Journal of Southeast University (English Edition) Vol. 22, No. 4, pp. 510 – 513 Dec. 2006 ISSN 1003—7985

# Application of fast wavelet transformation in signal processing of MEMS gyroscope

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**Abstract:** Decomposition and reconstruction of Mallat fast wavelet transformation (WT) is described. A fast algorithm, which can greatly decrease the processing burden and can be very easy for hardware implementation in real-time, is analyzed. The algorithm will no longer have the processing of decimation and interpolation of usual WT. The formulae of the decomposition and the reconstruction are given. Simulation results of the MEMS (micro-electro mechanical systems) gyroscope drift signal show that the algorithm spends much less processing time to finish the de-noising process than the usual WT. And the de-noising effect is the same. The fast algorithm has been implemented in a TMS320C6713 digital signal processor. The standard variance of the gyroscope static drift signal decreases from 78.435 5 (°)/h to 36.763 5 (°)/h. It takes 0.014 ms to process all input data and can meet the real-time analysis of signal.

**Key words:** wavelet transformation; signal processing; gyroscope; threshold

Recently, the gyroscope technology of the micro-electro mechanical systems (MEMS) has been developed greatly, and it is an important branch in the MEMS field and in the inertial technology field. Due to its low-cost, small dimension, light weight and high reliability, the MEMS gyroscope is widely used in low-price inertial systems nowadays [1-2]. Limited by MEMS fabrication precision, there are great drift and noise in the MEMS gyroscope measurement. A soft-threshold fast WT algorithm, without decimation and interpolation, can be used on-line to decrease noise of the drift signal and increase accuracy of the measurement[3-5].

## 1 Fast Wavelet Transformation Algorithm

Fast wavelet transformation includes fast wavelet decomposition and fast wavelet reconstruction.

### 1.1 Fast wavelet decomposition

Equations of the Mallat fast wavelet decomposition are[4]

$$C_n^j = \sum_{k \in \mathbf{Z}} h_{k-2n} C_k^{j-1} \tag{1}$$

$$D_n^j = \sum_{k \in \mathbf{Z}} g_{k-2n} C_k^{j-1} \tag{2}$$

where $k = 1, 2, \ldots, N$, $N$ is the number of the input signals; $C_n^j$ is the low frequency part of the decomposed gyroscope signal; $D_n^j$ is the high frequency component of the decomposed gyroscope signal; $j$ is the level of wavelet decomposition; $h_{k-2n}$ is the coefficient of the high-pass filter, which corresponds to the scale transformation; $g_{k-2n}$ is the coefficient of the low-pass filter, which corresponds to the wavelet transformation. All the filter coefficients can be obtained by the function wfilters("wanme", "d") in the Matlab wavelet analysis toolbox[6]. The procedure of the wavelet decomposition is shown in Fig. 1. The filtered original signal is decimated once to get the low-frequency series $C_n^{j+1}$ by the low-pass decomposition filter Lo_D and the high-frequency series $D_n^{j+1}$ by the high-pass decomposition filter Hi_D. Signals of the odd number are eliminated and the number of the decomposed signals is half of the pre-decomposed signals. Eqs. (1) and (2) can be simplified as[5]

$$y(n) = \sum_{l=1}^{l_{\max}} f(l) x(l + 2(n-1)) \tag{3}$$

where $f(l)$ is the filter coefficient of the high-pass filter and the low-pass filter.

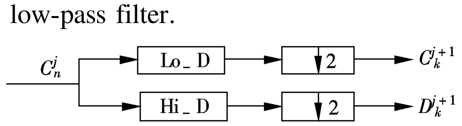


**Fig. 1** Wavelet decomposition

**Received** 2006-03-31.

**Foundation item:** The National High Technology Research and Development Program of China (863 Program) (No. 2002AA812038).

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### 1.2 Fast wavelet reconstruction

The reconstruction equation of the WT is[4]

$$C_n^{j-1} = \sum_{k \in \mathbf{Z}} h_{n-2k} C_k^j + \sum_{k \in \mathbf{Z}} g_{n-2k} D_k^j \quad (4)$$

where $C_k^j$ is the low frequency series and $D_k^j$ is the high frequency series from the wavelet decomposition; $h_{n-2k}$ and $g_{n-2k}$ are the coefficients of the high-pass filter and the low-pass filter of the wavelet reconstruction, which can be obtained by the function wfilters("wanme", "r") in the Matlab wavelet transformation toolbox[6]. The procedure of the wavelet reconstruction is shown in Fig. 2.

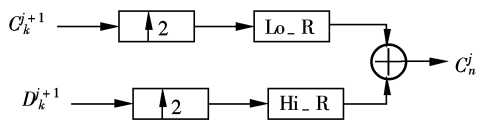


**Fig. 2** Wavelet reconstruction

During the signal reconstruction, the low-frequency signal $C_k^j$ and the high-frequency signal $D_k^j$ will be interpolated once before they are filtered by the low-pass reconstructed filter Lo_R and by the high-pass reconstructed filter Hi_R. The data rate is twice that of the original data rate. The data change from $C_k^j$ and $D_k^j$ into $C_{2k}^j$ and $D_{2k}^j$. Thus, the reconstruction formula after the up-sampling of $C_k^j$ and $D_k^j$ will become

$$C_n^{j-1} = \sum_{k \in \mathbf{Z}} h_{n-2k} C_{2k}^j + \sum_{k \in \mathbf{Z}} g_{n-2k} D_{2k}^j \quad (5)$$

The formula can be written as

$$C_n^{j-1} = \sum_{k \in \mathbf{Z}} h_l C_{n-l}^j + \sum_{k \in \mathbf{Z}} g_l D_{n-l}^j \quad (6)$$

The decomposed signal will be reconstructed only when $n$ and $l$ are even or odd at the same time. Eq. (6) will become[5]

$$C_{2m}^j = \sum_{l=0,2,\ldots} h_l C_{(2m-l)/2}^{j-1} + \sum_{l=0,2,\ldots} g_l C_{(2m-l)/2}^{j-1} \quad (7)$$

$$C_{2m+1}^j = \sum_{l=1,3,\ldots} h_l C_{(2m+1-l)/2}^{j-1} + \sum_{l=0,2,\ldots} g_l C_{(2m+1-l)/2}^{j-1} \quad (8)$$

Input data of the odd index are only filtered by the odd index filter coefficients to generate the odd index out data. Input data of the even index are also only filtered by the even index filter coefficients to generate the even index out data. Thus, interpolation will no longer be needed. The burden of the computation will be decreased further. The length of the output series will become twice that of the input series.

## 2 Selection of Threshold in Wavelet Reconstruction

Effect and real-time attribution of the WT are mainly decided by the threshold selection. The final signal will be obtained after the wavelet coefficients are processed by the fitful threshold. The often-used threshold function includes the soft-threshold function and the hard-threshold function. The often-used Donoho hard-threshold function is[7]

$$\hat{D}_k^j = \begin{cases} 0 & |D_k^j| < \lambda \\ D_k^j & |D_k^j| > \lambda \end{cases} \quad (9)$$

And the Donoho soft-threshold function is[7]

$$\hat{D}_k^j = \begin{cases} 0 & |D_k^j| < \lambda \\ \operatorname{sgn}(D_k^j)\,|D_k^j - \lambda| & |D_k^j| > \lambda \end{cases} \quad (10)$$

where sgn( * ) is the sign function; the threshold $\lambda$ is $\sigma(2\log N)^{1/2}$, $\sigma$ is median ( $|D_j^k|$ )/0.674 5, median( * ) is the median function. The soft-threshold method is more widely used in the signal processing than the hard-threshold method. There are great drift and noise in the MEMS gyroscope measurement. Adaptability of the threshold function can increase the effect of the WT. After comparing many threshold functions, the following function is selected as the threshold function in this paper. $N$ is decided by the practical signal[8-10].

$$\hat{D}_k^j = \begin{cases} 0 & |D_k^j| < \lambda \\ \operatorname{sgn}(D_k^j)\left(|D_k^j| - \dfrac{\lambda}{\exp((|D_k^j| - \lambda)/N)}\right) & |D_k^j| > \lambda \end{cases} \quad (11)$$

By the flexible selection of $N$, the practical threshold function can be obtained.

## 3 Simulation Result from MEMS Gyroscope

The method of de-noising in the MEMS gyroscope measure includes FIR filtering, time series analysis, multi-sensor information fusion, WT and so on[9-11]. Limited by effect and real-time of these processing methods, many of them cannot be applied in the technical project. The fast WT algorithm has the effect of the normal WT, but has better real-time than the normal WT. It can be used in the MEMS gyroscope signal processing.

The static drift signal is from the measurement of the MEMS ADXRS150 gyroscope and has been processed by the fast WT algorithm, where the db4 wavelet basis is selected. The signal sampling frequency is 50 Hz. The original signal and the WT signal are shown in Fig. 3. After the WT, the standard variance of the signal decreases from 78.435 5 (°)/h to 36.763 5 (°)/h.

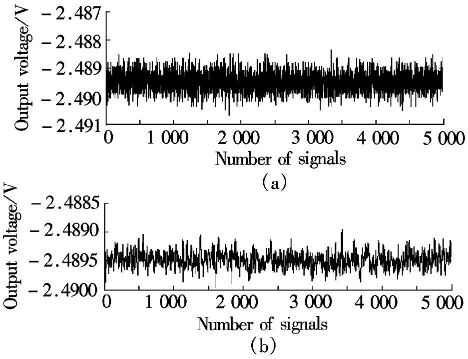


**Fig. 3** De-noise effect comparison of the static drift signal. (a) The original signal; (b) The wavelet transformed signal

A sine signal and white noise is added to the static drift signal to emulate a true dynamic signal. The result of the WT of the dynamics is shown in Fig. 4.

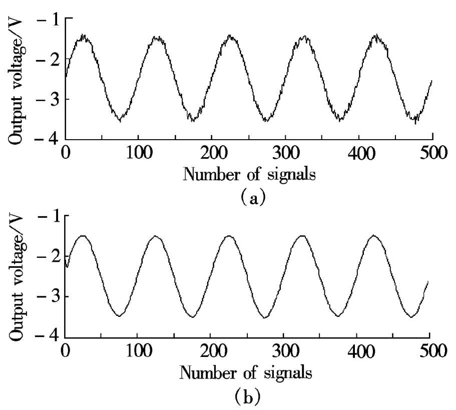


**Fig. 4** De-noise effect comparison of the dynamics signal. (a) The original dynamic signal; (b) The wavelet transformed dynamic signal

If the drift is also regarded as noise, the signal-to-noise ratio (SNR) will increase 4. 7 dB by the WT. The maximum amplitude of the white noise is 5% of the bias of the gyroscope. The amplitude of the sine signal is equal to the bias. Seen from simulation of the sine signal, added with the white noise, the fast WT algorithm can also effectively de-noise the noise of the MEMS gyroscope measurement.

The fast algorithm of the WT has been described by C programming first. Using the tool of Code Composer Studio (CCS), the algorithm is assembled and loaded into the digital signal processor (DSP) TMS320C6713. To the 50 input signal, the algorithm has 3 184 instructions and occupies 160 933 clock periods. The processing frequency of the processor is 225 MHz. It takes 0. 014 ms to generate an output signal. The processing speed can meet the practical need. To the often-used WT, the processing time is 0. 125 ms per signal.

## 4 Conclusion

Based on the WT, a fast wavelet transformation algorithm without interpolation and decimation is analyzed. A fitful soft threshold function is selected in the wavelet reconstruction. Simulation of the MEMS gyroscope measurement verifies the validity of the algorithm. The algorithm has also been implemented in the DSP and can meet the technical need.

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# 快速小波变换在 MEMS 陀螺信号处理中的应用

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**摘要:**介绍了 Mallat 快速小波分解和重构算法,分析了一种可以大大降低运算负担,并且十分易于硬件实时实现的快速算法. 该算法不再需要小波变换过程中的内插和抽取步骤,给出了相应的分解和重构过程的公式. 对 MEMS 陀螺仪测量信号的仿真结果表明:算法只需更短的处理时间就可以完成去噪声过程,并且可以取得同样的去噪效果. 在 TMS320C6713 芯片上实现了该算法,每个数据的处理时间只需 0. 014 ms,静态漂移信号的标准差也从 78. 435 5 (°)/h 降到 36. 763 5 (°)/h,完全可以满足信号实时处理的需要.

**关键词:**小波变换;信号处理;陀螺;阈值

**中图分类号:**TP206$^+$. 1