

Integrated continuous replenishment planning in VMI system of retail industry

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Abstract: In order to let suppliers monitor customers' inventory levels and make supply decisions regarding order quantities and delivery time, an integrated continuous replenishment planning (CRP) strategy of fresh food oriented to retail industry is presented, which integrates the consolidation of shipment and inventory replenishment considering the deterioration of items to guarantee the quality safety of fresh food during the replenishment cycle. A vendor managed inventory (VMI) mathematical model to compute upper-level inventory and delivery cycle is built. Based on the real-time sales data exchange, it enables suppliers to make the decision of the optimal time and the quantities of replenishment for retailers during a delivery cycle, in order to replenish the consumers' stock initiatively and minimize the long-run average cost.

Key words: vendor managed inventory; continuous replenishment planning; integrated replenishment; fresh food

Traditionally, the retail industry employs a “push” replenishment policy to manage its stock, based on demand forecast but not the real-time demand from point-of-sale (POS). In order to reduce the total operational cost with a condition of relatively high customer service level, many suppliers prefer to adopt a “pull” replenishment policy in the VMI system (see Fig. 1). In VMI, the supplier controls the retailer's inventory level, so as to ensure that predetermined customer service levels are maintained. In such a relationship, the supplier takes the replenishment decisions for the buyer, dispatching a quantity of product that may be fixed (to maximize production or transport efficiency, for instance) or variable^[1]. Retailers working alone are generally not able to achieve the same productivity increases because the vendor is the one able to provide a more responsive replenishment system based on more precise demand information^[2].

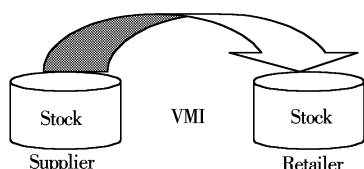


Fig. 1 Model of VMI

Continuous replenishment planning (CRP) is one of the key technologies of support for VMI. In some cases, it has been proved that both suppliers and retailers can achieve major improvements by lowering in-

ventories and reducing stockouts^[3]. CRP is an operation pushing products from inventory holding areas to pulling goods onto retailer shelves based on the consumer demand^[4]. In the CRP strategy, vendors receive point of sale data by electronic data interchange (EDI) and use this data to prepare shipments at previously agreed intervals to maintain specific inventory levels. Skouri et al.^[5] studied a continuous review inventory model, and Tyan et al.^[6] presented a consecutive improvement approach to solve the problem of determining the timing and size of replenishments. Ref. [7] investigated optimal properties of the replenishment policy for a two-echelon outsourcing system. Ref. [8] studied two strategies focusing on the substantial economies of scale in transportation. Ref. [9] built a model for stock replenishment and shipment for minimizing the total costs. In the supermarket retail business, to meet the increasing demands for raw and fresh food and to ensure its safety, it is necessary to establish an applied mode of CRP. The model presented in this paper is evolved from Ref. [9] and extended to figure out the optimal time and the quantities of replenishment for retailers.

1 Structure of Integrated CRP Model

1.1 Description of the model of shipment

The main idea of the consolidation of replenishment is to reduce the costs of replenishment by combining several consolidation cycles into a single larger shipment under the assumption that customers can wait^[10]. Also, it is assumed that partial shipment is allowed; i. e., if a shipment is due but the stock is insufficient to fill all the outstanding demand, then the vendor will ship whatever quantity is available. Under

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the VMI management mode, vendors have the full rights to make the decision of the time and the quantities of replenishment for retailers, so they can adopt a flexible, integrated replenishment strategy according to the retailers' current inventory and destination of shipments, then choose the corresponding strategy of production planning and inventory organization by the mode of shipments. To fulfill the requirements of retailers, vendors consider a kind of advantageous replenishment strategy based on time. Namely, adopt the way of replenishment that distributes only once the accumulated demands per constant interval (e. g. one day). Under this kind of strategy, it perhaps cannot get large-scale economic profits of replenishment, but it can guarantee that each demand can be satisfied in a certain time in advance, consequently, it can implement continuous replenishment to improve vendors' service levels and create business opportunities for both parties in VMI systems.

To describe the model, it is necessary to set up a graphical model as shown in Fig. 2, which is improved from Ref. [9].

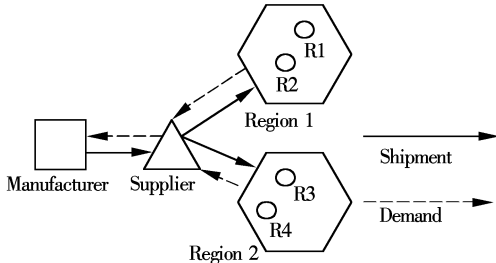


Fig. 2 Model of shipment

There are one manufacturer, one supplier and a group of retailers in some regions. Customers' demands are assumed following the compound Poisson process in the same region. Manufacturers and suppliers monitor every retailers' sales data and respond to the customers' demands following the information flows. When a customer's demand arrives, the supplier may choose not to ship the unit immediately, but consolidate the orders from several customers to form a large quantity of deliveries and wait for shipment per cycle. The supplier's inventory must satisfy the demands of delivery when it comes to the point of shipment. In order to simplify the analysis, it is assumed zero lead-time for replenishment orders regardless of the delay of replenishment. In this paper, it is supposed that the deterioration of items occurs at a fixed rate independent of time and the supplier uses the record point or the (R, nQ) policy to replenish stock. Under the (R, nQ) policy, a replenishment order of size Q is issued whenever the inventory position drops to the reorder level R .

The supplier employs the (R, nQ) policy to replenish its inventory; i. e., when the inventory level I drops to or below R , and n satisfies $R + 1 \leq I + nQ \leq R + Q$, a quantity of Q is ordered.

It is easy to show that the optimal reorder point must be negative under the assumption of zero lead-time. Without loss of generality, it is only considered those reorder points $R \geq -Q$.

1.2 Foundation of the integrated CRP model

The subsequent derivation is based on Fig. 3, which displays the dynamics of inventory levels. Let λ be the rate of the Poisson demand process, Q represent the supplier's objective inventory level after replenishment, T represent the consolidation cycle length, and T_j represent the length of the period during the j -th replenishment cycle ($j = 1, 2, \dots$). S_n is the time that the n -th demand arrives, and it can be inferred that

$$E[S_n] = \frac{n}{\lambda} \quad n = 1, 2, \dots \quad (1)$$

Then, let $N(t)$ represent $\sup\{n: S_n \leq t\}$, denoting the number of requested shipment during the period t , and it can be inferred that

$$E[N(t)] = \lambda t \quad (2)$$

The supplier will not dispatch its products until t unit arrives.

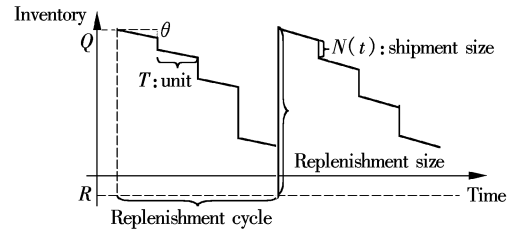


Fig. 3 Dynamics of inventory levels

As shown in Fig. 3, the object of supplier is to minimize the long-run average cost by determining the suitable Q and T by integrating the management of inventory and shipment. Let $C(Q, T)$ represent the long-run expected average costs, $E[R_1]$ represent the expected replenishment cost per cycle, and $E[R_2]$ represent the expected replenishment cycle length. Then the objective function can be written as

$$C(Q, T) = \frac{E[R_1]}{E[R_2]} \quad (3)$$

The objective function of this model derived from Eq. (3) can be written as

$$\min C(Q, T) \quad \text{s. t } Q \geq 0, T \geq 0 \quad (4)$$

From Fig. 3, when the number of the requested replenishments during the cycle is greater than Q , the consumer's stock needs to be replenished, so define the amount of shipment during the cycle as $K = \inf\{k: \sum_{j=1}^k N_j > Q\}$, and then $E[K] = \frac{Q+1}{\lambda T}$. The

expected replenishment cycle length is now presented as $E[K]T$.

Now define each of five cost components: replenishment, shipment, holding, waiting and deterioration costs.

1) Replenishment and dispatching costs per cycle

Let A_R represent the fixed cost of the replenishing inventory, and A_D represents the fixed cost of dispatching. It is clear to see that the inventory replenishment cost per replenishment cycle is A_R . And the dispatching cost per replenishment cycle is

$$E[K]A_D = \frac{Q+1}{\lambda T}A_D \quad (5)$$

2) Inventory holding and deterioration costs per cycle

From the characteristics of the inventory pattern, let $I(t)$ represent the inventory level at time t , N_j represent the number of shipments during the j -th replenishment cycle ($j = 1, 2, \dots$), and θ represent the fixed on-hand inventory deterioration rate per unit per unit time. It can be seen that the deterioration of items affects inventory holding costs, and then it can be inferred that

$$N_1(T) \sum_{j=1}^{K-1} N_j(T) \quad (6)$$

It can be seen that $N_j \equiv N(T)$, $j = 1, 2, \dots$. According to Ref. [9], let h represent inventory carrying cost per unit per unit time, and $E[H]$ represent the inventory holding expected average cost per cycle, then

$$E[H] = h \int_0^{KT} I(t) dt = h(1-\theta) \frac{[QT + (\lambda T - 1)T][Q+1]}{2\lambda T} \quad (7)$$

Let d represent the average costs of deterioration per unit per unit time, and $E[D]$ represent the expected average cost of deterioration per replenishment cycle, then

$$E[D] = d\theta \frac{[QT + (\lambda T - 1)T][Q+1]}{2\lambda T} \quad (8)$$

3) Waiting cost per cycle

Let w represent customer waiting cost per unit per unit time, and $E[W_1]$ represent the expected waiting cost of consolidation shipment cycle, then

$$E[W_1] = wE[(T - S_1) + (T - S_2) + \dots + (T - S_{N_j})] \quad (9)$$

Let $E[W_2]$ represent the expected waiting cost per replenishment cycle, then

$$E[W_2] = E[K]E[W_1] = \frac{Q+1}{\lambda T}w \left[\frac{\lambda T^2 - T}{2} \right] \quad (10)$$

From Eqs. (5), (7), (8) and (10), it can be obtained that

$$C(Q, T) = \frac{A_R \lambda}{Q+1} + [h(1-\theta) + d\theta] \cdot$$

$$\left[\frac{Q}{2} + \frac{\lambda T}{2} - \frac{1}{2} \right] + \frac{A_D}{T} + \frac{w\lambda T}{2} - \frac{w}{2} \quad (11)$$

To make $C(Q, T)$ minimal, the optimal (Q^*, T^*) must be satisfied:

$$\left. \begin{aligned} \frac{\partial C(Q, T)}{\partial Q} &= 0 \\ \frac{\partial C(Q, T)}{\partial T} &= 0 \end{aligned} \right\} \quad Q \geq 0, T \geq 0 \quad (12)$$

Obviously, Q must be greater than 0, then

$$\left. \begin{aligned} Q^* &= \max \left\{ \left[\sqrt{\frac{2A_R \lambda}{h(1-\theta) + d\theta}} - 1 \right], 0 \right\} \\ T^* &= \sqrt{\frac{2A_D}{[h(1-\theta) + d\theta + w]\lambda}} \end{aligned} \right\} \quad (13)$$

2 Numerical Study

In this section, an example of the CRP model is presented, and then a figure describing the model while defining one of the variables is given. Finally, a sensitivity study of two parameters related with deterioration is given. Now let the base values of model parameters $\lambda = 10$, $A_R = 125$, $A_D = 20$, $h = 2$, $d = 3$, $\theta = 0.15$, $w = 10$, and then figure out the optima.

From Fig. 4, it clearly shows the trend of $C(Q, T)$ and Q . The value of $C(Q, T)$ reaches the bottom of the curve, namely the optimal solution while the value of Q is growing, then the trend of $C(Q, T)$ rises slowly.

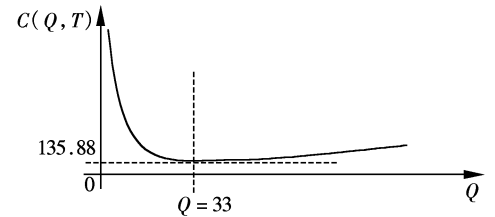


Fig. 4 Curve of C and Q if $T = 0.57$

Though the actual curve is quite different, adopting different parameters, the trend of curve is similar, and it reveals the relationships of objective inventory, shipment timing and total costs. Given by some parameters in this model, it can obtain the optimal objective inventory levels and the optimal shipment intervals.

Obviously, the curve of $C(Q, T)$ vs. T has the similar shape. The value of total costs during the replenishment cycle is decreasing quickly from infinitude with the increase in values of inventory levels and replenishment time, until the total costs reach the minimal limit. Then the value of total costs increases slowly while the values of inventory levels and replenishment time increase continuously.

In Tabs. 1 and 2, it provides additional numerical

examples by varying one of two deterioration-related parameters at a time while keeping others at base values.

Tab.1 Solutions for various θ values

| θ | Q | T | $C(Q, T)$ |
|----------|-----|------|-----------|
| 0.05 | 34 | 0.58 | 133.97 |
| 0.30 | 32 | 0.57 | 138.67 |

Tab.2 Solutions for various d values

| d | Q | T | $C(Q, T)$ |
|-----|-----|------|-----------|
| 1 | 36 | 0.58 | 130.01 |
| 5 | 31 | 0.57 | 141.38 |

From Tabs. 1 and 2, the numerical examples illustrate the sensitivity of the solution relative to the model deterioration-related parameters. For example,

- As θ increases, the resulting Q decreases, and the cost value increases while T decreases slightly;
- As d increases, the resulting Q decreases, and the cost value increases while T decreases slightly.

For other parameters sensitivity studies, one can refer to Refs. [9, 11].

3 Conclusion

Considering the deterioration of fresh food, and giving by certain assumptions of circumstance, a continuous replenishment planning model is presented. It can make the total costs minimal during a cycle by determining the optimal inventory levels and frequencies of replenishment. To obtain the optimal dispatching time and quantities of shipment, the proposed CRP can fulfill consumers' requirements as soon as possible.

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零售企业 VMI 系统集成持续补库计划

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摘要:为了能让供应商监测客户的库存水平,并协助供应商根据供货数量和配送时间进行供货决策,提出了一种面向零售企业生鲜食品的集成持续补库计划策略.该策略充分考虑了生鲜食品的腐坏特性,集成了整合发货和库存补货,以确保零售企业销售生鲜食品的质量安全.构建了用以计算最高库存量和配送周期的供应商管理库存模式下的补货数学模型.在实时交换销售数据的基础上,供应商能够依据模型在发货周期内确定最佳的货物配送时间和配送数量,主动补货,实现总的长期平均成本最小化.

关键词:供应商管理库存;持续补库计划;集成补货;生鲜食品

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