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Design of super-orthogonal space-time trellis codes based on trace criterion

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Abstract: A design of super-orthogonal space-time trellis codes (SOSTTCs) based on the trace criterion (TC) is proposed for improving the design of SOSTTCs. The shortcomings of the rank and determinant criteria based design and the advantages of the TC-based design are analyzed. The optimization principle of four factors is presented, which includes the space-time block coding (STBC) scheme, set partitioning, trellis structure, and the assignment of signal subsets and STBC schemes in the trellis. According to this principle, systematical and handcrafted design steps are given in detail. By constellation expansion, the code performance can be further improved. The code design results are given, and the new codes outperform others in the simulation.

Key words: super-orthogonal space-time trellis codes; trace criterion; handcrafted and systematical design

Space-time codes can be divided into two types: space-time trellis codes[1] (STTCs) and space-time block codes[2–4] (STBCs). By combining the advantages of STTCs and STBCs, super-orthogonal space-time trellis codes (SOSTTCs) were proposed in Ref. [5].

However, the rank and determinant criteria[1] (RDC) based design in Ref. [5] has some shortcomings. So a new design of SOSTTC based on the trace criterion (TC) is proposed in this paper.

1 Shortcomings of RDC Based Design

The performance of SOSTTC is determined by fives factors: design criterion; set partitioning, similar to TCM; trellis structure; STBC scheme; assignment of signal subsets and STBC schemes in the trellis. Among these, the design criterion is the most important.

In Ref. [5] the RDC was used to guide the design of SOSTTC. However this design method has two shortcomings: First, the rule proposed in Ref. [5] is only a sufficient but not necessary condition to guarantee the full rank of matrix $\boldsymbol{A}$ ($\boldsymbol{A}$ is defined in Ref. [1]). Some codes in Ref. [5] do not obey that rule. Secondly, the calculation of $|\boldsymbol{A}|$ in Ref. [5] is too complicated for handcrafted design.

2 Advantages of the TC Based Design

The TC[6–7] is more suitable than the RDC when the antennas number is large. The main idea of the TC is to evaluate the code performance by the trace of $\boldsymbol{A}$, which is just equal to the squared Euclidean distance between two codewords (c, e). Briefly, the Euclidean distance is only called distance in the following text.

The advantages of the design of SOSTTC based on the TC include:

1) The trace of $\boldsymbol{A}$, shortened as $\mathrm{tr}(\boldsymbol{A})$, is easier to calculate than the determinant of $\boldsymbol{A}$.

2) Let (c, e) be the corresponding codewords of the pairwise paths (P_c, P_e) with length L. It is easy to prove that

$$\mathrm{tr}(\boldsymbol{A}(c, e)) = \sum_{i=1}^{L} \mathrm{tr}(\boldsymbol{A}(c_i, e_i)) \tag{1}$$

where c_i and e_i are the codewords corresponding to the i-th branch of P_c and P_e, respectively. Eq. (1) shows that the squared distance between any pairwise paths can be gained by summing up the squared distances between all the pairwise branches of the paths. Therefore, if another pair (c', e') is the same as (c, e) in part, the same parts only need to be calculated once. Another advantage from Eq. (1) is that the optimization for the whole code can be realized branch by branch. This makes the design more flexible.

3) Owing to the two advantages mentioned above, the amount of handcrafted calculation of squared distance spectrum is acceptable.

4) The codes from the TC-based design are still good when there is only one receive antenna. This is due to the using of STBC in SOSTTC.

5) The codes based on the TC perform well in both slow and fast fading channels, because the TC is suitable for two types of fading.

3 Factors Optimization in TC-Based Design

3.1 STBC scheme

In order to get more STBC schemes, different from the Alamouti scheme[4], additional phase rotations are added to the signals transmitted from the first transmit antenna in Ref. [5]. Then if the signal phases

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corresponding to both the transmit antennae are rotated, more STBC schemes will be gained. It can be written as

$$\begin{bmatrix} x_1 e^{j\theta_1} & -x_2^* e^{j\theta_1} \\ x_2 e^{j\theta_2} & x_1^* e^{j\theta_2} \end{bmatrix} \tag{2}$$

3.2 Set partitioning

It is easy to prove $(\mathrm{tr}(\boldsymbol{A}))^2 = 4|\boldsymbol{A}|$, so the partitioning results are the same for both the TC and the RDC.

3.3 Trellis structure

The trellises in Ref. [5] are from TCM. These trellises can also be used in our design.

3.4 Assignment of signal subsets and STBC schemes in trellis

The idea is to assign different signal subsets and STBC schemes branch by branch, and to make the distances between all the paths maximized. This will be explained in detail in the following.

4 Design Example

How to design SOSTTC based on the TC in a systematical and handcrafted way is explained here through an example, assuming BPSK modulation, 4-state half-connection trellis and two transmit antennas. The band efficiency is 1 bit/(s·Hz).

The transmitted signals corresponding to each branch in SOSTTC are determined by the combination $(\boldsymbol{I}, \theta_1, \theta_2)$ when (2) is used for STBC coding. $\boldsymbol{I}$ is a signal in set S, which consists of 00, 01, 10 and 11 for BPSK modulation. Based on Euclidean distance, S can be partitioned into S_0 and S_1, and each can be further partitioned into two smaller subsets S_{00}, S_{01} and S_{10}, S_{11} respectively. The partition results are the same as those based on the RDC, and the detail can be found in Ref. [5]. Both θ_1 and θ_2 can pick 0 or π. The squared distance d^2 between any $(\boldsymbol{I}, \theta_1, \theta_2)$ and $(\hat{\boldsymbol{I}}, \hat{\theta}_1, \hat{\theta}_2)$ is summarized easily in theorem 1. The proof is omitted here.

Theorem 1

① If $\theta_1 = \hat{\theta}_1$ and $\theta_2 = \hat{\theta}_2$, then

$$d^2 = \begin{cases} 0 & \text{when } \boldsymbol{I} = \hat{\boldsymbol{I}} \\ 16 & \text{when } \boldsymbol{I}, \hat{\boldsymbol{I}} \text{ both in } S_0 \text{ or } S_1 \text{, and } \boldsymbol{I} \neq \hat{\boldsymbol{I}} \\ 8 & \text{when } \boldsymbol{I}, \hat{\boldsymbol{I}} \text{ one in } S_0 \text{, the other in } S_1 \end{cases}$$

② If $\theta_1 = \hat{\theta}_1, \theta_2 \neq \hat{\theta}_2$ or $\theta_1 \neq \hat{\theta}_1, \theta_2 = \hat{\theta}_2$, then

$$d^2 \equiv 8$$

③ If $\theta_1 \neq \hat{\theta}_1$ and $\theta_2 \neq \hat{\theta}_2$, then

$$d^2 = \begin{cases} 0 & \text{when } \boldsymbol{I}, \hat{\boldsymbol{I}} \text{ both in } S_0 \text{ or } S_1 \text{, and } \boldsymbol{I} \neq \hat{\boldsymbol{I}} \\ 16 & \text{when } \boldsymbol{I} = \hat{\boldsymbol{I}} \\ 8 & \text{when } \boldsymbol{I}, \hat{\boldsymbol{I}} \text{ one in } S_0 \text{, the other in } S_1 \end{cases}$$

The 4-state half-connection trellis is shown in Fig. 1. The transitions between states are denoted as P_1 to P_8. Since the band efficiency is 1 bit/(s·Hz), there must be two parallel branches for each P_i. However, only one branch of each P_i is drawn for brevity. Similar to TCM, the signals of parallel branches should have the largest distance. From theorem 1 this can be realized by picking two elements of S_0 (or S_1) for $\boldsymbol{I}$ and $\hat{\boldsymbol{I}}$, respectively, and making $\theta_1 = \hat{\theta}_1$, $\theta_2 = \hat{\theta}_2$. Since there are two and only two elements in S_0 (or S_1), the signals of parallel branches corresponding to any P_i can be written as $(S_x, \theta_1, \theta_2)$, where $S_x = S_0$ or S_1. $(S_x, \theta_1, \theta_2)$ for each P_i should be different at best, in order to decrease the number of the branches, between which the distance is zero.

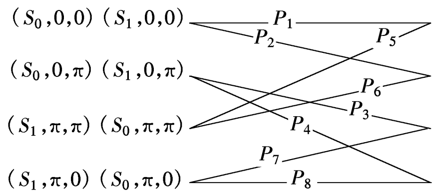


Fig. 1 Code 1: a 4-state BPSK modulated code with half-connecting trellis of 1 bit/(s·Hz)

There are eight pairs of parallel branches in Fig. 1. $(S_x, \theta_1, \theta_2)$ can also choose eight different values ($S_x = S_0$ or S_1, $\theta_1 = 0$ or π, $\theta_2 = 0$ or π). Just one $(S_x, \theta_1, \theta_2)$ value is assigned to one pair of parallel branches. However, how the different $(S_x, \theta_1, \theta_2)$ are assigned is a key problem. At the beginning, $(S_0, 0, 0)$ is to be assigned. As regards the first, $(S_0, 0, 0)$ can be assigned to any pair of parallel branches, for example P_1. Then the next is (S_0, π, π). The distance between $(S_{00}, 0, 0)$ and (S_{01}, π, π) is zero. The distance between $(S_{01}, 0, 0)$ and (S_{00}, π, π) is also zero. Therefore, $(S_0, 0, 0)$, which consists of $(S_{00}, 0, 0)$ and $(S_{01}, 0, 0)$, cannot appear with (S_0, π, π), which consists of (S_{01}, π, π) and (S_{00}, π, π), in pairwise paths at same time slot. Otherwise the distance between two paths will be decreased. If this cannot be avoided, the length of the pairwise paths should be as long as possible. When P_6 (or P_8) appears with P_1 in pairwise paths at the same time slot, the minimal length of such paths is 5, which is already the largest one of all P_i. Thus according to the rule mentioned above, when $(S_0, 0, 0)$ is assigned to P_1, (S_0, π, π) should be assigned to P_6 or P_8, and this time P_6 is selected.

Next $(S_1, 0, 0)$ is to be assigned. Any squared distance between $(S_1, 0, 0)$ and $(S_0, 0, 0)$, or between $(S_1, 0, 0)$ and (S_0, π, π) is 8. Consequently $(S_1, 0, 0)$ can be assigned discretionarily. This time, $(S_1, 0, 0)$ is assigned to P_2. Now for (S_1, π, π), its relation to $(S_1, 0, 0)$ is just like the relation between $(S_0, 0, 0)$ and (S_0, π, π). Through the similar analysis mentioned above, (S_1, π, π) can be assigned to P_5 and P_7, and this time P_5 is picked. In the same way, $(S_0,$

0, π), (S_0, π, 0), (S_1, 0, π) and (S_1, π, 0) can be assigned one by one. The final result, Code 1, is shown in Fig. 1.

It happens that, in the view of squared distance spectrum, Code 1 is equivalent to the code proposed in Fig. 5 of Ref. [5], though they are different codes designed by different criteria. The reason is that the number of states and the constellation size of these codes are small. This phenomenon also exists in the STTC design. However the way Code 1 is designed here is more systematic and easier to use than that in Ref. [5].

5 Performance Improvement by Constellation Expansion

Constellation expansion is a key point of TCM. It can also be used in SOSTTC to improve performance. Some examples are given as follows.

5.1 Realizing 1 bit/(s·Hz) with QPSK

5.1.1 4-state half-connection trellis

In Fig. 1, θ only can be 0 or π, so there are not enough different (S_x, θ_1, θ_2). This leads to the existence of branches with zero distance, which causes performance degradation. If QPSK modulation is used, such degradation can be avoided.

In Fig. 1, the signals corresponding to the branches started from state 1 are (S_0, 0, 0) and (S_1, 0, 0), which only appear on the real axis in the constellation map. For (S_0, $\pi/2$, $\pi/2$) and (S_1, $\pi/2$, $\pi/2$), these signals only appear on the imaginary axis. Thus the squared distance between any signal of (S_i, 0, 0) (i = 0 or 1) and any signal of (S_j, $\pi/2$, $\pi/2$) (j = 0 or 1) is always 8. When (S_1, π, π) and (S_0, π, π) corresponding to the branches started from state 3 are replaced by (S_0, $\pi/2$, $\pi/2$) and (S_1, $\pi/2$, $\pi/2$), the distance between branches started from state 1 and state 3, respectively will never be zero. In the same way, (S_1, π, 0) and (S_0, π, 0) corresponding to branches started from state 4 can be replaced by (S_0, $\pi/2$, $3\pi/2$) and (S_1, $\pi/2$, $3\pi/2$). This code is denoted as Code 2, shown in Fig. 2. It is easy to draw the conclusion that the squared distance between any non parallel branches is 8 in Code 2. The distance, which is 0 in Code 1, is enlarged to 8 in Code 2, while other distances have not been changed. Therefore in the view of squared distance spectrum, Code 2 should be better than Code 1.

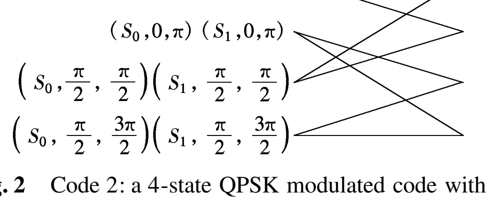


Fig. 2 Code 2: a 4-state QPSK modulated code with half-connection trellis of 1 bit/(s·Hz)

5.1.2 4-state full-connection trellis

The trellis is shown in Fig. 3(a), without parallel branches. Due to the symmetry of the trellis, the signal subsets and STBC schemes should also be assigned symmetrically. One of such assignments is shown in Fig. 3(a). Each of the four branches diverging from the same state is assigned with a different signal subset from S_{00}, S_{01}, S_{10} and S_{11}, but with the same rotation phases (STBC scheme). All the states differ in rotation phases. At the beginning of design, the phases are unknown. They can be determined as follows. In Fig. 3(b), one type of the shortest pairwise paths is shown, which can represent other pairwise paths for symmetry. The calculations of distances can be divided into left and right parts. Due to the same (θ_1, θ_2) for all the left branches, the squared distances between each other have nothing to do with (θ_1, θ_2). The spectrum is: 16 for 2 times, and 8 for 4 times. For QPSK modulation, the squared distance between the right branches may be 4, 8, 12 or 16, but it is impossible to make all the squared distances larger than or equal to 12. By matching the right branches with squared distance 8 and 12 to the left branches with squared distance 16 and 8 complementally, the minimal squared distance between any of the paths can be improved to $\min(8+16, 12+8) = 20$, larger than 16 in Code 1 and Code 2. This new code is denoted as Code 3 and shown in Fig. 3(a).

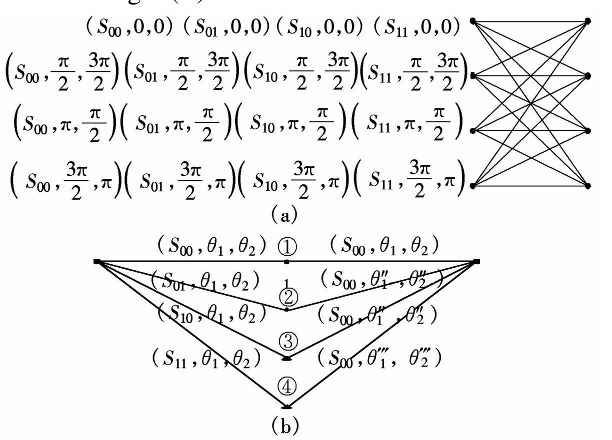


Fig. 3 A 4-state QPSK modulated code with full-connection trellis of 1 bit/(s·Hz). (a) Code 3; (b) Pairwise paths

Constellation expansion was also proposed in Ref. [8]. Different from our design, the design in Ref. [8] is based on the RDC, and the signals transmitted at the same time slot have the same rotation phases. In the view of the RDC and in terms of the whole frame, this is not equivalent to adding the same rotations to the signals transmitted from the same antenna, which is proposed in our design. In addition, the codes in Ref. [8] were gained by computer search. However, it happened that a code proposed in Ref. [8] has the same squared distance spectrum as Code 3, though the two codes are not completely the same. This chance will never exist when the number of states or the constellation size is large. For example, it is impossible to do a computer search for good 8PSK modulated 16-state codes. The advantages of our design will stand out.

5.2 Realizing 2 bit/(s·Hz) with 8PSK

When using 8PSK instead of QPSK to realize 2 bit/(s·Hz), performance can be improved. In order to get rid of the suppression on performance from parallel branches, only a full-connection trellis is considered here. The design idea and procedure are just like those of Code 3, the detail is omitted here. The result is shown in Fig. 4, and denoted as Code 4. The minimal squared distance of Code 4 is 10.34. While for Code 5 shown in Fig. 5, which is gained by the idea in Ref. [5] with a half-connection trellis and QPSK modulation, its minimal squared distance is only 8. Therefore Code 4 should outperform Code 5.

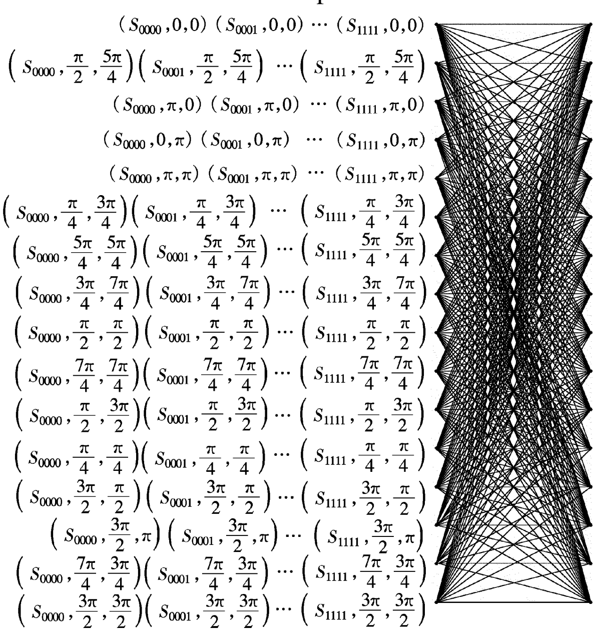


Fig. 4 Code 4: 8PSK modulation with full-connection trellis

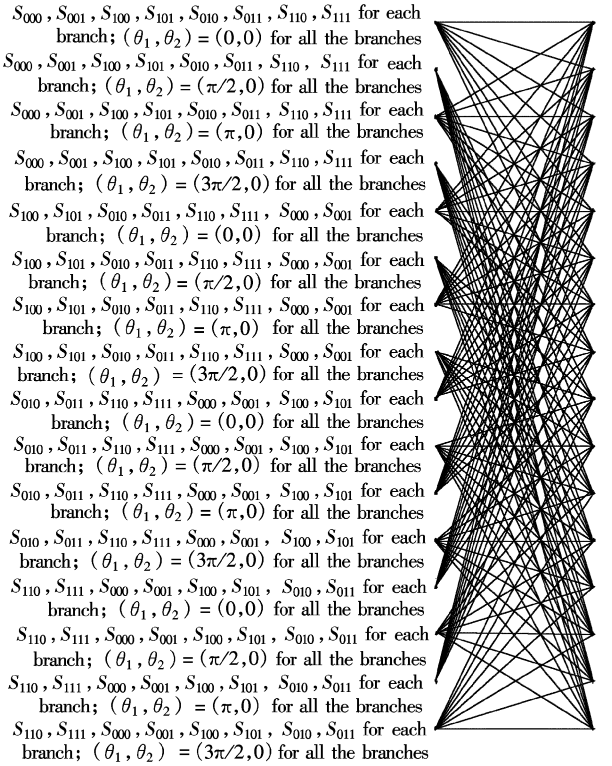


Fig. 5 Code 5: QPSK modulation with half-connection trellis

6 Simulation

The simulation results under Rayleigh fading with ideal channel state information are shown in Fig. 6 and Fig. 7, where R denotes the number of receive antennae. In these simulations, each frame consists of 130 time slots (65 STBC blocks), and all the antennae are independent. For slow fading, the channel is constant during one frame and varies independently from one frame to another. For fast fading, the channel is constant during one STBC block and varies independently from one STBC block to another. In Fig. 6 and Fig. 7, the gap between different criteria based codes in fast fading channels is much bigger than that in slow fading channels. The reason is that the TC is suitable for both types of fading, while the RDC is only suitable for slow fading.

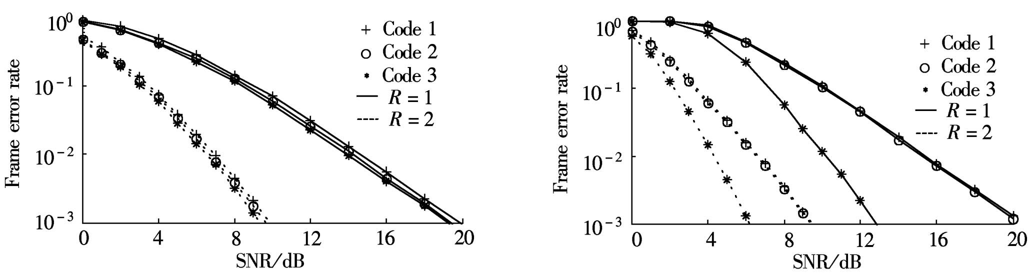
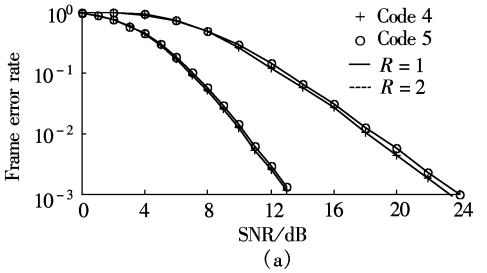


Fig. 6 Comparison of three 4-state codes of 1 bit/(s·Hz). (a) Slow fading; (b) Fast fading



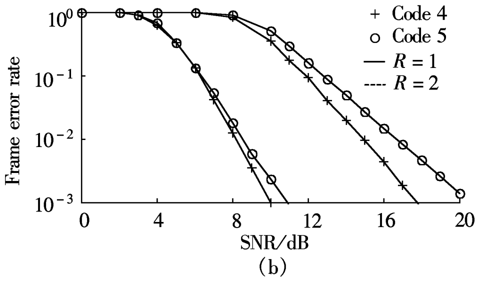


Fig. 7 Comparison of two 16-state codes of 2 bit/(s·Hz). (a) Slow fading; (b) Fast fading

7 Conclusion

The design of SOSTTC based on the trace criterion is proposed. This design has many advantages. The main one is that it can design the codes in a handcrafted and systematic way easily. The design procedure is illustrated, and the design results are also given. By constellation expansion, new codes can be found, which are better than other codes as known.

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基于迹准则的超正交空时格码的设计

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摘要:为了改进超正交空时格码的设计,提出了基于迹准则的设计方法.分析了传统基于秩-行列式准则设计方法的缺点,以及基于迹准则设计的优点.研究了基于迹准则的设计方法中STBC方案、集分割、网格结构、信号子集与STBC方案在网格中的分配这四大要素的优化原则,并依据这一原则给出了码的系统化手工设计的具体步骤.再通过星座扩展的方法,可以使设计出的码性能进一步提高.文中给出了码的设计结果,仿真表明得到的新码具有更好的性能.

关键词:超正交空时格码;迹准则;手工系统设计

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