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Improved reduced-complexity bit and power allocation algorithms for multicarrier systems

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Abstract: Based on the iterative bit-filling procedure, a computationally efficient bit and power allocation algorithm is presented. The algorithm improves the conventional bit-filling algorithms by maintaining only a subset of subcarriers for computation in each iteration, which reduces the complexity without any performance degradation. Moreover, a modified algorithm with even lower complexity is developed, and equal power allocation is introduced as an initial allocation to accelerate its convergence. Simulation results show that the modified algorithm achieves a considerable complexity reduction while causing only a minor drop in performance.

Key words: multicarrier modulation; allocation algorithm; bit loading; computational complexity

Research in multicarrier modulation has grown tremendously in recent years due to its advantages, especially for its high immunity to inter symbol interference (ISI). For many applications, such as digital audio and video broadcasting (DAB, DVB), and WLAN, multicarrier modulation is an attractive alternative to single-carrier systems.

In a multicarrier system, to improve performance, different bits and power can be assigned across the subcarriers according to their channel gains[1]. Some efficient algorithms, based on the assumption of infinite granularity in constellation sizes, were presented in Refs. [2 – 3] to implement adaptive bit and power allocation for multicarrier systems. However, the performance degradation arises when the continuous solutions of bit allocation are rounded to integers. The iterative algorithms exploiting a greedy bit-filling procedure were developed in Refs. [4 – 5] to obtain the optimal performance, which also increased the computational complexity. In Ref. [6], Krongold et al. designed an algorithm with low complexity using the Lagrange multiplier method. Ref. [7] studied the discrete bit-loading algorithms that tried to balance the tradeoff between the performance and implementation complexities. Evidently, it is still a key issue to design an adequately performing algorithm with low complexity for adaptive allocation techniques. In this paper, we present a new approach to reduce the complexity of greedy adaptive loading (GAL) algorithms without performance degradation.

1 System Model

The key problem of the adaptive discrete bit and power allocation is formulated as

$$\max_{m_n} C \triangleq \sum_{i=1}^{N} m_i \quad \text{subject to} \sum_{i=1}^{N} p_i \leqslant P_{\text{total}} \tag{1}$$

where N denotes the total number of subcarriers; m_i and p_i represent the number of bits and the allocated power of the i-th subcarrier, respectively; P_{total} is a total power constraint. Let B denote the maximum number of bits that can be assigned to each subcarrier. The value of m_i should be an integer number not greater than B. Additionally, for the m-bit constellation, denote S^m as the signal-to-noise ratio (SNR) required to maintain BER_{th}, the target bit error rate (BER). We thus allocate p_i to guarantee the system BER performance using

$$p_i = \frac{S^{m_i}\sigma^2}{g_i^2} \tag{2}$$

where g_i is the channel gain at the i-th subcarrier and σ^2 is the subcarrier noise power.

So far, the allocation problem is described as the optimization problem in Eq. (1); however, it is difficult to obtain the optimal solution using the classical Lagrange multiplier method. Hence, a greedy method based on an iterative bit-loading procedure is developed, which iteratively assigns one more bits to the subcarrier requiring the smallest incremental power until the total power constraint is violated. In this way, the optimal performance can be achieved at a cost of high computational complexity [4].

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2 Proposed GAL-Based Algorithms

2.1 Properties of optimal allocation scheme

It can be easily proved that the optimal allocation system has the maximal power efficiency, when transforming the objective in Eq. (1) as

$$\min_{m_n} \frac{P_{\text{total}}}{C} \tag{3}$$

Considering any two subcarriers in the system with the channel gains g_i^2 and g_j^2, we allocate two modulation modes m_i and m_j, and denote p_i and p_j as the corresponding allocated power to guarantee BER_{th} on the i-th and j-th subcarrier, respectively. From Eq. (2), the total transmit power is

$$p_{i+j} = \sigma^2 \left(\frac{S^{m_i}}{g_i^2} + \frac{S^{m_j}}{g_j^2} \right) \tag{4}$$

Then we exchange the allocated modulation modes of the two subcarriers and recalculate the total transmit power p'_{i+j}. The two different allocation schemes are compared:

$$p_{i+j} - p'_{i+j} = \sigma^2 \frac{(S^{m_i} - S^{m_j})(g_j^2 - g_i^2)}{g_i^2 \, g_j^2} \tag{5}$$

Assuming $g_i^2 \geqslant g_j^2$ and $m_i \geqslant m_j$, so $S^{m_i} \geqslant S^{m_j}$ and the result in Eq. (5) will be negative. It indicates that lower transmit power is required to transmit a certain number of bits by allocating higher-order modulation on subcarriers with larger channel frequency responses. Therefore, the optimal allocation pattern should be designed as shown in Fig. 1. In this figure, the available modulation set is {0, 1, 2, 4}, which represents {unused, BPSK, QAM, 16QAM}.

$g_0^2 \leqslant \cdots \leqslant g_{n_0}^2 \leqslant \cdots \leqslant g_{n_1}^2 \leqslant \cdots \leqslant g_{n_2}^2 \leqslant \cdots \leqslant g_{n_4}^2$

unused	n_0	BPSK	n_1	QAM	n_2	16QAM	n_4

Fig. 1 Optimal allocation pattern

Denote $\Delta p_i^{m_i}$ as the power efficiency, namely the required power to transmit one bit on the i-th subcarrier employing modulation m_i. We have

$$\Delta p_i^{m_i} = \frac{\sigma^2 S^{m_i}}{m_i g_i^2} \tag{6}$$

Given that there is another allocation scheme to transmit symbols on the j-th subcarrier using m_j, the difference of the power efficiency is

$$\Delta p_i^{m_i} - \Delta p_j^{m_j} = \sigma^2 \left(\frac{\alpha_{m_i}}{g_i^2} - \frac{\alpha_{m_j}}{g_j^2} \right) \qquad \alpha_m = \frac{S^m}{m} \tag{7}$$

where α_m is defined as a multiplier factor of modulation mode m, because it is a constant to a certain modulation mode when S^m is given. It is indicated in Eq. (7) that the allocation with higher power efficiency will surely satisfy the condition below:

$$\Delta p_i^{m_i} \leqslant \Delta p_j^{m_j} \Leftrightarrow \frac{g_i^2}{g_j^2} \geqslant \frac{\alpha_{m_i}}{\alpha_{m_j}} \tag{8}$$

Two observations can be obtained from (8). First, the cases of allocating different modulation modes on a single subcarrier are compared:

$$\Delta p_i^{m_i} \leqslant \Delta p_j^{m_j} \Leftrightarrow \alpha_{m_i} \leqslant \alpha_{m_j} \tag{9}$$

It reveals that allocating the modulation mode with a smaller multiplier is more efficient than with a larger one. Generally, higher-order modulation modes always have larger multiplier factors. Second, the condition of allocating the same modulation mode to different subcarriers is considered by

$$\Delta p_i^{m_i} \leqslant \Delta p_j^{m_j} \Leftrightarrow g_i^2 \leqslant g_j^2 \tag{10}$$

It shows that the subcarriers with larger channel gains require less power when employing the same modulation mode. Based on the above observations, a simplified GAL scheme (S-GAL) is proposed. In each iteration, only subcarriers with the largest channel responses in each group employing the same modulation modes have the chance to upgrade their modulation modes. Take Fig. 1 for example, only one subcarrier among n_0, n_1, n_2, and n_4 is selected to upgrade its modulation mode.

2.2 Simplified GAL algorithm

Let m_i and m_j denote the currently employed modulations on the i-th and j-th subcarriers, respectively. m' is the available adjacent higher-order modulation mode to m. The required incremental power efficiency for upgrading m_i to m'_i can be obtained by

$$\Delta p_i^{\text{I}} = \frac{1}{m'_i - m_i}(p_i^{m'_i} - p_i^{m_i}) = \frac{\sigma^2 (S^{m'_i} - S^{m_i})}{(m'_i - m_i) g_i^2} \tag{11}$$

By comparing Δp_i^{I} and Δp_j^{I}, the better allocation scheme can be determined according to

$$\Delta p_i^{\text{I}} \leqslant \Delta p_j^{\text{I}} \Leftrightarrow \frac{g_i^2}{g_j^2} \geqslant \frac{\beta(m_i, m'_i)}{\beta(m_j, m'_j)} \tag{12}$$

where $\beta(m, m') = (S^{m'} - S^m)/(m' - m)$ is a constant of a modulation mode pair (m, m').

This provides an easy way to compare the incremental power of different subcarriers. The right hand side of (12) can be precalculated, and thus the optimal subcarrier can be selected using lookup tables. The details of the proposed simplified GAL (S-GAL) algorithm are described in algorithm 1. It is important to note that i^m can be directly obtained in each iteration since all subcarriers are sorted at the beginning of our algorithm with the complexity of $O(N\log_2 N)$.

Algorithm 1 S-GAL

① All the subcarriers initially employ no modulation, and are sorted according to g_i.

② Sort the subcarriers using the same constellation into groups. Denote G^m as the subcarrier group using an m-bit constellation. Thus $G^0 = \{1, 2, \dots, N\}$ and $G^m = \varnothing (1 \leqslant m \leqslant B)$, and $P_t = 0$.

③ Sort out i^m, which denotes the subcarrier with the maximal channel gain in G^m. Note that the subcarriers in G^B, which have employed the maximum constellation size, are not considered.

④ Use (12) to find the optimal subcarrier i^* among the i^m's. Assume that i^* belongs to G^{m^*}.

⑤ Calculate the required incremental power $\Delta p_{i*}^{\mathrm{I}}$ and check the total power constraint. If $P_t + \Delta p_{i*}^{\mathrm{I}} > P_{\mathrm{total}}$, the procedure is terminated; otherwise go to ⑥.

⑥ Remove i^* from G^{m^*} to G^{m^*+1}, then update $P_t = P_t + \Delta p_{i*}^{\mathrm{I}}$. Go back ③.

2.3 Modified GAL algorithm

Although the S-GAL algorithm has low complexity in each iteration, its convergence is slow. A simple modification for accelerating the convergence is to select proper initial allocations. A proper initial allocation step is introduced to accelerate its convergence. Instead of all-zero initialization, the modulation sizes are initially allocated based on equi-power assumption, and then the power is adjusted using (2). The proposed modified GAL (M-GAL) algorithm is redescribed in algorithm 2.

Algorithm 2 M-GAL

① Sort the subcarriers in ascending order according to g_i.

② For each subcarrier, allocate $m_i = m$ when $S^m \leqslant \frac{g_i^2}{\sigma^2} < S^{m+1}$. Calculate p_i from (2).

③ Calculate $P_t = \sum_{i=1}^{N} p_i$. Since the transmit power is normalized, thus $P_t < P_{\mathrm{total}}$ is avoided.

④ Obtain $G^m (0 \leqslant m \leqslant B)$ by grouping the subcarriers according to their initialized constellations. And the i^m's are determined.

⑤ Use (12) to select the optimal subcarrier i^* among the subset made up of $i^m (0 \leqslant m \leqslant B)$.

⑥ If the required total power overflows, end the algorithm.

⑦ Remove i^* from G^{m^*} to G^{m^*+1}, then update i^{m^*} from G^{m^*}, $P_t = P_t + \Delta p_{i*}^{\mathrm{I}}$. Go to ⑤.

The proposed M-GAL algorithm reduces both the computational complexity in each iteration and the number of iterations. It will be shown in the next section that the performance gap between the optimal allocation and the proposed M-GAL algorithm is negligible.

3 Numerical Results

3.1 Performance of proposed algorithms

We simulate S-GAL and M-GAL algorithms in an OFDM system with 256 subcarriers and $\mathrm{BER_{th}} = 10^{-4}$. The wireless channel is modeled as a six-tap multipath channel with its power profile decaying exponentially. M-QAM is employed with the constellation size set as $\{0, 1, 2, 3, 4, 5, 6\}$. Then the S^m's are determined using the BER expressions for M-QAM given in Ref. [8].

In Fig. 2, the throughput of different bit and power allocation algorithms are compared. The S-GAL algorithm achieves the same performance as the algorithm in Ref. [5], and the performance of the M-GAL algorithm is very close to them. The performance of the Wyglinski's algorithm using equal power allocation is also shown for comparison. All these algorithms outperform the Leke's algorithm. It should be noted that we choose a proper value of SNR gap denoted as Γ in Ref. [3] to guarantee the BER constraint. As given in Ref. [8], it can be calculated from $\Gamma = -\ln(5\mathrm{BER_{th}})/1.5$.

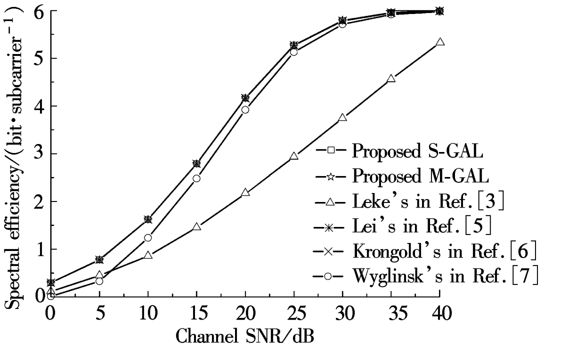

Fig. 2 Throughput of different adaptive allocation algorithms

3.2 Quantitative complexity analysis

In this subsection, we analyze the complexity of our proposed algorithms with conventional ones. The quantitative complexity comparison is listed in Tab. 1. Denote I_X as the required number of iterations of different algorithms. Fig. 3 shows the simulation results of I_X. Because the numbers of iterations of different algorithms are independent, the value of I_X in Fig. 3 is normalized. As shown in Fig. 3, the numbers of iterations of the S-GAL and the M-GAL algorithms are normalized by NB.

For instance, the numerical complexity is calculated in the scenario as $N = 256$, $B = 6$ and I_X at SNR $= 20$ dB in Fig. 3. In this case, the Lei's optimal algorithm requires more than 3.8×10^6 operations, which is much greater than 38 290, the number of operations

Tab. 1 Quantitative complexity comparison

Algorithms	Quantitative complexity
Leke's in Ref. [3]	$N\log_2 N+(5I_{\mathrm{Leke}}+11)N$
Lei's in Ref. [5]	$I_{\mathrm{Lei}}(3B-4)N$
Krongold's in Ref. [6]	$I_{\mathrm{K}}(B+4)N$
Wyglinski's in Ref. [7]	$N\log_2 N+(2B+7+I_{\mathrm{W}}(B+2))N$
Proposed S-GAL	$N\log_2 N+I_{\mathrm{S}}(5B+4)$
Proposed M-GAL	$N\log_2 N+2N+I_{\mathrm{M}}(5B+4)$

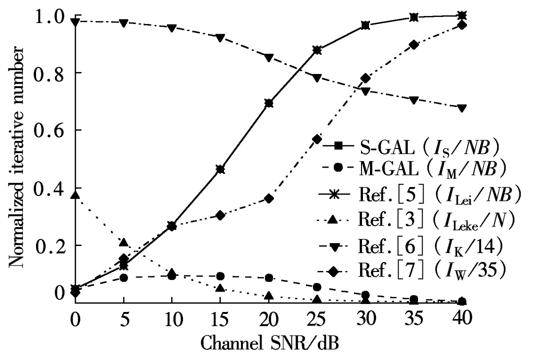

Fig. 3 Normalized number of iterations I_X of different adaptive allocation algorithms

for the S-GAL algorithm. Meanwhile, the Leke's, the Krongold's, and the Wyglinski's algorithms require 12 381, 25 561, and 30 939 operations, respectively. And the proposed M-GAL algorithm requires only 7 133 operations. It indicates that the S-GAL algorithm is still more complex than conventional suboptimal algorithms due to its optimality. However, the M-GAL algorithm outperforms the others while requiring approximately half of the operations of Leke's or a quarter of Krongold's.

4 Conclusion

In this paper, two efficient bit and power allocation algorithms are proposed for multicarrier systems. Compared with conventional GAL-based algorithms, the S-GAL algorithm reduces the complexity from $O(N)$ to $O(B)$ in each iteration. The complexity is further reduced in the M-GAL algorithm by introducing an efficient initial allocation. This algorithm requires no more than half of the number of operations of some existing algorithms. And its performance approaches very closely that of the optimal one. Simulation results verify the effectiveness of our proposed algorithms.

References

[1] Bingham A C. Multicarrier modulation for data transmission: an idea whose time has coming [J]. *IEEE Communications Magazine*, 1990, **28**(5): 5 − 14.

[2] Chow P S, Cioffi J M, Bingham A C. A practical discrete multitone transceiver loading algorithm for data transmission over spectrally shaped channels [J]. *IEEE Transactions on Communications*, 1995, **43**(2): 773 − 775.

[3] Leke A, Cioffi J M. A maximum rate loading algorithm for discrete multitone modulation systems [C]//*IEEE Globecom Proceedings*. Phoenix, AZ, 1997, **3**: 1514 − 1518.

[4] Fasano A. On the optimal discrete bit loading for multicarrier systems with constraints [C]//*IEEE VTC Proceedings*. Seoul, Korea, 2003, **2**: 915 − 919.

[5] Lei M, Zhang P, Harada H, et al. An adaptive power distribution algorithm for improving spectral efficiency in OFDM [J]. *IEEE Transactions on Broadcasting*, 2004, **50**(3): 347 − 351.

[6] Krongold B S, Ramchandran K, Jones D L. Computationally efficient optimal power allocation algorithms for multicarrier communication systems [J]. *IEEE Transactions on Communications*, 2000, **48**(1): 23 − 27.

[7] Wyglinski A M, Labeau F, Kabal P. Bit loading with BER-constraint for multicarrier systems [J]. *IEEE Transactions on Wireless Communications*, 2005, **4**(4): 1383 − 1387.

[8] Goldsmith A J, Chua S G. Variable-rate variable-power MQAM for fading channels [J]. *IEEE Transactions on Communications*, 1997, **45**(10): 1218 − 1230.

多载波系统中低复杂度的改进比特和功率分配算法

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摘要:基于迭代的比特和功率分配机制,提出了一种低复杂度的比特和功率分配算法. 与传统的迭代分配算法不同,该算法在每次迭代中只需要比较几个特定的子载波. 该方法在保持传统迭代算法性能的前提下极大地减小了迭代分配算法的复杂度. 此外,通过选择等功率分配方案作为初始方案加快了算法的收敛速度,进一步降低了算法复杂度. 仿真结果表明,提出的改进算法在基本不牺牲系统性能的前提下有效地降低了算法复杂度.

关键词:多载波调制;分配算法;比特分配;算法复杂度

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