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Description logics for fuzzy ontologies on semantic web

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Abstract: To enable representation and reasoning for fuzzy ontologies with expressive fuzzy knowledge on the semantic web, a new fuzzy extension of description logics called the fuzzy description logics with comparison expressions (FCDLs) is presented. The syntax and semantics of FCDLs are formally defined, and the forms of axioms and assertions in FCDLs knowledge bases are specified. FCDLs combine both fuzzy concepts from the fuzzy description logics (FDLs) and cut concepts from the extended fuzzy description logics (EFDLs) in the same theory. Furthermore, cut concepts are extended into comparison cut concepts in FCDLs to represent comparison expressions between fuzzy membership degrees, which are often used in practice but not supported by the other fuzzy extensions of description logics. FCDLs have more expressive power than FDLs and EFDLs, and are able to represent expressive fuzzy knowledge and to perform reasoning tasks based on them. Therefore, FCDLs can enable representation and reasoning for fuzzy ontologies with expressive fuzzy knowledge on the semantic web.

Key words: semantic web; ontology; description logics; fuzzy

Ontology is the basis of sharing and reusing knowledge on the semantic web. Description logics (DLs)[1] are a family of knowledge representation languages, with a simple well-established declarative semantics to capture the meaning of popular features in ontologies. Nowadays, many knowledge representation systems have been built by description logics in a variety of applications. The popular ontology languages DAML + OIL and OWL are both based on description logics to enable reasoning.

The fuzzy knowledge plays an important role in many domains that face a huge amount of imprecise and vague knowledge and information, such as text mining, multimedia information system, medical informatics, machine learning, and human natural language processing[2]. When facing such fuzzy knowledge, it may be uncertain whether an individual belongs to a concept or not, as well as whether two individuals have a role or not. However, classic description logics interpret concepts as crisp sets of individuals and interpret roles as crisp sets of individual pairs. They are insufficient to represent such uncertain information. For example, in classic description logics, we can only say a person is either tall or short, but cannot say how tall the person is.

The fuzzy description logics[3] interpret concepts or roles as fuzzy sets of individuals or individual pairs. Such concepts and roles are called fuzzy concepts and fuzzy roles. Then we can generate fuzzy ontologies, which contain fuzzy concepts and fuzzy roles. The fuzzy ontologies are capable of dealing with fuzzy knowledge[4], and are efficient in text and multimedia object representation and retrieval[5]. By using the fuzzy logic, we can say one is tall to a degree of 0.7, another is tall to a degree of 0.9.

Yen provided a structural subsumption algorithm for a fuzzy extension of a sub-language of ALC[6]. Tresp et al.[7] presented a fuzzy extension of ALC, ALC_{FM} with a reasoning method for computing the degree of subsumption between concepts. Straccia presented fuzzy ALC and an algorithm for reasoning[3]; he also transformed fuzzy ALC into classical ALC[8]. Hölldobler et al.[9] introduced the membership manipulator constructor to present ALC_{FH}. Sanchez et al.[10] generalized the quantification in fuzzy ALCQ. Stoilos et al. provided pure ABoxes reasoning algorithms for the fuzzy extensions of SHIN[11]. Straccia presented fuzzy SHOIN(D) for a fuzzy OWL[12]. Li et al.[13]

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presented a family of extended fuzzy description logics.

This paper introduces three different kinds of fuzzy extensions of description logics: fuzzy description logics, extended fuzzy description logics, and fuzzy description logics with comparison cuts. It shows that fuzzy description logics with comparison cuts are the most expressive. They enable representation and reasoning for expressive fuzzy knowledge on the semantic web.

1 Fuzzy Description Logics

The fuzzy description logics (FDLs) do not change the syntax of concept descriptions and axioms in classic description logics, but use the fuzzy semantics to interpret concepts and roles, and have a special form of assertions. FALC[3] is a fuzzy extension of description logic ALC by adopting fuzzy interpretation to redefine the semantics and extend the assertion forms. Compared with concepts and roles in classical description logics, which describe crisp sets of individuals and their relationships, fuzzy description logics contain fuzzy concepts and fuzzy roles that describe fuzzy sets. Let B be an atomic concept, R be an atomic role, and concepts C and D of FALC are inductively defined with the application of ALC concept constructors:

$$C, D ::= \top \mid \bot \mid B \mid \neg C \mid C \cup D \mid C \cap D \mid \forall R.C \mid \exists R.C$$

The fuzzy interpretation for FALC is a pair $I = \langle \Delta^I, \cdot^I \rangle$, where Δ^I is a nonempty domain, and $\cdot^I$ is an interpretation function which maps every individual a into an element $a^I \in \Delta^I$, maps every atomic concept B into a function $B^I: \Delta^I \to [0, 1]$, and maps every atomic role R into a function $R^I: \Delta^I \times \Delta^I \to [0, 1]$. Furthermore, for any element $d \in \Delta^I$, $\cdot^I$ satisfies the following equations:

$$\left.\begin{array}{l} \top^I(d) = 1;\ \ \bot^I(d) = 0;\ \ (\neg C)^I(d) = 1 - C^I(d) \\ (C \cap D)^I(d) = \min\{C^I(d), D^I(d)\} \\ (C \cup D)^I(d) = \max\{C^I(d), D^I(d)\} \\ (\exists R.C)^I(d) = \sup\limits_{d' \in \Delta^I}\{\min\{R^I(d,d'), C^I(d')\}\} \\ (\forall R.C)^I(d) = \inf\limits_{d' \in \Delta^I}\{\max\{1 - R^I(d,d'), C^I(d')\}\} \end{array}\right\} \quad (1)$$

A knowledge base of FALC contains fuzzy axioms and fuzzy assertions. A fuzzy axiom is in the form of $C \subseteq D$. $C \subseteq D$ means D subsumes C. An interpretation I satisfies $C \subseteq D$ iff $\forall d \in \Delta^I$, $C^I(d) \leqslant D^I(d)$. A fuzzy assertion is in the form of $\alpha \geqslant n$ or $\alpha \leqslant n$, where α is an expression: a: C or (a, b): R. I satisfies a fuzzy assertion $a: C \geqslant (\leqslant) n$ (respectively $(a, b): R \geqslant (\leqslant) n$) iff $C^I(a^I) \geqslant (\leqslant) n$ (respectively $R^I(a^I, b^I) \geqslant (\leqslant) n$).

The fuzzy extension method used in FALC can be applied to many other description logics, such as ALCQ[10], SHIN[11] and SHOIN(D)[12]. They form a family of fuzzy description logics. There are already reasoning algorithms for many FDLs[3, 10-11]. However, FDLs only support limited expressive power of fuzzy knowledge. For example, FDLs cannot describe an individual a such that $\mathrm{Tall}^I(a^I) \geqslant 0.7$ or $\mathrm{Strong}^I(a^I) \geqslant 0.9$, or an axiom $\mathrm{Cub}^I(d) \geqslant 0.6 \to \mathrm{Young}^I(d) \geqslant 0.9$. The reason is that FDLs can only describe one given degree in a fuzzy assertion and no degree can be specified in axioms.

2 Extended Fuzzy Description Logics

To extend the expressive power of FDLs, we propose a new fuzzy extension of description logics, called extended fuzzy description logics (EFDLs). The main idea is from the cut sets of fuzzy sets. In fuzzy set theory, a fuzzy set S with respect to a universe U is defined as a function $\mu_S: U \to [0, 1]$, and the n-cut set of S is defined as $S_{[n]} = \{d \in U \mid \mu_S(d) \geqslant n\}$, where $0 \leqslant n \leqslant 1$.

Based on the idea that the cut sets are indeed crisp sets, but facilitate a normative theory for formalizing fuzzy set theory, our fuzzy extension of description logics uses cut concepts and cut roles instead of fuzzy concepts and fuzzy roles. Consider a set N_C of fuzzy concept names and a set N_R of fuzzy role names. For any $A \in N_C$, $R \in N_R$ and $0 \leqslant n \leqslant 1$, we call their cuts $A_{[n]}$ an atomic cut concept and $R_{[n]}$ an atomic cut role, where A or R is the prefix of n, and n is the suffix of A or R. Let N_C^E and N_R^E be the sets of atomic cut concepts and atomic cut role, respectively. For any cut concept $A_{[n]}$ and cut roles $R_{[n]}$ such that $0 \leqslant n \leqslant 1$, the interpretation function $\cdot^I$ maps $A_{[n]}$ and $R_{[n]}$ into sets over Δ^I and $\Delta^I \times \Delta^I$:

$$\left.\begin{array}{l} (A_{[n]})^I = \{d \mid d \in \Delta^I \wedge A^I(d) \geqslant n\} \\ (R_{[n]})^I = \{(d, d') \mid d, d' \in \Delta^I \wedge R^I(d, d') \geqslant n\} \end{array}\right\} \quad (2)$$

From Eq. (2), for any n_1, n_2 such that $0 \leqslant n_2 \leqslant n_1 \leqslant 1$, it must be true that $(A_{[n_1]})^I \subseteq (A_{[n_2]})^I$ and $(R_{[n_1]})^I \subseteq (R_{[n_2]})^I$ for any interpretation I. A^I and R^I define fuzzy sets over Δ^I and $\Delta^I \times \Delta^I$, while their cuts $(A_{[n]})^I$ and $(R_{[n]})^I$ are actually crisp sets. Generally, a collection of $(A_{[n_1]})^I$, $(A_{[n_2]})^I$, …, $(A_{[n_k]})^I$ and $(R_{[n_{k+1}]})^I$, $(R_{[n_{k+2}]})^I$, …, $(R_{[n_h]})^I$ is able to describe the semantic of A^I and R^I completely or to an acceptable degree. It facilitates a classical description logic theory for simulating the fuzzy description logic theory.

For any classical DL, if its atomic concepts and atomic roles are cut concepts and cut roles respectively, then it is called extended fuzzy description logic (EFDL), and its concepts and roles are called cut concepts and cut roles. It has been proved that any FDL ABox can be transformed to an EFDL ABox. The reasoning algorithms for EFDL ALCN and ALCH are proposed in Refs. [13 – 14].

In EFDLs, the limitations in fuzzy description logics can be naturally overcome. For example, $\mathrm{Tall}^I(a^I) \geqslant 0.7$ or $\mathrm{Strong}^I(a^I) \geqslant 0.9$ can be represented by a: $\mathrm{Tall}_{[0.7]} \cup \mathrm{Strong}_{[0.9]}$; $\forall d \in \Delta^I$, $\mathrm{Cub}^I(d) \geqslant 0.6 \rightarrow \mathrm{Young}^I(d) \geqslant 0.9$ can be written down as $\mathrm{Cub}_{[0.6]} \subseteq \mathrm{Young}_{[0.9]}$.

3 Fuzzy Description Logics with Comparison Expressions

It is a familiar description that "Tom is taller than Mike." It can be regarded as a comparison between fuzzy membership degrees, and means Tom is tall to a degree greater than the degree to which Mike is tall. For example, in Fig. 1, we know Tom:Tall > 0.8, Mike:Tall < 0.9 and Tom:Tall > Mike:Tall. The last assertion is a comparison between fuzzy membership degrees. The comparison between the fuzzy membership degrees is very useful in practice. For example, facing goods of the same quality, we are always interested in the cheapest one. There are more complicated comparison expressions such as "Tom is quite tall and taller than Mike," and "No close friend of Tom is taller and stronger than he." However, FDLs and EFDLs do not support the expression of comparisons between fuzzy membership degrees.

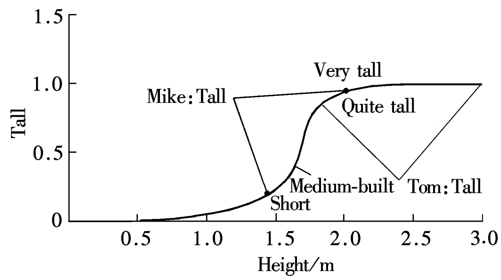

Fig. 1 The fuzzy concept "Tall"

We extend cut concepts to comparison cut concepts (cuts for short) to express the comparison expressions. First, individuals can be divided into different classes based on simple comparisons. Such classes are called comparison cut concepts. For example,

- [Tall > 0.9] means very tall; Tom: [Tall > 0.9] means Tom is very tall;
- [Absolutist < Liberalist] means anyone who prefers liberalism to absolutism. Tom: [Absolutist < Liberalist] means Tom prefers liberalism to absolutism;
- [Tall >] means taller than a given thing; Tom: [Tall >](Mike) means Tom is taller than Mike.

Secondly, complex comparison cut concepts can be built from cuts inductively with constructors. Here we only consider the Boolean constructors, AND ($\cap$), OR ($\cup$) and NOT ($\neg$). For example,

- [Tall > 0.9] $\cup$ [Strong > 0.8] means very tall or quite strong;
- [Tall > 0.9] $\cap$ [Tall >] means very tall and taller than a given thing;
- $\neg$ [Tall > 0.9] means not very tall, i. e. [Tall $\leqslant$ 0.9].

Finally, new fuzzy concepts can be defined from the cuts with restrictions on the fuzzy roles. For example,

- $\exists$hasFriend.Tall is a fuzzy concept of person who has tall friends;
- $\exists$hasFriend.[Tall > 0.9] is a fuzzy concept of person who has very tall friends;
- $\exists$hasFriend.[Tall >] is a fuzzy concept of person who has a friend taller than him.

This extension can also be applied to other kind of restrictions on fuzzy roles, e. g., $\forall R.C$. Furthermore, the more expressive fuzzy concepts can also have their cuts.

The formal syntax and semantics of cuts and new fuzzy concepts are as follows.

The comparison cut concepts are defined as

- If C, D are fuzzy concepts, $n \in [0, 1]$ and $* \in \{=, \neq, >, \geqslant, <, \leqslant\}$, then $[C * n]$, $[C * D]$ and $[C * D^{\uparrow}]$ are cuts;
- If P, Q are cuts, then $\neg P$, $P \cap Q$ and $P \cup Q$ are cuts.

The interpretation function $\cdot^I$ maps, additionally, every cut P into a function $P^I: \Delta^I \rightarrow 2^{\Delta^I}$:

$$\left.\begin{aligned}
&[C * n]^I(s) = \{t \mid C^I(t) * n\} \\
&[C * D]^I(s) = \{t \mid C^I(t) * D^I(t)\} \\
&[C * D^{\uparrow}]^I(s) = \{t \mid C^I(t) * D^I(s)\} \\
&(\neg P)^I(s) = \Delta^I \backslash P^I(s) \\
&(P \cap Q)^I(s) = P^I(s) \cap Q^I(s) \\
&(P \cup Q)^I(s) = P^I(s) \cup Q^I(s)
\end{aligned}\right\} \qquad (3)$$

Obviously, if a cut P contains no up-arrow, then for any s, t, $P^I(s) = P^I(t)$. We simply use P^I to denote them. In addition, there are new fuzzy concepts: if R is a fuzzy role, P is a cut, then $\exists R.P$ and $\forall R.P$ are fuzzy concepts, and they satisfy

$$\left.\begin{aligned} (\exists R.P)^I(s) &= \sup_{t \in P^I(s)} (s,t) \\ (\forall R.P)^I(s) &= \inf_{t \in (\neg P)^I(s)} (s,t)) \end{aligned}\right\} \quad (4)$$

There are also new forms of assertions and axioms: a: P, a: $P'(b)$ and $P \subseteq Q$, where P, Q are cuts without an up-arrow, and P' is a cut with up-arrows. An interpretation I satisfies a: P iff a^I: $P^{\#}$, I satisfies a: $P'(b)$ iff a^I: $P^{\#}(b^I)$, I satisfies $P \subseteq Q$ iff $P^I \subseteq Q^I$.

The above new language elements are called comparison expressions. A family of fuzzy description logics with comparison expressions (FCDLs) can be defined by importing the comparison expressions into FDLs. FCDLs are extensions of FDLs, so they are more expressive. FCDLs are also more expressive than EFDLs. For example, a: $\text{Tall}_{[0.7]} \cup \text{Strong}_{[0.9]}$ can be rewritten as a: $[\text{Tall} \geqslant 0.7] \cup [\text{Strong} \geqslant 0.9]$. $\text{Cub}_{[0.6]} \subseteq \text{Young}_{[0.9]}$ is the same as $[\text{Cub} \geqslant 0.6] \subseteq [\text{Young} \geqslant 0.9]$.

FCDLs enable representation of very expressive fuzzy knowledge. For example, we can write down axioms like $[\text{Small} < 0.3] \subseteq [\text{Big} > 0.8]$ to show the relationship between the fuzzy concepts small and big. Mike: $([\text{Tall} > 0.9] \cup [\text{Tall} >)(\text{Tom})$ means "Mike is either very tall or taller than Tom." Mike: $\forall$hasFriend. $[\text{Tall} <] \geqslant 1$ means "no friend of Mike is taller than him." However, the reasoning for FCDLs is also more complicated. Recently, we have completed the reasoning algorithm for FCDL ALC_{FC}, a fuzzy extension of DL ALC with comparison expressions[15].

4 Conclusion

This paper introduces three different kinds of fuzzy description logics. The FDLs define fuzzy concepts and fuzzy roles to enable representation and reasoning for fuzzy knowledge. The EFDLs use cut concepts and cut roles to represent fuzzy concepts and fuzzy roles. The FCDLs extend cut concepts to comparison cut concepts, and combine fuzzy concepts and comparison cut concepts together to enable representation and reasoning for very expressive fuzzy knowledge on the semantic web. More representation features and efficient reasoning algorithms for FCDLs are the future work.

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支持语义 web 模糊本体的描述逻辑

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摘要:为实现语义 web 上包含复杂模糊知识的模糊本体的表示和推理,提出了一种描述逻辑的模糊扩展——支持比较表达式的模糊描述逻辑(FCDLs). 给出 FCDLs 语法和语义的形式化定义,并规定 FCDLs 知识库中的公理和断言形式. FCDLs 将模糊描述逻辑(FDLs)中的模糊概念和扩展模糊描述逻辑(EFDLs)中的截概念结合在同一理论中,并将截概念扩展为比较截概念,从而支持对实际中经常用到的模糊隶属度之间比较表达式的描述,而其他的描述逻辑模糊扩展均不支持比较表达式. FCDLs 具有比 FDLs 和 EFDLs 更强的表达能力,能够表示复杂的模糊知识并基于它们完成推理任务. 因此 FCDLs 可实现语义 web 上包含复杂模糊知识的模糊本体的表示和推理.

关键词:语义 web;本体;描述逻辑;模糊

中图分类号:TP181