Journal of Southeast University (English Edition) Vol. 23, No. 1, pp. 1 – 6 Mar. 2007 ISSN 1003—7985

Design and performance analysis of adaptive ordered LDPC coded OFDM system

Wang Fang[1] Cheng Shixin[1] Chen Ming[1] Wang Haifeng[2]

([1] National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China)

([2]Nokia Technology Platform, Oulu 90570, Finland)

Abstract: A novel adaptive ordered LDPC (low-density parity-check) coded OFDM (orthogonal frequency-division multiplexing) transmission technique is proposed to exploit different error probabilities of irregular LDPC coded bits in OFDM systems. Assuming that the CSI (channel state information) is known at the transmitter, the irregular LDPC coded bits are ordered according to their degrees and then allocated into subcarriers adaptively. Bits with higher degrees are allocated into less attenuated subcarriers and bits with lower degrees are allocated into deep attenuated subcarriers. Quantization on CSI feedback can be applied to minimize the signaling overhead. Performance of this strategy is analyzed by density evolution and numerical simulation. Simulation results show that about a 1 to 1.5 dB gain in terms of SNR (signal to noise ratio) can be achieved over frequency-selective fading channels compared to conventional LDPC coded OFDM systems without ordering, and the proposed scheme is robust to CSI quantization.

Key words: low-density parity-check codes; orthogonal frequency-division multiplexing; ordering; adaptive; density evolution

OFDM (orthogonal frequency-division multiplexing) as become a key technology in many communication systems. It increases the robustness against frequency-selective fading. By using a cyclic-prefix (CP) in an OFDM system, channel distortion can be easily complemented by a simple one-tap equalizer in the frequency domain[1].

Recent research has proved that using the message passing decoding algorithm (also known as the belief propagation (BP) algorithm) the irregular low-density parity-check (LDPC) codes exhibit performance extremely close to the Shannon limit in noisy channels[2]. With other advantages such as low complexity and full parallelizable decoders, and detectable decoding errors, LDPC codes have become a powerful candidate for future wireless communication systems.

How to efficiently utilize the advantages of LDPC codes flexibly in OFDM systems is a challenging question. Some studies have been carried out regarding this issue. For example, Ref. [3] analyzed the LDPC coded OFDM system on both an AWGN and a frequency-selective fading channel. Lu[4] considered the performance analysis and design optimization of LDPC coded MIMO OFDM systems by density evolution[5-6]. Ref. [7] presented an optimized LDPC coding scheme for OFDM systems over frequency-selective fading channels. But the codes should be re-optimized once the channel condition changed, which will bring high time and hardware overhead to the systems.

By the definition of irregular LDPC codes[8], we say degree of a variable node (corresponding to a coded bit) is i, when this node is checked by i parity-check equations. Correspondingly, it also provides messages to i parity-check equations. So a variable node of higher degree will contribute more to the bipartite graph than a lower degree variable node. In the initialization step of the BP decoding algorithm[5], each variable node calculates its LLR (log-likelihood-ratio) values from the channel and transports them to their neighbors. Hence, if there is a chance to allocate LLR values to variable nodes, assigning the larger LLR to the higher degree variable node will bring more reliability messages to the bipartite graph. This idea can be realized in OFDM systems. Because of different frequency responses of each subcarrier, the variable nodes with different degrees can be distributed among the subcarriers to enlarge the total reliability message of the bipartite graph. Considering the above analysis, a novel coded bit allocation algorithm with CSI knowledge at the transmitter is proposed for OFDM systems,

Received 2006-04-11.

Foundation item: The National High Technology Research and Development Program of China (863 Program) (No. 2002AA123031).

Biographies: Wang Fang (1978—), female, graduate, wangfangg@seu.edu.cn; Cheng Shixin (corresponding author), male, professor, sxcheng@seu.edu.cn.

where only an addition ordering operation is needed. At the transmitter, LDPC coded bits are ordered by their degrees and allocated to the subcarriers correspondingly, where the coded bits with higher degrees are allocated to subcarriers with less channel attenuation. Simulation results show that the proposed scheme can noticeably improve the bit error rate (BER) performance of OFDM systems over a frequency-selective fading channel.

1 System Description

The transceiver structure of the proposed adaptive ordered LDPC coded OFDM system is illustrated in Fig. 1. At the transmitter, K_c information bits are encoded by the LDPC encoder into N_c coded bits X_{k_c}, $k_c = 1, 2, \dots, N_c$ with code rate $R = K_c / N_c$. Assuming the subcarrier number is N and the modulation constellation size is M, the encoded bits are segmented into blocks with size of $N\log_2 M$ and mapped into N symbols S_k, $k = 0, 1, \dots, N-1$. For the purpose of simplicity, we set $N_c = N\log_2 M$. Coded bits are ordered according to their corresponding degrees and the channel attenuation of each subcarrier with the CSI knowledge, respectively. After modulation, the mapped symbols with higher degrees are allocated to the subcarriers with less attenuation. The ordering operation rules are described as follows:

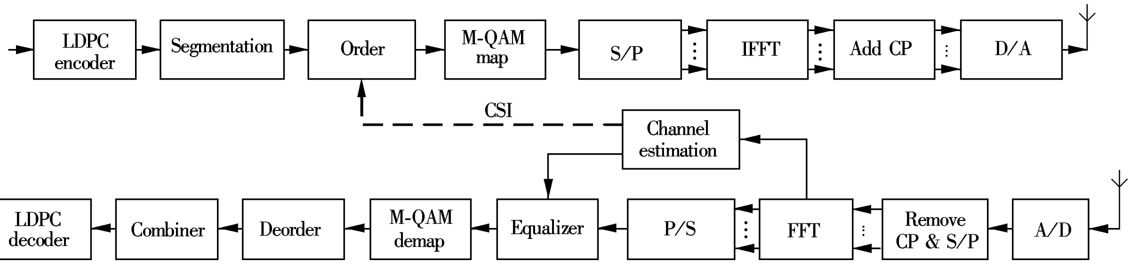

Fig. 1 Block diagram of adaptive ordered LDPC coded OFDM system

If the $\tilde{k}$-th encoded symbol $S_{\tilde{k}}$ becomes the $\tilde{i}$-th symbol after ordering, we write it as $\tilde{i} = O_S(S_{\tilde{k}})$. Its inverse function is denoted by $S_{\tilde{k}} = O_S^{-1}(\tilde{i})$. Given the $\tilde{k}_1$-th and $\tilde{k}_2$-th encoded symbol $S_{\tilde{k}_1}$ and $S_{\tilde{k}_2}$ with degree i_1 and i_2, respectively, $i_1 > i_2$, we have $\tilde{i}_1 > \tilde{i}_2$ after ordering, where $\tilde{i}_1 = O_S(S_{\tilde{k}_1})$ and $\tilde{i}_2 = O_S(S_{\tilde{k}_2})$. Similarly, we denote the serial number of the $\tilde{q}$-th subcarrier after ordering by $\tilde{j} = O_H(\tilde{q})$, and $\tilde{q} = O_H^{-1}(\tilde{j})$. Given the above definition, if the serial number of the $\tilde{q}$-th subcarrier after ordering is $\tilde{j}$, then we can write the symbol which is transmitted in the $\tilde{q}$-th subcarrier as $S_{\tilde{k}} = O_S^{-1}(\tilde{j}) = O_S^{-1}(O_H(\tilde{q}))$. The subscript S and H mean that the function is defined on frequency domain signal S_k and channel frequency response $H_{\tilde{q}}$.

The ordered symbols are processed by IFFT operation. After being sampled every T_c second and converted from parallel to serial (P/S), where T_c is the sampling period, the transmitted symbols in time domain can be expressed as Eq. (1). Finally, s_n are upconverted and transmitted.

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} O_S^{-1}(O_H(k)) \exp\left(\mathrm{j}\,\frac{2\pi kn}{N}\right) \qquad n = 0, 1, \dots, N-1 \tag{1}$$

The discrete-time received signal can be written as Eq. (2), where s_n, h_n, n_n are the transmitted signal, channel impulse response and the additional white Gaussian noise (AWGN), respectively. It is assumed that the channel impulse response is unchangeable during an OFDM symbol period. The received signal with CP removal is converted into frequency domain by FFT operation for channel equalization. The frequency-domain signal can be written as Eq. (3). Here H_k is the channel frequency domain response, and N_k is the AWGN at the k-th subcarrier. Y_k can be equalized by a one-tap equalizer based on the CSI estimation in frequency domain, demapped from symbol level to bit level, re-ordered, and then decoded by the LDPC decoder with the BP decoding algorithm.

$$y_n = s_n \otimes h_n + n_n \qquad n = 0, 1, \dots, N-1 \tag{2}$$

$$Y_k = S_k H_k + N_k \qquad k = 0, 1, \dots, N-1 \tag{3}$$

The CSI has to be signaled to the transmitter via a feedback signaling channel for the purpose of ordering. To reduce the signaling overhead on CSI feedback from receiver to transmitter, quantization of feedback CSI can be applied. It is proved by analysis and simulation that the quantization induced performance loss is negligible.

2 Performance Analysis

For uncorrelated Rayleigh fading channel, each path of channel impulse response h_n, $n = 1, 2, \dots, L$, is an independent and identically distributed (i. i. d) variable, whose amplitude has probability density function (pdf) as Eq. (4), where $2\sigma_n^2$ is the variance of the n-th path.

$$p_{h_n}(\alpha) = \frac{\alpha}{\sigma_n^2}\exp\left(-\frac{\alpha^2}{2\sigma_n^2}\right) \qquad \alpha \geqslant 0; n = 1, 2, \dots, L \tag{4}$$

Frequency domain response of each subcarrier can be expressed as

$$H_k = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} h_n \exp\left(-\mathrm{j}\frac{2\pi kn}{N}\right) \qquad k = 0, 1, \dots, N-1 \tag{5}$$

H_k is also complex Gaussian with variance of real/imaginary part $\sigma_{\mathrm{H}}^2 = \frac{1}{N}\sum_{n=1}^{L}\sigma_n^2$, but may not be independent. Pdf and cumulative distribution function (cdf) of H_k are

$$p_{\mathrm{H}}(\alpha) = \frac{\alpha}{\sigma_{\mathrm{H}}^2}\exp\left(-\frac{\alpha^2}{2\sigma_{\mathrm{H}}^2}\right) \qquad \alpha \geqslant 0 \tag{6}$$

$$F_{\mathrm{H}}(\alpha) = 1 - \exp\left(-\frac{\alpha^2}{2\sigma_{\mathrm{H}}^2}\right) \qquad \alpha \geqslant 0 \tag{7}$$

Considering the coded bit X is mapped into signal $S = 1 - 2X$, $X = 0, 1$ for BPSK modulation, received signal in the k-th subcarrier, Y_k, has the conditional pdf in Eq. (8),

$$p_{Y_k}(y \mid S, \alpha) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(y - S\alpha)^2}{2\sigma^2}\right) \qquad S = \pm 1 \tag{8}$$

where α is the Rayleigh distributed subcarrier fading factor with pdf as in Eq. (6), σ^2 is the variance of the AWGN. Assuming $p(S = 1) = p(S = -1) = 0.5$, LLR observed from the channel, denoted by u^0, can be written as[9]

$$u^0 = \log\left(\frac{p(S = 1 \mid y, \alpha)}{p(S = -1 \mid y, \alpha)}\right) = \log\left(\frac{p_{Y_k}(y \mid S = 1, \alpha)}{p_{Y_k}(y \mid S = -1, \alpha)}\right) = \frac{2\alpha}{\sigma^2}y \tag{9}$$

Given $S = 1$, i. e., all-zero word is transmitted, the conditional pdf of u^0 is

$$p_{u^0}(q \mid a) = \frac{\sigma}{\sqrt{2\pi}2\alpha}\exp\left(\frac{-(q - 2\alpha^2/\sigma^2)^2}{8\alpha^2/\sigma^2}\right) \tag{10}$$

In the ordered LDPC coded OFDM systems, subcarriers to transmit bits with different degrees may have distinct statistical characteristics. Given an LDPC code with the variable nodes degree distribution $\lambda(x) = \sum_{i=2}^{d_{\mathrm{v}}}\lambda_i x^{i-1}$, we consider the simplest case that the LDPC block N_{c} is equal to the subcarrier number N. Let $\boldsymbol{Q}(i)$ denote the aggregate of bits whose degree is i, $\boldsymbol{C}(\boldsymbol{Q}(i))$ denote the aggregate of subcarriers who transmit $\boldsymbol{Q}(i)$. For convenience, we also write it as $\boldsymbol{C}(i)$. Given the block length N, the number of degree-i variable nodes is $n_i = \frac{\lambda_i}{i}\left(N \Big/ \sum_{i=1}^{d_{\mathrm{v}}}\frac{\lambda_i}{i}\right)$, then the serial numbers of $\boldsymbol{Q}(i)$ after ordering are from $o_{\mathrm{n},i} = n_{i-1} + 1$ to $o_{\mathrm{e},i} = n_{i-1} + n_i$.

Independent ordered random variables can be analyzed by the ordered statistics theory[10]. Suppose that there exist w independent random variables, Z_1, Z_2, …, Z_w, each with the same cdf $F_Z(z)$ and pdf $p_Z(z)$. Ordering these variables as $Z_{(1)}$, $Z_{(2)}$, …, $Z_{(w)}$, where $Z_{(r)}$, $r = 1, 2, \dots, w$, denotes the r-th ordered variable. Then the pdf of the ordered random variable $Z_{(r)}$ is

$$p_r(z) = \frac{w!}{(r-1)!(w-r)!}[F_Z(z)]^{r-1}[1 - F_Z(z)]^{w-r}p_Z(z) \tag{11}$$

Assuming that the subcarriers are independent of each other, the approximate pdf of $\boldsymbol{C}(i)$ can be expressed as

$$p'_{\mathrm{H},i}(\alpha) = \frac{1}{o_{\mathrm{e},i} - o_{\mathrm{b},i} + 1}\sum_{r=o_{\mathrm{b},i}}^{o_{\mathrm{e},i}} p_{\mathrm{H},r}(\alpha) \tag{12}$$

where $p_{\mathrm{H},r}(\alpha)$ is the pdf of the r-th subcarrier of the ordered N subcarriers, which can be written as

$$p_{\mathrm{H},r}(\alpha) = \frac{N!}{(r-1)!(N-r)!}[F_{\mathrm{H}}(\alpha)]^{r-1} \cdot [1 - F_{\mathrm{H}}(\alpha)]^{N-r}p_{\mathrm{H}}(\alpha) \tag{13}$$

Let v^l and u^l denote the output messages of a variable node and a check node up to the l-th iteration, respectively, $p_{v^l}(q)$ and $p_{u^l}(q)$ are their pdf. Then v^0 represents the average output message of variable nodes, and also is the incoming LLR of check nodes at the first iteration. In conventional unordered systems, $v^0 = u^0$ and $p_{v^0}(q) = p_{u^0}(q)$. But in ordered systems, $p_{v^0}(q)$ is not equal to $p_{u^0}(q)$ anymore. For a check node, its incoming LLR message has a probability λ_i coming from a degree-i variable node, so the average incoming LLR of check nodes along each edge is $v^0 = \sum_{i=2}^{d_v}\lambda_i u_i^0$. Here u_i^0 denotes the average initial LLR of $\boldsymbol{Q}(i)$. For the independency assumption among different $\boldsymbol{Q}(i)$, $p_{v^0}(q)$ becomes

$$p_{v^0}(q) = \sum_{i=2}^{d_{\mathrm{v}}}\lambda_i p_{u_i^0}(q) \tag{14}$$

where $p_{u_i^0}(q)$ is pdf of u_i^0 written as

$$p_{u_i^0}(q) = \int_0^{+\infty} p_{u_i^0}(q \mid \alpha)p'_{\mathrm{H},i}(\alpha)\,\mathrm{d}\alpha = \int_0^{+\infty}\frac{\sigma}{\sqrt{2\pi}2\alpha}\exp\left(\frac{-(q - 2\alpha^2/\sigma^2)^2}{8\alpha^3/\sigma^2}\right)p'_{\mathrm{H},i}(\alpha)\,\mathrm{d}\alpha \tag{15}$$

Eqs. (10) and (14) give the expression of initial input to variable nodes and check nodes in density evolution respectively. With them, density evolution can observe how $p_{v^l}(q)$ and $p_{u^l}(q)$ evolve during the course of iterative decoding. Error probability up to the l-th iteration can be determined as $p_e = \int_{-\infty}^{0-} p_{v^l}(q)\,\mathrm{d}q + \frac{1}{2}p_{v^l}(0)$. When the number of iterations tends to infinity, $p_{v^l}(q)$ may tend to Δ_∞ (equivalently, p_e tends to zero) or may converge to a density with a finite p_e. The so-called threshold is defined as the maximum noise level such that p_e tends to zero as the number of iterations tends to infinity. Thus, only by comparing $p_{v^0}(q)$ between unordered and ordered systems can we predict whether the ordered systems outperform the unordered systems.

3 Simulation Results

Performance of the proposed adaptive ordered LDPC coded OFDM system is simulated and evaluated in a quasi-static frequency-selective fading channel with perfect channel estimation. The simulation specifications are summarized in Tab. 1. The parity-check matrices of LDPC code are generated by the PEG method[11], in which the variable degrees are in nonincreasing order from systematic bits to parity bits. LDPC codes are decoded by the BP decoding algorithm with 100 iterations.

Tab. 1 Simulation parameters

Parameter	Value
Subcarrier number N	1 024, 512
LDPC code length N_c	1 024
Carrier frequency f_c/GHz	5
Sample frequency f_s/MHz	10
CP number	64
Channel	ITU-R M. 1225
Mobile velocity/(km·h^{-1})	100
Time delay/ns	0, 310, 710, 1 090, 1 730, 2 510
Power spectrum/dB	0, −1, −9, −10, −15, −20
$\lambda(x)$	$0.276\,84x + 0.283\,42x^2 + 0.439\,74x^8$ [8]

Fig. 2 (a) presents a real-time channel impulse response in frequency domain. Fig. 2 (b) is the degree distribution of the symbols after ordering operations in all the subcarriers.

In Fig. 3, the real line shows $p_{v^0}(q)$ and the dashed line denotes $p_{u^0}(q)$ of code in an ordered system with parameters shown in Tab. 1 at SNR = 0 dB. With the expression of p_e in density evolution, we can draw the conclusion that the BER of an ordered system is less than that of an unordered one.

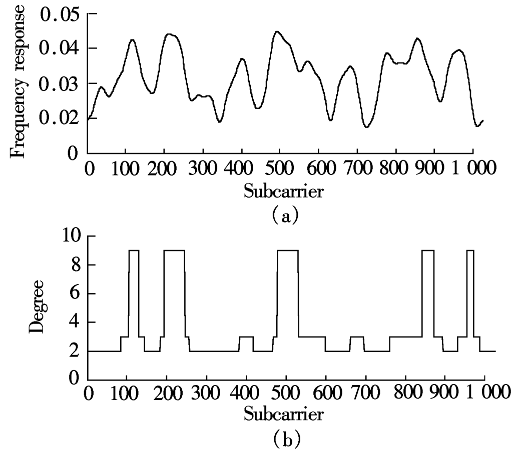

Fig. 2 An instance of degree distribution in subcarriers. (a) An instance of the channel frequency domain response; (b) Degree distribution in subcarriers after ordering

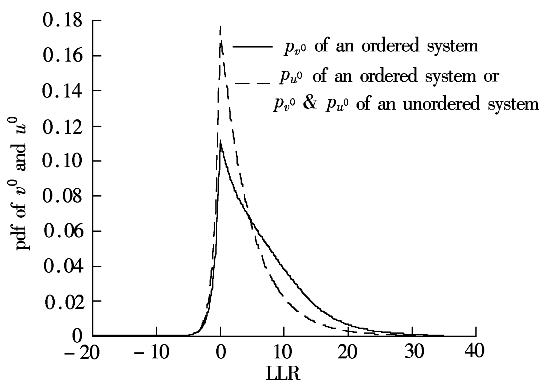

Fig. 3 $p_{u^0}(q)$ and $p_{v^0}(q)$ of ordered LDPC coded OFDM system

Fig. 4 displays the corresponding density evolution of p_v^l in both ordered and unordered systems, where p_v^l is the pdf of a variable node output LLR value after the l-th iteration. The marked line and unmarked denote p_v in ordered systems and unordered systems respectively after 1, 2 and 6 iterations.

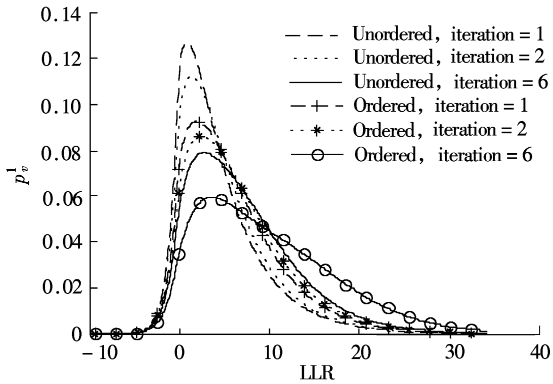

Fig. 4 $p_v(q)$ after 1, 2 and 6 iterations, ordered and unordered

Fig. 5 illustrates BER performance of the proposed ordered LDPC coded OFDM system with BPSK modulation ($M = 2$), code rate $R = 1/2$ and $N_c = N = 1\,024$. The line with triangle mark denotes the BER versus SNR of the ordered LDPC coded OFDM system, and the circle marked curve represents the BER of the unordered system with the same parameters. It can be noticed that the proposed scheme improves the system performance by around 1 to 1.5 dB compared with the conventional scheme without ordering. The plus marked curve is the BER of the ordered system, ordering the coded symbols according to CSI of the last block.

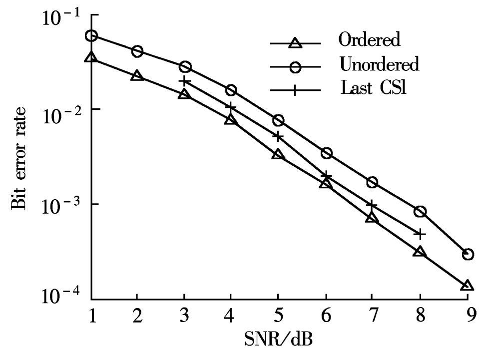

Fig. 5 BER performance of ordered LDPC coded OFDM system with BPSK modulation

Fig. 6 presents the impacts of the quantization on CSI feedback signaling channel. Different quantization bits, from 1 to 3, are adopted. We can see from Fig. 6 that, for 2 or 3 bits quantization, although the CSI is no longer accurate for the transmitter, the BER performance suffers almost no loss compared with the system which obtains the accurate CSI. But for only 1 bit quantization, obvious performance loss appears. It can be seen that the proposed scheme is robust regarding

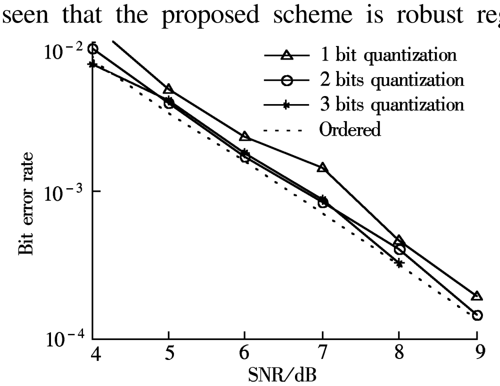

Fig. 6 BER performance of ordered LDPC coded OFDM system with BPSK and quantization

CSI feedback errors. Generally, due to the fair treatment to coded bits which have the same degree in ordering operation, the number of quantization bit $\tilde{\omega}$ should satisfy the condition that $2^{\tilde{\omega}} \geqslant \delta(d_v)$, where $\delta(d_v)$ is the class number of variable degrees. For example, $\delta(d_v)=4$ implies that $\tilde{\omega}=2$ is sufficient.

Fig. 7 illustrates the BER performance of the proposed system with QPSK modulation ($M=4$) and $N_c=512$. Also about 1.5 dB gain can be achieved. Fig. 8 presents the impacts of the quantization on CSI feedback signaling channel for this case.

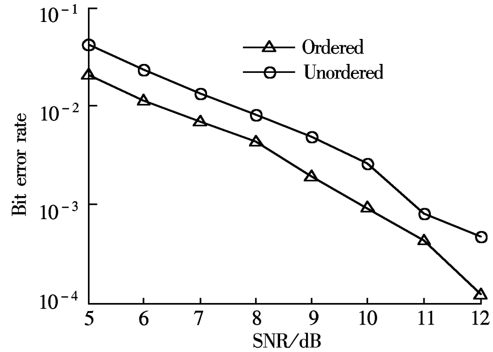

Fig. 7 BER performance of ordered LDPC coded OFDM system with QPSK modulation

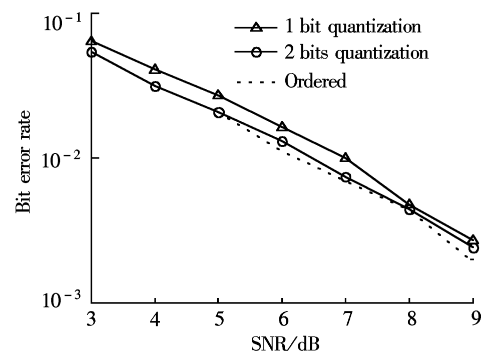

Fig. 8 BER performance of ordered LDPC coded OFDM system with QPSK and quantization

4 Conclusion

A novel adaptive ordered LDPC coded OFDM transmission technique is proposed in this paper. The LDPC coded symbols are allocated to subcarriers according to the LDPC degrees and corresponding channel attenuation. The symbols with higher LDPC degree are allocated at subcarriers with less channel attenuation. Simulation results show that the system performance can be noticeably improved by the proposed scheme with robustness regarding CSI feedback errors. It is proved that the quantization on CSI feedback can be applied to minimize the signaling overhead. The proposed scheme is well-suited for enhancing system throughput especially for quasi-static fading environments so that the channel does not vary fast and there is no need to signal CSI so often.

References

[1] Wang Zhengdao, Giannakis G B. Wireless multicarrier communications: where Fourier meets Shannon [J]. *IEEE Signal Processing Magazine*, 2000, **17**(3): 29 − 48.

[2] Gallager R G. *Low density parity check codes*[M]. Cambridge, MA: MIT Press, 1963.

[3] Hisashi Futaki, Tomoaki Ohtsuki. Performance of low-density parity-check (LDPC) coded OFDM systems [C]// *Proceedings of ICC* 2002. New York, 2002: 1696 - 1700.

[4] Lu Ben, Yue Guosen, Wang Xiaodong. Performance analysis and design optimization of LDPC-coded MIMO OFDM systems [J]. *IEEE Transactions on Signal Processing*, 2004, **52**(2): 348 - 361.

[5] Richardson T J, Urbanke R. The capacity of low-density parity-check codes under message-passing decoding [J]. *IEEE Transactions on Information Theroy*, 2001, **47**(2): 599 - 618.

[6] Chung S Y. On the construction of some capacity-approaching coding schemes [D]. Cambridge, MA: MIT, 2000.

[7] Mannoni V, Declercq D, Gelle G. Optimized irregular Gallager codes for OFDM transmission[C]//*Proceedings of PIMRC* 2002. Lisbon, Portugal, 2002: 222 - 226.

[8] Richardson T J, Shokrollahi A, Urbanke R. Design of capacity approaching irregular low-density parity-check codes [J]. *IEEE Transactions on Information Theroy*, 2001, **47** (2): 619 - 637.

[9] Hou J, Siege P H, Milstein L B. Performance analysis and code optimization of low density parity-check codes on Rayleigh fading channels [J]. *IEEE J Select Areas Commun*, 2001, **19**(5): 924 - 934.

[10] David H A. *Order statistics* [M]. New York: John Wiley, 1981.

[11] Hu X Y, Eleftheriou E, Arnold D M. Regular and irregular progressive edge-growth Tanner graphs [J]. *IEEE Transactions on Information Theroy*, 2005, **51**(1): 376 - 398.

基于自适应排序 LDPC 码的 OFDM 系统及其性能分析

王　芳[1]　程时昕[1]　陈　明[1]　王海峰[2]

([1]东南大学移动通信国家重点实验室,南京 210096)

([2]诺基亚奥卢技术平台,芬兰奥卢 90570)

摘要:为了在正交频分复用(OFDM)系统中充分利用非正则低密度奇偶校验(LDPC)码编码比特错误概率不同的特性,提出了一种新的基于自适应排序 LDPC 码的 OFDM 传输技术. 假设发射机已知信道状态信息(CSI),非正则 LDPC 码的编码比特根据度进行排序,并被分配到相应的子载波上,度高的比特分配到轻度衰落的子载波,度低的节点分配到深度衰落的子载波. 为了降低反馈信道的开销,可以对反馈的 CSI 进行量化. 该传输技术的性能可以由密度演进算法进行分析. 分析和仿真结果表明,相对于传统的无排序 LDPC 码编码 OFDM 系统,所提出的基于自适应排序 LDPC 码的 OFDM 系统可以获得 1 ~1. 5 dB 的性能增益,并且对 CSI 的量化具有鲁棒性.

关键词:LDPC 码;正交频分复用;排序;自适应;密度演进

中图分类号:TN929. 533