texture images and real images are taken as the test images. Fig. 4 shows the results performed on a texture image with the additive Gaussian white noise (The variance is 0. 1). Fig. 5 shows similar results on the Lena image to which is added the Gaussian white noise (The noise mean is 0, the variance is 0. 02). All the experiments are implemented on Matlab 6. 0 with similar default values[6]. In Fig. 5, the edge map detected directly with the Canny detector is also presented.

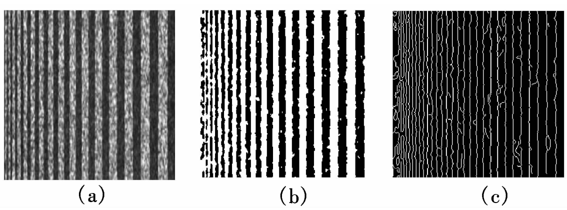

Fig. 4 Edge detection on a typical texture image. (a) The original texture image with additive Gaussian white noise, variance is 0. 1; (b) The segmented image; (c) The edge map detected with the proposed detector

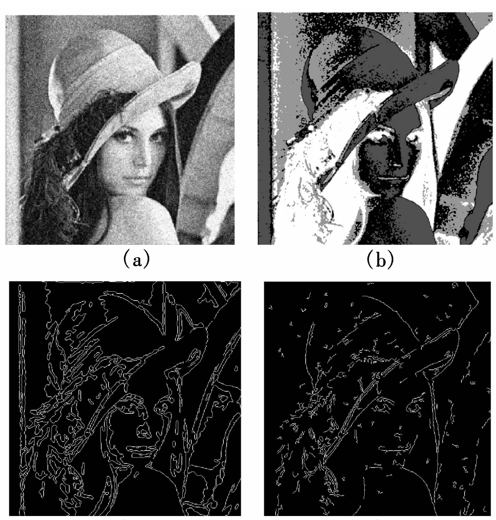

Fig. 5 Experiments on the Lena image. (a) The original Lena image with additive Gaussian white noise (variance is 0. 2); (b) The segmented image; (c) The edge map detected with the proposed detector; (d) The edge map detected directly with the Canny detector

Fig. 4 and Fig. 5 demonstrate that the algorithm succeeds in detecting edges from different images contaminated with noise, and the detected edges are legible and precise. Figs. 5 (c) and (d) show that the algorithm has some advantages over the direct Canny detector when dealing with images contaminated with noise. Under similar conditions, the edge map detected with the novel algorithm contains more details than that detected directly with the Canny detector.

In order to test the robusticity of the novel edge detection algorithm, a series of Lena images contaminated with varying intensities of white noises have been tested. The results in Fig. 6 show clearly that the novel algorithm can detect legible edges until the noise variance is above 0. 5 when the noise pollution is very serious, and it possesses a fairly strong capability of "immunity" from the noise.

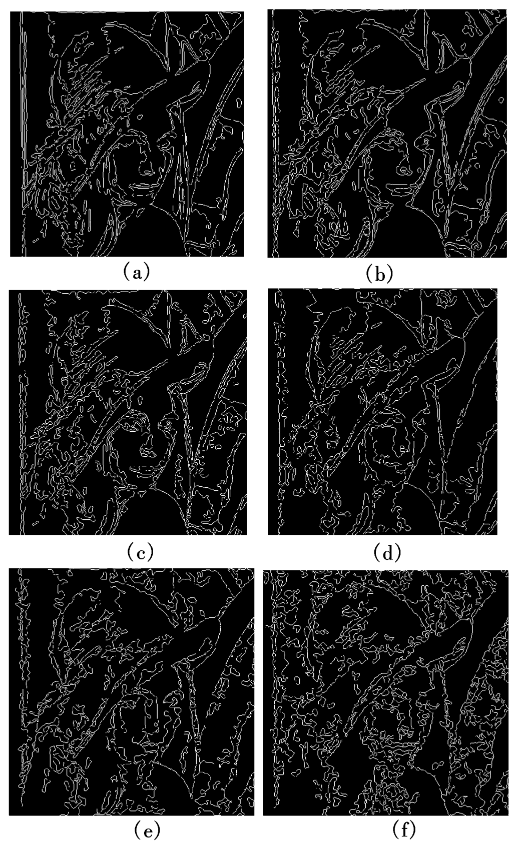

Fig. 6 Detected edge map on Lena images with different intensive white noises. (a) Original image; (b) With additive Gaussian white noise, variance is 0. 01; (c) With additive Gaussian white noise, variance is 0. 05; (d) With additive Gaussian white noise, variance is 0. 10; (e) With additive Gaussian white noise, variance is 0. 20; (f) With additive Gaussian white noise, variance is 0. 50

4 Conclusion

In this paper, stationary wavelet and fuzzy c-means are combined in order to develop a novel robust image edge detection algorithm. Information on all sub-images decomposed with stationary wavelet is utilized to form feature vectors in the algorithm, theoretically it may obtain more precise and robust image edges. Many experiments on different images with different intensive noises have shown that it is really an effective and robust edge detection algorithm, and appears more robust than the direct Canny edge detector when dealing with images contaminated with intensive noise. Further study on comparing the novel algorithm with the other edge detection algorithms will be continued in the future.

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基于平稳小波变换的高鲁棒性的边缘提取

章国宝　　刘　泉

(东南大学自动化研究所,南京 210096)

摘要:借鉴平稳小波变换的多尺度分析思想,结合模糊聚类均值法,提出了一种高鲁棒性的图像边缘提取算法. 该算法利用平稳小波变换的位移不变性,将小波分解后的分量进行配准构成一像素的特征向量,然后利用模糊 c-均值进行无监督分类,分割图像,最后用 Canny 算子提取图像边缘. 用一系列附加不同强度的高斯白噪声图像测试了该算法的有效性. 实验证明:在图像受到较强噪声(如附加高斯白噪声)污染时,该算法仍可检测到较好的边缘效果,展现出良好的鲁棒性.

关键词:边缘检测;平稳小波;多尺度;模糊 c-均值

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