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Exponential stability and existence of periodic solutions for a class of recurrent neural networks with delays

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Abstract: Both the global exponential stability and the existence of periodic solutions for a class of recurrent neural networks with continuously distributed delays (RNNs) are studied. By employing the inequality $a\prod_{k=1}^{m} b_k^{q_k} \leqslant \frac{1}{r}\sum_{k=1}^{m} q_k b_k^r + \frac{1}{r}a^r (a \geqslant 0, b_k \geqslant 0, q_k > 0$, with $\sum_{k=1}^{m} q_k = r-1$ and $r \geqslant 1)$, constructing suitable Lyapunov functions and applying the homeomorphism theory, a family of simple and new sufficient conditions are given ensuring the global exponential stability and the existence of periodic solutions of RNNs. The results extend and improve the results of earlier publications.

Key words: recurrent neural network; global exponential stability; periodic solution; delay; homeomorphism; Lyapunov function

As is well known, the stability of recurrent neural networks (RNNs), including cellular neural networks (CNNs) and Hopfield neural networks (HNNs), plays an important role in their potential applications such as associative content-addressable memories, pattern recognition and optimization. Since significant time delays are ubiquitous both in neural processing and in signal transmission, it is necessary to introduce delays into communication channels which lead to delayed RNNs models. Moreover, because a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desirable to model them by introducing continuously distributed delays over a certain duration of time such that the distant past has less influence compared to the recent behavior of the condition. In recent years, the stability of the resulting RNNs models with continuously distributed delays has been extensively studied and various stability conditions have been obtained for these models of neural networks, see, for example, Refs. [1 – 12], and references therein.

To the best of our knowledge, however, there are few results about the stability properties and the existence of periodic solutions of neural networks with continuously distributed delays, in the literature today. In this paper, we intend to investigate the stability and the existence of periodic solutions for RNNs with continuously distributed delays. By using inequality techniques and constructing suitable Lyapunov functions, the sufficient conditions to guarantee the global exponential stability (GES) and the existence of periodic solutions are given. Our methods are different from what is found in the above mentioned literature.

In this paper, we consider the global exponential stability and periodic solutions of the DCNNs model described by delayed intergro-differential equations

$$\left.\begin{aligned} x_i'(t) &= -c_i(t)h_i(x_i(t)) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)g_j(x_j(s))\,\mathrm{d}s + I_i(t) \qquad t \geqslant 0 \\ x_i(t) &= \phi_i(t) \qquad t \leqslant 0 \end{aligned}\right\} \tag{1}$$

where $i=1,2,\ldots,n$; $a_{ij}(t)$ and $b_{ij}(t)$ are the synaptic connection strengths; $c_i(t)>0$; f_j and g_j represent the neuronal output signal functions; $I_i(t)$ are the exogenous inputs; ϕ_i are assumed to be bounded and continuous functions on $(-\infty, 0]$; k_{ij} are non-negative continuous functions on $[0, +\infty)$ and $\int_0^{+\infty} k_{ij}(s)\,\mathrm{d}s = 1$.

Assume that system (1) has a continuous solution denoted by $x(t,\phi)$ or simply $x(t)$ if no confusion should occur, where $x(t) = \mathrm{col}\{x_i(t)\}$. For $x \in \mathbf{R}^n$, we define the vector norm $\|\cdot\|$ and $\|\cdot\|_\infty$ respectively by $\|x\| =$

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$\left(\sum_{i=1}^{n}|x_i|^2\right)^{\frac{1}{2}}$, $\|x\|_\infty = \max_i\{|x_i|\}$. For any $\phi = \mathrm{col}\{\phi_i\} \in C: \triangleq C((-\infty, 0], \mathbf{R}^n)$, we define a norm in C by $\|\phi\|_\tau = \sup_{\theta\leqslant 0}\|\phi(\theta)\|_\infty$.

The signum function $\mathrm{sgn}(\rho)$ ($\rho\in\mathbf{R}$) is defined as 1 if $\rho>0$; 0 if $\rho=0$; -1 if $\rho<0$.

Definition 1 The equilibrium point $x^* = \mathrm{col}\{x_i^*\}$ of system (1) is said to be the global exponential stability if there are constants $\lambda>0$ and $M\geqslant 1$ such that $\|x(t)-x^*\|_\infty \leqslant M\|\phi - x^*\|_\tau \mathrm{e}^{-\lambda t}$, for $\forall t\geqslant 0$.

Definition 2[13] A map $H: \mathbf{R}^n\to\mathbf{R}^n$ is a homeomorphism of $\mathbf{R}^n$ onto itself if H is continuous and one-to-one and its inverse map H^{-1} is also continuous.

Lemma 1[13] Let $H: \mathbf{R}^n\to\mathbf{R}^n$ be continuous. If H satisfies the following conditions: ① $H(x)$ is injective on $\mathbf{R}^n$; ② $\|H(x)\|\to\infty$ as $\|x\|\to\infty$; then H is a homeomorphism.

Lemma 2[14] For $a\geqslant 0, b_k\geqslant 0$ ($k=1,2,\dots,m$), the following inequality holds

$$a\prod_{k=1}^{m} b_k^{q_k} \leqslant \frac{1}{r}\sum_{k=1}^{m} q_k b_k^r + \frac{1}{r}a^r$$

where $q_k>0$ ($k=1,2,\dots,m$) is some constant, $\sum_{k=1}^{m} q_k = r-1$ and $r\geqslant 1$.

For system (1), we introduce the following assumptions:

(H1) h_i are differentiable, $\gamma_i: = \inf_{x\in\mathbf{R}}\{h_i'(x)\}>0$ and $h_i(0)=0$ ($i=1,2,\dots,n$).

(H2) There are constants $\mu_j>0$ and $\sigma_j>0$ ($j=1,2,\dots,n$) such that

$$|f_j(u)-f_j(v)|\leqslant\mu_j|u-v|, \quad |g_j(u)-g_j(v)|\leqslant\sigma_j|u-v|$$

for any $u, v\in\mathbf{R}$ and $j=1,2,\dots,n$.

(H3) There exist constants $\alpha_{kj}, \beta_{kj}\in\mathbf{R}$, $q_k>0$ and $d_i>0$ ($i,j=1,2,\dots,n$; $k=1,2,\dots,m+1$) such that

$$r\underline{c_i}\gamma_i > \sum_{j=1}^{n}|\bar{a}_{ij}|\sum_{k=1}^{m} q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}} + \sum_{j=1}^{n}|\bar{b}_{ij}|\sum_{k=1}^{m} q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}} + \frac{1}{d_i}\left(\sum_{j=1}^{n}|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}} + \sum_{j=1}^{n}|\bar{b}_{ji}|d_j\sigma_i^{r\alpha_{m+1,i}}\right)$$

where $\sum_{k=1}^{m+1}\alpha_{kj} = 1$; $\sum_{k=1}^{m+1}\beta_{kj} = 1$; $\sum_{k=1}^{m} q_k = r-1$; $r\geqslant 1$; $i,j = 1,2,\dots,n$; $\underline{c_i} = \inf_t\{c_i(t)\}$; $|\bar{a}_{ij}| = \sup_t|a_{ij}(t)|$; $|\bar{b}_{ij}| = \sup_t|b_{ij}(t)|$.

(H4) There exists a constant $\mu>0$ such that $\int_0^{+\infty}\mathrm{e}^{\mu s}k_{ij}(s)\mathrm{d}s < +\infty$.

1 Existence and Uniqueness of the Equilibrium Point

If $x^* = \mathrm{col}\{x_i^*\}$ is an equilibrium point of system (1), from the condition $\int_0^{+\infty}k_{ij}(s)\mathrm{d}s = 1$, then x^* satisfies the following algebraic equation:

$$-c_i(t)h_i(x_i^*) + \sum_{j=1}^{n}a_{ij}(t)f_j(x_j^*) + \sum_{j=1}^{n}b_{ij}(t)g_j(x_j^*) + I_i(t) = 0 \qquad i = 1,2,\dots,n$$

Define the map H as

$$H(x) = \mathrm{col}\{H_i(x_i)\} \tag{2}$$

where $H_i(x_i) = -c_i(t)h_i(x_i) + \sum_{j=1}^{n}a_{ij}(t)f_j(x_j) + \sum_{j=1}^{n}b_{ij}(t)g_j(x_j) + I_i(t)$.

Theorem 1 Assume that (H1) to (H3) hold, then the map H defined by (2) is an injective.

Proof We prove that if $x\neq x'$ then $H(x)\neq H(x')$ holds for any $x, x'\in\mathbf{R}^n$. The component $H_i(x_i)-H_i(x_i')$ of the vector $H(x)-H(x')$ is as

$$H_i(x_i) - H_i(x_i') = -c_i(t)(h_i(x_i)-h_i(x'_i)) + \sum_{j=1}^{n}a_{ij}(t)(f_j(x_j)-f_j(x'_j)) + \sum_{j=1}^{n}b_{ij}(t)(g_j(x_j)-g_j(x_j'))$$

By (H1), (H2) and lemma 2, we obtain

$$\sum_{i=1}^{n}\mathrm{sgn}(x_i-x_i')rd_i[H_i(x_i)-H_i(x_i')]|x_i-x_i'|^{r-1} = \sum_{i=1}^{n}\mathrm{sgn}(x_i-x_i')rd_i|x_i-x_i'|^{r-1}\cdot$$

$$\left[-c_i(t)(h_i(x_i)-h_i(x_i')) + \sum_{j=1}^{n}a_{ij}(t)(f_j(x_j)-f_j(x_j')) + \sum_{j=1}^{n}b_{ij}(t)(g_j(x_j)-g_j(x_j'))\right] \leqslant$$

$$\sum_{i=1}^{n} rd_i\left\{-\underline{c}_i\gamma_i|x_i-x_i'|^r+\sum_{j=1}^{n}|\bar{a}_{ij}|\mu_j|x_j-x_j'||x_i-x_i'|^{r-1}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j|x_j-x_j'||x_i-x_i'|^{r-1}\right\}=$$
$$\sum_{i=1}^{n} rd_i\left\{-\underline{c}_i\gamma_i|x_i-x_i'|^r+\sum_{j=1}^{n}|\bar{a}_{ij}||\mu_j^{\alpha_{m+1,j}}(x_j-x_j')|\prod_{k=1}^{m}|\mu_j^{\frac{\alpha_{kj}}{q_k}}(x_i-x_i')|^{q_k}+\right.$$
$$\left.\sum_{j=1}^{n}|\bar{b}_{ij}||\sigma_j^{\beta_{m+1,j}}(x_j-x_j')|\prod_{k=1}^{m}|\sigma_j^{\frac{\beta_{kj}}{q_k}}(x_i-x_i')|^{q_k}\right\}\leqslant$$
$$\sum_{i=1}^{n} rd_i\left\{-\underline{c}_i\gamma_i|x_i-x_i'|^r+\sum_{j=1}^{n}|\bar{a}_{ij}|\frac{1}{r}\left[\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}|x_i-x_i'|^r+\mu_j^{r\alpha_{m+1,j}}|x_j-x_j'|^r\right]+\right.$$
$$\left.\sum_{j=1}^{n}|\bar{b}_{ij}|\frac{1}{r}\left[\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}|x_i-x_i'|^r+\sigma_j^{r\beta_{m+1,j}}|x_j-x_j'|^r\right]\right\}=$$
$$\sum_{i=1}^{n}\left\{-rd_i\underline{c}_i\gamma_i|x_i-x_i'|^r+d_i\left[\sum_{j=1}^{n}|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}\right]|x_i-x_i'|^r+\right.$$
$$\left.d_i\left[\sum_{j=1}^{n}|\bar{a}_{ij}|\mu_j^{r\alpha_{m+1,j}}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\right]|x_j-x_j'|^r\right\}=$$
$$\sum_{i=1}^{n}\left\{-rd_i\underline{c}_i\gamma_i|x_i-x_i'|^r+d_i\left[\sum_{j=1}^{n}|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}\right]|x_i-x_i'|^r+\right.$$
$$\left.\left[\sum_{j=1}^{n}|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}}+\sum_{j=1}^{n}|\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\right]|x_i-x_i'|^r\right\}=-\sum_{i=1}^{n}d_i\left\{r\underline{c}_i\gamma_i-\left[\sum_{j=1}^{n}|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+\right.\right.$$
$$\left.\left.\sum_{j=1}^{n}|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}\right]-\frac{1}{d_i}\left[\sum_{j=1}^{n}|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}}+\sum_{j=1}^{n}|\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\right]\right\}|x_i-x_i'|^r$$

From (H3) we derive

$$\sum_{i=1}^{n}\operatorname{sgn}(x_i-x_i')rd_i\left[H_i(x_i)-H_i(x_i')\right]|x_i-x_i'|^{r-1}<0$$

which implies that there exists at least one index i such that $H_i(x_i)-H_i(x_i')\neq 0$. It directly follows that $H(x)\neq H(x')$. Hence, the map H is an injective.

Theorem 2 Assume that (H1) to (H3) hold, then the map H defined by (2) is a homeomorphism on $\mathbf{R}^n$.

Proof By lemma 1 and theorem 1, we only need to prove that $\|H(x)\|\to\infty$ as $\|x\|\to\infty$. Let $H^*(x)=\mathrm{col}\{H_i^*(x_i)\}$, where

$$H_i^*(x_i)=-c_i(t)(h_i(x_i)-h_i(0))+\sum_{j=1}^{n}a_{ij}(t)(f_j(x_j)-f_j(0))+\sum_{j=1}^{n}b_{ij}(t)(g_j(x_j)-g_j(0))$$

To prove that $\|H(x)\|\to\infty$ as $\|x\|\to\infty$, it suffices to show that $\|H^*(x)\|\to\infty$ as $\|x\|\to\infty$. Similar to the proof of theorem 1, we have

$$\sum_{i=1}^{n}rd_i\operatorname{sgn}(x_i)H_i^*(x_i)|x_i|^{r-1}\leqslant r\sum_{i=1}^{n}d_i\left\{-\underline{c}_i\gamma_i|x_i|^r+\sum_{j=1}^{n}|\bar{a}_{ij}|\mu_j|x_j||x_i|^{r-1}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j|x_j||x_i|^{r-1}\right.=$$
$$r\sum_{i=1}^{n}d_i\left\{-\underline{c}_i\gamma_i|x_i|^r+\sum_{j=1}^{n}|\bar{a}_{ij}||\mu_j^{\alpha_{m+1,j}}x_j|\prod_{k=1}^{m}|\mu_j^{\frac{\alpha_{kj}}{q_k}}x_i|^{q_k}+\sum_{j=1}^{n}|\bar{b}_{ij}||\sigma_j^{\beta_{m+1,j}}x_j|\prod_{k=1}^{m}|\sigma_j^{\frac{\beta_{kj}}{q_k}}x_i|^{q_k}\right\}\leqslant$$
$$r\sum_{i=1}^{n}d_i\left\{-\underline{c}_i\gamma_i|x_i|^r+\sum_{j=1}^{n}|\bar{a}_{ij}|\frac{1}{r}\left[\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}|x_i|^r+\mu_j^{r\alpha_{m+1,j}}|x_j|^r\right]+\right.$$
$$\left.\sum_{j=1}^{n}|\bar{b}_{ij}|\frac{1}{r}\left[\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}|x_i|^r+\sigma_j^{r\beta_{m+1,j}}|x_j|^r\right]\right\}=-\sum_{i=1}^{n}d_i\left\{r\underline{c}_i\gamma_i-\left[\sum_{j=1}^{n}|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+\right.\right.$$
$$\left.\left.\sum_{j=1}^{n}|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}\right]-\frac{1}{d_i}\left[\sum_{j=1}^{n}|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}}+\sum_{j=1}^{n}|\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\right]\right\}|x_i|^r\leqslant-\gamma\sum_{i=1}^{n}d_i|x_i|^r$$

where

$$\gamma=\min_i\left\{r\underline{c}_i\gamma_i-\left[\sum_{j=1}^{n}|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{\beta_{kj}}{q_k}}\right]-\frac{1}{d_i}\left[\sum_{j=1}^{n}|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}}+\sum_{j=1}^{n}|\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\right]\right\}>0$$

Thus, we obtain

$$\gamma d_i|x_i|^r\leqslant\gamma\sum_{i=1}^{n}d_i|x_i|^r\leqslant\left|\sum_{i=1}^{n}rd_i\operatorname{sgn}(x_i)H_i^*(x_i)|x_i|^{r-1}\right|\leqslant\sum_{i=1}^{n}rd_i|H_i^*(x_i)||x_i|^{r-1}\leqslant\|H^*(x)\|_\infty\|x\|_\infty^{\,r-1}\sum_{i=1}^{n}rd_i$$

That is $\gamma \underline{d}\|x\|_\infty \leqslant \left(\sum_{i=1}^{n} rd_i\right)\|H^*(x)\|_\infty$, where $\underline{d}=\min_i\{d_i\}$.

Hence, it follows that $\|H^*(x)\|_\infty \geqslant \gamma\, \underline{d}\Big/\left(\sum_{i=1}^{n} rd_i\right)\|x\|_\infty$, which directly implies that $\|H^*(x)\|\to\infty$ as $\|x\|\to\infty$ in view of the equivalence of the norms $\|\cdot\|_\infty$ and $\|\cdot\|$, thus H is a homeomorphism on $\mathbf{R}^n$.

Theorem 3 If the conditions (H1) to (H3) hold, then the system (1) has a unique equilibrium point x^*.

Proof Theorem 2 ensures that H is a homeomorphism. Hence there is a unique equilibrium point $x=x^*$ such that $H(x^*)=0$.

2 Global Exponential Stability of the Equilibrium Point

Let $x^*=\mathrm{col}\{x_i^*\}$ be the equilibrium point of system (1). Making a transformation for system (1): $y_i(t)=x_i(t)-x_i^*\ (i=1,2,\ldots,n)$, we have

$$\left.\begin{aligned} y_i'(t) &= -c_i(t)K_i(y_i(t)) + \sum_{j=1}^{n} a_{ij}(t)F_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}(t)\int_{-\infty}^{t} k_{ij}(t-s)G_j(y_j(s))\,\mathrm{d}s \qquad t\geqslant 0\\ y_i(t) &= \Phi_i(t) \qquad t\leqslant 0\end{aligned}\right\} \tag{3}$$

where $K_i(y_i)=h_i(x_i)-h_i(x_i^*), F_j(y_j)=f_j(x_j)-f_j(x_j^*), G_j(y_j)=g_j(x_j)-g_j(x_j^*), \Phi_i(t)=\phi_i(t)-x_i^*\,(i,j=1,2,\ldots,n)$.

Clearly, the equilibrium point x^* of system (1) is global exponential stable if and only if the equilibrium point 0 of system (3) is global exponential stable. Thus in the following, we only consider the GES of the equilibrium point 0 for system (3).

Theorem 4 If the conditions (H1) to (H4) hold, then system (3) has a unique equilibrium point 0 which is GES in the sense that every solution $y(t)$ of system(3) satisfies

$$\|y(t)\|_\infty \leqslant M\mathrm{e}^{-\bar{\mu}t}\|\Phi\|_\tau \tag{4}$$

where $M=\left\{\frac{1}{\underline{d}}\left[\sum_{i=1}^{n} d_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{d_i|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}}{r\bar{\mu}}\left(\int_0^{+\infty}\mathrm{e}^{r\bar{\mu}s}k_{ij}(s)\,\mathrm{d}s-1\right)\right]\right\}^{\frac{1}{r}}\geqslant 1$, $y(t)=\mathrm{col}\{y_i(t)\}$ and $\Phi=\mathrm{col}\{\Phi_i\}$.

Proof The existence and uniqueness of an equilibrium point 0 is guaranteed by theorem 3, so we only need to prove that inequality (4) holds. From the conditions $\int_0^{+\infty} k_{ij}(s)\,\mathrm{d}s=1$ and $\int_0^{+\infty}\mathrm{e}^{\mu s}k_{ij}(s)\,\mathrm{d}s<+\infty$, we can find a critical value $\mu^*>0$, such that $\int_0^{+\infty}\mathrm{e}^{\mu s}k_{ij}(s)\,\mathrm{d}s<+\infty$ can be held for arbitrary $\mu\in(0,\mu^*)$. Define the function $G(\mu)$ as

$$\begin{aligned} G(\mu) = {} & r(\underline{c}_i\gamma_i-\mu)-\sum_{j=1}^{n}\left[|\bar{a}_{ij}|\sum_{k=1}^{m} q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}} + |\bar{b}_{ij}|\sum_{k=1}^{m} q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}\right] - \\ & \frac{1}{d_i}\sum_{j=1}^{n}\left[|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}} + |\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\mu s}\mathrm{d}s\right]\end{aligned}$$

From (H3), $G(0)>0$ and $G(\mu)\to-\infty$ as $\mu\to\mu_-^*$. Thus there exists a $\bar{\mu}\in(0,\mu^*)$ such that $G(\bar{\mu})\geqslant 0$, i.e.,

$$r(\underline{c}_i\gamma_i-\bar{\mu})-\sum_{j=1}^{n}\left[|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}\right]-\frac{1}{d_i}\sum_{j=1}^{n}\left[|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}}+|\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s\right]\geqslant 0$$

Now we consider the following Lyapunov function:

$$V(t)=\sum_{i=1}^{n} d_i\left[|y_i(t)|^r\mathrm{e}^{r\bar{\mu}t}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\int_0^{+\infty}k_{ij}(s)\left(\int_{t-s}^{t}|y_j(\tau)|^r\mathrm{e}^{r\bar{\mu}(s+\tau)}\mathrm{d}\tau\right)\mathrm{d}s\right] \tag{5}$$

Calculating the upper right derivation of V along system (3) and using lemma 2, we obtain

$$\begin{aligned}\frac{D^+V(t)}{\mathrm{d}t} \leqslant {} & \sum_{i=1}^{n} d_i\left\{r|y_i(t)|^{r-1}D^+|y_i(t)|\mathrm{e}^{r\bar{\mu}t}+|y_i(t)|^r r\bar{\mu}\mathrm{e}^{r\bar{\mu}t}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\cdot\right.\\ & \left.\left[|y_j(t)|^r\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}(t+s)}\mathrm{d}s-\int_0^{+\infty}k_{ij}(s)|y_j(t-s)|^r\mathrm{e}^{r\bar{\mu}t}\mathrm{d}s\right]\right\}\leqslant\sum_{i=1}^{n} rd_i\left\{\mathrm{e}^{r\bar{\mu}t}\left[-\underline{c}_i\gamma_i|y_i(t)|^r+\right.\right.\\ & \sum_{j=1}^{n}|\bar{a}_{ij}|\mu_j|y_i(t)|^{r-1}|y_j(t)|+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j|y_i(t)|^{r-1}\int_{-\infty}^{t}k_{ij}(t-s)|y_j(s)|\mathrm{d}s\left.\right]+|y_i(t)|^r\bar{\mu}\mathrm{e}^{r\bar{\mu}t}+\end{aligned}$$

$$\frac{1}{r}\Big[\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}|y_j(t)|^r\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}(t+s)}\mathrm{d}s-\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\int_0^{+\infty}k_{ij}(s)|y_j(t-s)|^r\mathrm{e}^{r\bar{\mu}t}\mathrm{d}s\Big]\Big\} =$$

$$\sum_{i=1}^{n}rd_i\Big\{\mathrm{e}^{r\bar{\mu}t}\Big[-(\underline{c}_i\gamma_i-\bar{\mu})|y_i(t)|^r+\sum_{j=1}^{n}|\bar{a}_{ij}||\mu_j^{\alpha_{m+1,j}}y_j(t)|\prod_{k=1}^{m}|\mu_j^{\frac{\alpha_{kj}}{q_k}}y_i(t)|^{q_k}+$$

$$\sum_{j=1}^{n}|\bar{b}_{ij}|\left|\sigma_j^{\beta_{m+1,j}}\int_{-\infty}^{t}k_{ij}(t-s)|y_j(s)|\mathrm{d}s\right|\prod_{k=1}^{m}\left|\sigma_j^{\frac{\beta_{kj}}{q_k}}y_i(t)\right|^{q_k}\Big]+$$

$$\frac{1}{r}\Big[\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}|y_j(t)|^r\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}(t+s)}\mathrm{d}s-\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\int_0^{+\infty}k_{ij}(s)|y_j(t-s)|^r\mathrm{e}^{r\bar{\mu}t}\mathrm{d}s\Big]\Big\}\leqslant$$

$$\sum_{i=1}^{n}rd_i\mathrm{e}^{r\bar{\mu}t}\Big\{-(\underline{c}_i\gamma_i-\bar{\mu})|y_i(t)|^r+\frac{1}{r}\sum_{j=1}^{n}|\bar{a}_{ij}|\Big[\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}|y_i(t)|^r+\mu_j^{r\alpha_{m+1,j}}|y_j(t)|^r\Big]+$$

$$\frac{1}{r}\sum_{j=1}^{n}|\bar{b}_{ij}|\Big[\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}|y_i(t)|^r+\sigma_j^{r\beta_{m+1,j}}\Big(\int_{-\infty}^{t}|y_j(s)|k_{ij}(t-s)\mathrm{d}s\Big)^r\Big]+\frac{1}{r}\Big[\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}|y_j(t)|^r\cdot$$

$$\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s-\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\int_0^{+\infty}k_{ij}(s)|y_j(t-s)|^r\mathrm{d}s\Big]\Big\}\leqslant-\mathrm{e}^{r\bar{\mu}t}\sum_{i=1}^{n}d_i\Big\{r(\underline{c}_i\gamma_i-\bar{\mu})-$$

$$\sum_{j=1}^{n}\Big[|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}\Big]-\frac{1}{d_i}\sum_{j=1}^{n}\Big[|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}}+|\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s\Big]\Big\}|y_i(t)|^r\leqslant 0$$

So $V(t)\leqslant V(0)$. From Eq. (5), we derive

$$V(0)=\sum_{i=1}^{n}d_i\Big[|\Phi_i(0)|^r+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\int_0^{+\infty}k_{ij}(s)\Big(\int_{-s}^{0}|\Phi_j(\tau)|^r\mathrm{e}^{r\bar{\mu}(\tau+s)}\mathrm{d}\tau\Big)\mathrm{d}s\Big]\leqslant$$

$$\sum_{i=1}^{n}\Big[d_i+d_i\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\int_0^{+\infty}k_{ij}(s)\Big(\int_{-s}^{0}\mathrm{e}^{r\bar{\mu}(\tau+s)}\mathrm{d}\tau\Big)\mathrm{d}s\Big]\|\Phi\|_\tau^r=$$

$$\sum_{i=1}^{n}\Big[d_i+d_i\sum_{j=1}^{n}\frac{|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}}{r\bar{\mu}}\Big(\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s-1\Big)\Big]\|\Phi\|_\tau^r$$

and $V(t)\geqslant\sum_{i=1}^{n}d_i|y_i(t)|^r\mathrm{e}^{r\bar{\mu}t}\geqslant d_i|y_i(t)|^r\mathrm{e}^{r\bar{\mu}t}$. Hence

$$d_i|y_i(t)|^r\mathrm{e}^{r\bar{\mu}t}\leqslant\sum_{i=1}^{n}\Big[d_i+d_i\sum_{j=1}^{n}\frac{|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}}{r\bar{\mu}}\Big(\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s-1\Big)\Big]\|\Phi\|_\tau^{\ r}$$

which leads to $\|y(t)\|_\infty\leqslant M\mathrm{e}^{-\bar{\mu}t}\|\Phi\|_\tau$, $t\geqslant 0$, where

$$M=\Big\{\frac{1}{\underline{d}}\Big[\sum_{i=1}^{n}d_i+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{d_i|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}}{r\bar{\mu}}\Big(\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s-1\Big)\Big]\Big\}^{\frac{1}{r}}\geqslant 1$$

This implies that the equilibrium point 0 of system (3) is global exponential stable, i. e., the equilibrium point x^* of system (1) is global exponential stable. We complete the proof.

Theorem 5 Suppose that (H1), (H2) and (H4) hold. If there exist constants $\alpha_{kj},\beta_{kj}\in\mathbf{R}$, $q_k>0$ and $d_i>0$ $(i,j=1,2,\ldots,n;k=1,2,\ldots,m+1)$ such that

$$r\underline{c}_i\gamma_i>\sum_{j=1}^{n}\sum_{k=1}^{m}q_k|\bar{a}_{ij}|^{\frac{r\alpha_{kj}}{q_k}}\mu_j+\sum_{j=1}^{n}\sum_{k=1}^{m}q_k|\bar{b}_{ij}|^{\frac{r\beta_{kj}}{q_k}}\sigma_j+\frac{1}{d_i}\Big[\sum_{j=1}^{n}|\bar{a}_{ji}|^{r\alpha_{m+1,i}}d_j\mu_i+\sum_{j=1}^{n}|\bar{b}_{ji}|^{r\beta_{m+1,i}}d_j\sigma_i\Big]\tag{6}$$

where $\sum_{k=1}^{m+1}\alpha_{kj}=1$; $\sum_{k=1}^{m+1}\beta_{kj}=1$; $\sum_{k=1}^{m}q_k=r-1$; $r\geqslant 1$; $i,j=1,2,\ldots,n$; $\underline{c}_i=\inf_t\{c_i(t)\}$; $|\bar{a}_{ij}|=\sup_t|a_{ij}(t)|$; $|\bar{b}_{ij}|=\sup_t|\bar{b}_{ij}(t)|$. Then system (1) has a unique global exponential stable equilibrium point.

Proof Similar to the proof of theorem 3, system (1) has a unique equilibrium point. In the following, we prove the equilibrium point is global exponential stable. Similar to the proof of theorem 4, from (6) and (H4), there exists a constant $\bar{\mu}>0$, such that

$$r(\underline{c}_i\gamma_i-\bar{\mu})-\Big[\sum_{j=1}^{n}\sum_{k=1}^{m}q_k|\bar{a}_{ij}|^{\frac{r\alpha_{kj}}{q_k}}\mu_j+\sum_{j=1}^{n}\sum_{k=1}^{m}q_k|\bar{b}_{ij}|^{\frac{r\beta_{kj}}{q_k}}\sigma_j\Big]-\frac{1}{d_i}\sum_{j=1}^{n}\Big[|\bar{a}_{ji}|^{r\alpha_{m+1,i}}d_j\mu_i+|\bar{b}_{ji}|^{r\beta_{m+1,i}}d_j\sigma_i\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s\Big]\geqslant 0$$

Choose the Lyapunov function as

$$V(t)=\sum_{i=1}^{n}d_i\Big[|y_i(t)|^r\mathrm{e}^{r\bar{\mu}t}+\sum_{j=1}^{n}|\bar{b}_{ij}|^{r\beta_{m+1,j}}\sigma_j\int_0^{+\infty}k_{ij}(s)(\int_{t-s}^{t}|y_j(\tau)|^r\mathrm{e}^{r\bar{\mu}(s+\tau)}\mathrm{d}\tau)\mathrm{d}s\Big]\tag{7}$$

The remaining proof is similar to that of theorem 4, so we omit it here.

Corollary 1 Suppose that (H1), (H2) and (H4) hold. Furthermore, if one of the following conditions holds,

(A1) $\sum_{j=1}^{n}(r-1)|\bar{a}_{ij}|+\sum_{j=1}^{n}(r-1)|\bar{b}_{ij}|+\frac{1}{d_i}\left[\sum_{j=1}^{n}|\bar{a}_{ji}|d_j\mu_i^{\,r}+\sum_{j=1}^{n}|\bar{b}_{ji}|d_j\sigma_i^{\,r}\right] < r\underline{c}_i\gamma_i$;

(A2) $\sum_{j=1}^{n}(r-1)\mu_j+\sum_{j=1}^{n}(r-1)\sigma_j+\frac{1}{d_i}\left[\sum_{j=1}^{n}|\bar{a}_{ji}|^r d_j\mu_i+\sum_{j=1}^{n}|\bar{b}_{ji}|^r d_j\sigma_i\right] < r\underline{c}_i\gamma_i$;

(A3) $\frac{1}{d_i}\left[\sum_{j=1}^{n}|\bar{a}_{ji}|d_j\mu_i+\sum_{j=1}^{n}|\bar{b}_{ji}|d_j\sigma_i\right] < \underline{c}_i\gamma_i$;

(A4) $\sum_{j=1}^{n}(r-1)|\bar{a}_{ij}|^{\frac{r}{r-1}}\mu_j+\sum_{j=1}^{n}(r-1)|\bar{b}_{ij}|^{\frac{r}{r-1}}\sigma_j+\frac{1}{d_i}\left[\sum_{j=1}^{n}d_j\mu_i+\sum_{j=1}^{n}d_j\sigma_i\right] < r\underline{c}_i\gamma_i$;

(A5) $\sum_{j=1}^{n}|\bar{a}_{ij}|^2\mu_j+\sum_{j=1}^{n}|\bar{b}_{ij}|^2\sigma_j+\frac{1}{d_i}\left[\sum_{j=1}^{n}d_j\mu_i+\sum_{j=1}^{n}d_j\sigma_i\right] < 2\underline{c}_i\gamma_i$.

Then system (1) has a unique equilibrium point that is GES, independent of the delays.

Proof Let $\alpha_{kj}=\beta_{kj}=0$ $(k=1,2,\ldots,m; j=1,2,\ldots,n)$, and $\alpha_{m+1,j}=\beta_{m+1,j}=1$ in (H3), respectively, then (H3) turns to (A1).

Let $m=1$, $\alpha_{kj}=\beta_{kj}=0$ $(k=1,2,\ldots,m)$ and $\alpha_{m+1,j}=\beta_{m+1,j}=1$ $(j=1,2,\ldots,n)$ in (6), respectively, then (6) turns to (A2).

Let $r=1$, $m=1$, $\alpha_{kj}=\beta_{kj}=0$ $(k=1,2,\ldots,m)$ and $\alpha_{m+1,j}=\beta_{m+1,j}=1$ $(j=1,2,\ldots,n)$ in (6), respectively, then (6) turns to (A3).

Let $m=1$, $\alpha_{kj}=\beta_{kj}=1$ $(k=1,2,\ldots,m)$ and $\alpha_{m+1,j}=\beta_{m+1,j}=0$ $(j=1,2,\ldots,n)$ in (6), respectively, then (6) turns to (A4).

Let $r=2$, $m=1$, $\alpha_{kj}=\beta_{kj}=1$ $(k=1,2,\ldots,m)$ and $\alpha_{m+1,j}=\beta_{m+1,j}=0$ $(j=1,2,\ldots,n)$ in (6), respectively, then (6) turns to (A5). Thus, by theorems 3, 4 and 5, system (1) has a unique equilibrium point that is GES, independent of the delays.

Remark When $r=1$, the condition (A1) is equivalent to $C\gamma-(A^+\mu+B^+\sigma)$ being a nonsingular M-matrix, where $\boldsymbol{C}=\mathrm{diag}(\inf_t c_i(t))$, $\boldsymbol{A}^+=(\sup_t|\bar{a}_{ij}(t)|)_{n\times n}$, $\boldsymbol{B}^+=(\sup_t|\bar{b}_{ij}(t)|)_{n\times n}$, $\boldsymbol{\gamma}=\mathrm{diag}\{\gamma_i\}$, $\boldsymbol{\mu}=\mathrm{diag}\{\mu_j\}$, $\boldsymbol{\sigma}=\mathrm{diag}\{\sigma_j\}$.

3 Periodic Solutions of Recurrent Neural Networks

In this section, we consider the periodic solution of system (1), in which $I_i(t):\mathbf{R}^+\to\mathbf{R}$, $i=1,2,\ldots,n$ are continuously periodic functions with period ω, i.e., $I_i(t+\omega)=I_i(t)$.

Theorem 6 If the conditions (H1) to (H4) hold, then there exists exactly one ω-periodic solution of system (1) and all other solutions of (1) converge exponentially to it as $t\to+\infty$.

Proof For $\forall\phi,\psi\in C$, we denote the solutions of (1) as $x(t,\phi)=\mathrm{col}\{x_1(t,\phi),\ldots,x_n(t,\phi)\}$, $x(t,\psi)=\mathrm{col}\{x_1(t,\psi),\ldots,x_n(t,\psi)\}$, respectively.

Define $x_t(\phi)=x(t+\theta,\phi)$, $\theta\in(-\infty,0]$, $t\geqslant 0$, then $x_t(\phi)\in C$ for $\forall t\geqslant 0$. Thus we follow from system (1) that

$$(x_i(t,\phi)-x_i(t,\psi))' = -c_i(t)[h_i(x_i(t,\phi))-h_i(x_i(t,\psi))]+\sum_{j=1}^{n}a_{ij}(t)[(f_j(x_j(t,\phi))-f_j(x_j(t,\psi))]+\sum_{j=1}^{n}b_{ij}(t)\int_{-\infty}^{t}k_{ij}(t-s)[g_j(x_j(s,\phi))-g_j(x_j(s,\psi))]\mathrm{d}s$$

for $t\geqslant 0$, $i=1,2,\ldots,n$. From (H3) and (H4), there exists a small constant $\bar{\mu}>0$, such that

$$r(\underline{c}_i\gamma_i-\bar{\mu})-\sum_{j=1}^{n}\left[|\bar{a}_{ij}|\sum_{k=1}^{m}q_k\mu_j^{\frac{r\alpha_{kj}}{q_k}}+|\bar{b}_{ij}|\sum_{k=1}^{m}q_k\sigma_j^{\frac{r\beta_{kj}}{q_k}}-\frac{1}{d_i}\sum_{j=1}^{n}\left[|\bar{a}_{ji}|d_j\mu_i^{r\alpha_{m+1,i}}+|\bar{b}_{ji}|d_j\sigma_i^{r\beta_{m+1,i}}\int_0^{+\infty}k_{ij}(s)\mathrm{e}^{r\bar{\mu}s}\mathrm{d}s\right]\geqslant 0$$

We consider the following Lyapunov function

$$V(t)=\sum_{i=1}^{n}d_i\left[|x_i(t,\phi)-x_i(t,\psi)|^r\mathrm{e}^{r\bar{\mu}t}+\sum_{j=1}^{n}|\bar{b}_{ij}|\sigma_j^{r\beta_{m+1,j}}\int_0^{+\infty}k_{ij}(s)\left(\int_{t-s}^{t}|x_j(\tau,\phi)-x_j(\tau,\psi)|^r\mathrm{e}^{r\bar{\mu}(\tau+s)}\mathrm{d}\tau\right)\mathrm{d}s\right]$$

By a minor modification of the proof of theorem 4, we can easily get

$$\|x(t,\phi)-x(t,\psi)\|_\infty \leqslant M\mathrm{e}^{-\bar{\mu}t}\|\phi-\psi\|_\tau \qquad t\geqslant 0$$

where $M\geqslant 1$ is a constant. One can easily follow the formula above that

$$\|x_t(\phi)-x_t(\psi)\|_\tau \leqslant M\mathrm{e}^{-\bar{\mu}(t+\theta)}\|\phi-\psi\|_\tau \qquad \theta\in(-\infty,0] \tag{8}$$

because $\bar{\mu}$ satisfies the condition $\int_0^{+\infty}\mathrm{e}^{\mu s}k_{ij}(s)\,\mathrm{d}s<+\infty$, we can choose a positive integer m such that

$$M\mathrm{e}^{-\bar{\mu}(m\omega+\theta)}\leqslant\frac{1}{2} \qquad \theta\in(-\infty,0] \tag{9}$$

Now we define a *Poincáre* mapping T: $C\to C$ by $T\phi=x_\omega(\phi)$, then we can derive from (8) and (9) that

$$\|T^m\phi-T^m\psi\|_\tau\leqslant\frac{1}{2}\|\phi-\psi\|_\tau$$

This implies that T^m is a contraction mapping, hence there exists a unique fixed point $\phi^*\in C$ such that $T^m\phi^*=\phi^*$. Note that

$$T^m(T\phi^*)=T(T^m\phi^*)=T\phi^*$$

This shows that $T\phi^*\in C$ is also a fixed point of T^m, so $T\phi^*=\phi^*$, i. e., $x_\omega(\phi^*)=\phi^*$.

Let $x(t,\phi^*)$ be the solution of (1), obviously, $x(t+\omega,\phi^*)$ is also a solution of (1) and $x_{t+\omega}(\phi^*)=x_t(x_\omega(\phi^*))=x_t(\phi^*)$ for $t\geqslant 0$; therefore, $x(t+\omega,\phi^*)=x(t,\phi^*)$ for $t\geqslant 0$.

This shows that $x(t,\phi^*)$ is exactly one ω-periodic solution of (1), and it is easy to see that all other solutions of (1) converge exponentially to it as $t\to+\infty$.

Theorem 7 Suppose that all conditions of theorem 5 hold. Then there exists one ω-periodic solution of system (1) and all other solutions of (1) converge exponentially to it as $t\to+\infty$.

Proof By a minor modification of the proof of theorem 6, we can also prove theorem 7, which is omitted. This completes the proof.

Applying theorems 6 and 7, we can prove the following corollary.

Corollary 2 Suppose that (H1), (H2) and (H4) hold, furthermore, if one of the following conditions holds,

(B1) $\sum_{j=1}^n(r-1)|\bar{a}_{ij}|+\sum_{j=1}^n(r-1)|\bar{b}_{ij}|+\frac{1}{d_i}\left[\sum_{j=1}^n|\bar{a}_{ji}|d_j\mu_i^r+\sum_{j=1}^n|\bar{b}_{ji}|d_j\sigma_i^r\right]<r\underline{c}_i\gamma_i$;

(B2) $\sum_{j=1}^n(r-1)\mu_j+\sum_{j=1}^n(r-1)\sigma_j+\frac{1}{d_i}\left[\sum_{j=1}^n|\bar{a}_{ji}|^r d_j\mu_i+\sum_{j=1}^n|\bar{b}_{ji}|^r d_j\sigma_i\right]<r\underline{c}_i\gamma_i$;

(B3) $\frac{1}{d_i}\left[\sum_{j=1}^n|\bar{a}_{ji}|d_j\mu_i+\sum_{j=1}^n|\bar{b}_{ji}|d_j\sigma_i\right]<\underline{c}_i\gamma_i$;

(B4) $\sum_{j=1}^n(r-1)|\bar{a}_{ij}|^{\frac{r}{r-1}}\mu_j+\sum_{j=1}^n(r-1)|\bar{b}_{ij}|^{\frac{r}{r-1}}\sigma_j+\frac{1}{d_i}\left[\sum_{j=1}^n d_j\mu_i+\sum_{j=1}^n d_j\sigma_i\right]<r\underline{c}_i\gamma_i$;

(B5) $\sum_{j=1}^n|\bar{a}_{ij}|^2\mu_j+\sum_{j=1}^n|\bar{b}_{ij}|^2\sigma_j+\frac{1}{d_i}\left[\sum_{j=1}^n d_j\mu_i+\sum_{j=1}^n d_j\sigma_i\right]<2\underline{c}_i\gamma_i$.

Then system (1) exists one ω-periodic solution and all other solutions of (1) converge exponentially to it as $t\to+\infty$.

4 Conclusion

Some sufficient conditions are given ensuring the global exponential stability and the periodic solutions of RNNs by using a new analysis technique and constructing suitable Lyapunov functions. The conditions possess highly important significance in some application fields, for instance, they can be applied to design globally exponentially stable RNNs and periodic oscillatory RNNs and easily checked in practice by simple algebraic methods.

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时滞反馈神经网络模型的周期解的存在性和全局稳定性

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摘要:研究了一类具有连续分布延时的反馈神经网络模型的周期解的存在性和全局稳定性. 通过利用不等式 $a\prod_{k=1}^{m} b_k^{q_k} \leqslant \frac{1}{r}\sum_{k=1}^{m} q_k b_k^r + \frac{1}{r}a^r$ ($a \geqslant 0, b_k \geqslant 0, q_k > 0$, 且 $\sum_{k=1}^{m} q_k = r - 1$, $r \geqslant 1$, 构造适当的 Lyapunov 函数,以及运用同构定理,得到了一系列简单有用的条件,推广并完善了已有结论.

关键词:反馈神经网络;全局稳定性;周期解;时滞;同构;Lyapunov 函数
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