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Quantum stochastic filters for nonlinear time-domain filtering of communication signals

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Abstract: Principles and performances of quantum stochastic filters are studied for nonlinear time-domain filtering of communication signals. Filtering is realized by combining neural networks with the nonlinear Schroedinger equation and the time-variant probability density function of signals is estimated by solution of the equation. It is shown that obviously different performances can be achieved by the control of weight coefficients of potential fields. Based on this characteristic, a novel filtering algorithm is proposed, and utilizing this algorithm, the nonlinear waveform distortion of output signals and the denoising capability of the filters can be compromised. This will make the application of quantum stochastic filters be greatly extended, such as in applying the filters to the processing of communication signals. The predominant performance of quantum stochastic filters is shown by simulation results.

Key words: communication signals processing; nonlinear filtering; quantum stochastic filters

The question of signal processing by means of nonlinear systems is attracting increasing attention[1], and this paper focuses on nonlinear filtering of communication signals. A filter characterized by randomly time-varying parameters is known as a stochastic filter, and the essence of stochastic filtering is the computation of the time-dependent probability density function for the state of an observed system by performing operations on a historical ensemble of observed data[2]. Quantum neurodynamics is a mathematical model for analyzing biological neural systems by quantum mechanics, and, in the past years, some research on nonlinear filtering based on quantum mechanics has been done[3–5]. Based on the previous work, this paper studies the model and performance of quantum stochastic filters (QSF) and a novel filtering algorithm is proposed. Simulation results show that the performance of QSF can be greatly improved by the algorithm. In contrast to the previous literature which has focused on speech signals and target tracking, this paper focuses on communication signals, and the filtering algorithm proposed is novel.

1 Model of QSF

1.1 Nonlinear Schroedinger equation

The nonlinear Schroedinger equation has a time-dependent form, which is a partial differential equation[6],

$$\mathrm{i}\hbar\frac{\partial\psi(x,t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x,t)\right]\psi(x,t) \tag{1}$$

where $\hbar$ is the Planck constant divided by 2π, i is the imaginary unit, m is the mass of the particle, and $V(x,t)$ is a spatial potential field with a nonlinear component. $\psi(x,t)$ is the wave function associated with the particle at space-time point (x,t); $|\psi(x,t)|^2$ represents a probability density function for the location of the particle in the vector space and can be expressed as

$$\int_{-\infty}^{+\infty}|\psi(x,t)|^2\mathrm{d}x=1 \tag{2}$$

1.2 Principle of QSF

The architecture of the proposed quantum stochastic filters is shown in Fig. 1, where y is the input signal, $\hat{y}$ is the estimate of the actual signal, and N is the number of neurons. The synaptic weights $K(x,t)$ are an $N\times 1$ dimensional vector, and their values are expressed by $W_j(1\leqslant j\leqslant N)$. $\hat{y}$ and the potential field $V(x,t)$ can be expressed as

$$\hat{y}(t)=\int x\rho(x,t)\mathrm{d}x \tag{3}$$

$$V(x,t)=\zeta(U(x,t)+G(|\psi|^2)) \tag{4}$$

$$U(x,t)=-K(x,t)y(t) \tag{5}$$

$$\rho(x,t)=|\psi(x,t)|^2 \tag{6}$$

$$G(|\psi|^2)=K(x,t)\int x\rho(x,t)\mathrm{d}x \tag{7}$$

where ζ is the weight coefficient of the potential field $V(x,t)$, and weight $K(x,t)$ is updated using the Heb-

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bian learning algorithm:

$$v(t) = y(t) - \hat{y}(t) \tag{8}$$

$$\frac{\partial K(x, t)}{\partial t} = \beta v(t) \rho(x, t) \tag{9}$$

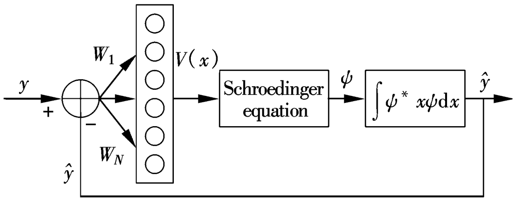

Fig. 1 The model of quantum stochastic filters

1.3 Integration of the Schroedinger wave equation

The Schroedinger wave equation is a partial differential equation in two variables: x and t, the equation can be converted to the finite difference form by dividing the x-axis into N mesh points so that x and t are represented as

$$x_j = j\Delta x \qquad -\frac{N-1}{2} \leqslant j \leqslant \frac{N-1}{2} \tag{10}$$

$$t_n = n\Delta t \qquad n \geqslant 0 \tag{11}$$

The finite-difference form of Eq. (1) is expressed as

$$\mathrm{i}\hbar \frac{\psi(x, t+\Delta t) - \psi(x, t)}{\Delta t} = -\frac{\hbar^2 \psi(x+\Delta x, t) - 2\psi(x, t) + \psi(x-\Delta x, t)}{2m\Delta x^2} + V(x, t)\psi(x, t) \tag{12}$$

2 Algorithm of Filtering

Eq. (1) involves four external parameters: $\hbar$, ζ, m, β. For simplicity, the parameter $\hbar$ is taken as unity, and the other three parameters are selected by optimization. It follows that quantum stochastic filters are near-optimal.

2.1 Optimizing parameters

Since the optimization of parameters ζ, m and β is a multivariate optimization problem, and the square error between output signals and reference signals is selected as the objective function. Here, we use a genetic algorithm (GA) based on the concept of the univariate marginal distribution algorithm (UMDA) [7]:

① Initialize population;

② Select better individuals;

③ Compute frequencies of gene values;

$$P_k = \frac{1}{n} \sum_{j=1}^{n} x_{jk} \qquad k = 1, 2, \ldots, l \tag{13}$$

where x_{jk} is the value of the k-th bit of the j-th individual, n is the number of the better individuals;

④ Generate new solutions;

⑤ If terminating conditions satisfy, then finish; otherwise, go to step ②.

2.2 Learning of neural network

After the parameters ζ, m, β are selected, a signal which is similar to the actual signal processed is selected as the training signal. In the processing of training, Eq. (12) is iteratively updated several times for the wave equation to reach a steady state to an instant sample. Since the neural network is a feedback network, weights $W_j (1 \leqslant j \leqslant N)$ must be normalized, and the probability density function $\rho(x, t)$ must also be normalized according to Eq. (2).

2.3 Filtering

The input signal is estimated by parameters ζ, m, β and $W_j (1 \leqslant j \leqslant N)$, and when compared with the previous algorithm in Ref. [5], the value of ζ can be adjusted for a tradeoff between the accuracy of the estimated signal amplitude and the capability of denoising of the filters. As far as the complexity of computation is concerned, the computation load in training and optimizaiton is heavy, but the load is small in filtering and it is $O(n)$.

3 Simulation Results

Parameters used for integration are selected as: $N = 401$, $\Delta x = 0.1$, $\Delta t = 0.001$. Since the simulation is focused on communication signals, a sinusoid signal with amplitude 1 is selected as a training signal. First, using GA to select the parameters, the obtained values are $\zeta = 2\,750$, $m = 0.168$, $\beta = 0.321\,8$, and the initial value of $P_k (1 \leqslant k \leqslant n)$ is 0.5. Next, fixing the parameters ζ, m, β, then weights $W_j (1 \leqslant j \leqslant N)$ are trained by the sinusoid signal, and the initial values of $W_j (1 \leqslant j \leqslant N)$ are selected randomly from $[-1, 1]$. Fig. 2 shows that the phase information can be resumed even though variance of noise is 4, and SINR (signal to interference noise rate) improvement of at least 11 dB can be achieved.

4 Control of Weight Coefficient ζ

In the above simulation, it is shown that ζ can be adjusted in a large range and the performance of the model can be improved by the control of ζ. The regulation of control is: the higher the value of ζ, the smaller the distortion of the output signal, but the capability of eliminating noise declines. Simulation results are shown in Fig. 3. From Fig. 3, it is shown that in the

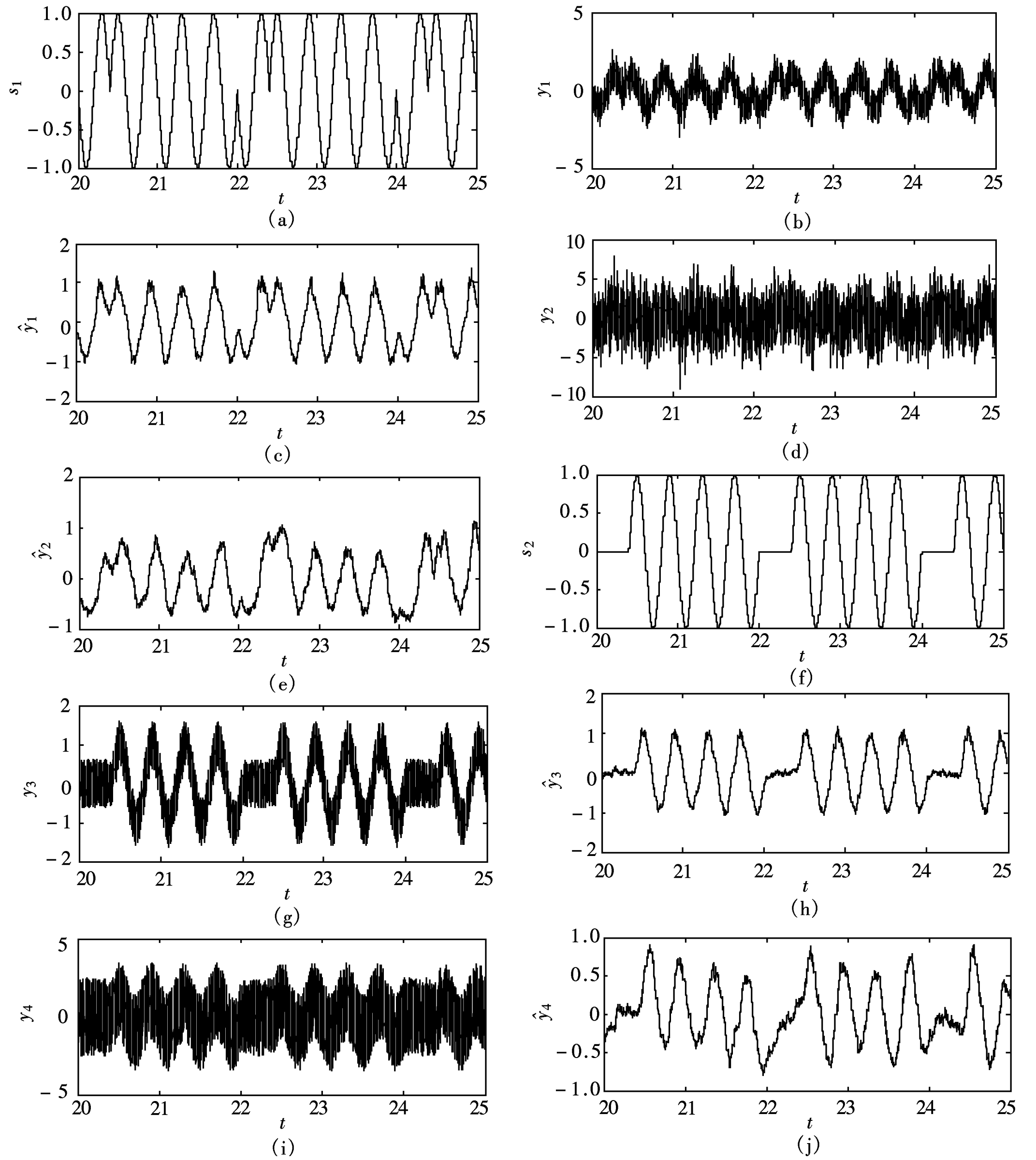

Fig. 2 Simulation results. (a) The actual signal s_1, which is a binary phase shifting keying(BPSK) signal; (b) y_1 is s_1 embedded in Gaussian noise with mean 0 and variance 0.25; (c) Filtering of $y_1(\zeta = 1\ 750)$; (d) y_2 is s_1 immersed in Gaussian noise with mean 0 and variance 4; (e) Filtering of $y_2(\zeta = 150)$; (f) The actual signal s_2, which is a pulse modulation signal; (g) y_3 is s_2 embedded in uniform distribution noise with mean 0 and variance 0.25; (h) Filtering of $y_3(\zeta = 1\ 750)$; (i) y_4 is s_2 immersed in uniform distribution noise with mean 0 and variance 4; (j) Filtering of $y_4(\zeta = 150)$

case of low noise, when $\zeta = 150$, the amplitude of output signal is around 0.8, and if $\zeta = 2\ 750$, the amplitude is around 1; in the case of high noise, when $\zeta = 150$, phase shift can be found in output signal, but if $\zeta = 2\ 750$, phase information cannot be resumed.

As the explanation of this phenomenon, one can find the answer from Eqs. (1) and (4). When ζ increases, wave function $\psi(x, t)$ becomes more sensitive to the change of $v(t)$, but if ζ decreases, $\psi(x, t)$ will be less sensitive.

5 Conclusion

Combining quantum mechanics with neuroscience, quantum stochastic filters can resume the signals contaminated by heavy noise. Although the computation complexity is great in training and optimization, it is small in filtering, and the filters also exhibit good performance when the nature of noise is unknown. Moreover, the capability of the filters will be much more predominant by adjusting ζ, and these advantages will make the proposed filters applicable to the filtering of communication signals.

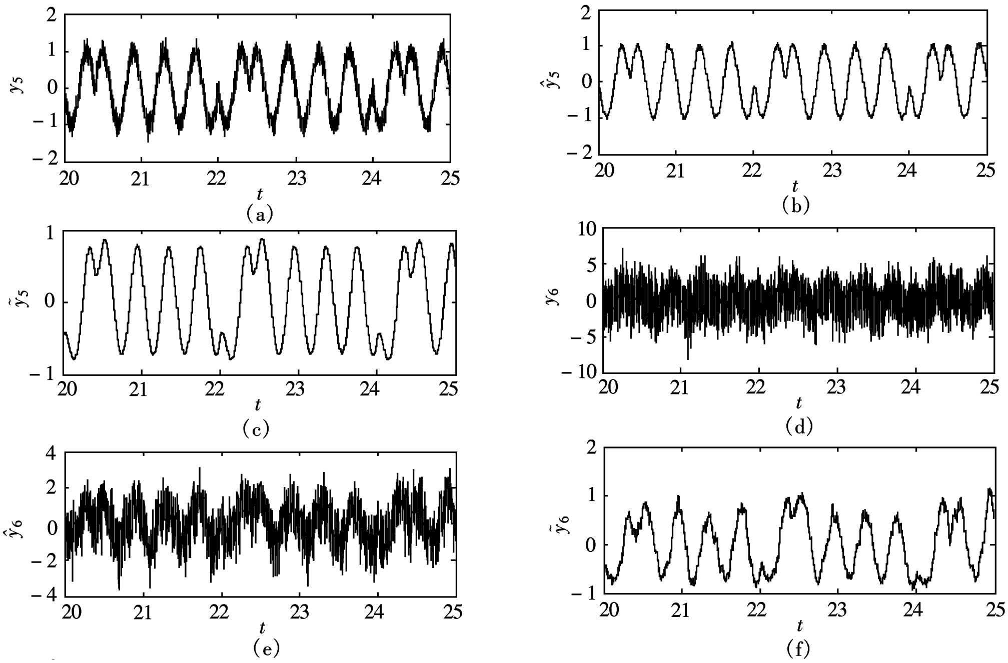

Fig. 3 Control of weight coefficient. (a) y_5 is s_1 embedded in Gaussian noise with mean 0 and variance 0.015 6; (b) Filtering of $y_5(\zeta = 2\,750)$; (c) Filtering of $y_5(\zeta = 150)$; (d) y_6 is s_1 immersed in Gaussian noise with mean 0 and variance 3.15; (e) Filtering of $y_6(\zeta = 2\,750)$; (f) Filtering of $y_6(\zeta = 150)$

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用于通信信号非线性时域滤波的量子随机滤波器

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摘要:针对通信信号的非线性时域滤波问题,研究了量子随机滤波器的原理和性能. 将神经网络与非线性 Schroedinger 方程相结合,把方程的解作为信号时变的概率密度函数,进而实现滤波功能. 研究发现,通过调整势场权系数的取值,可使滤波器具有明显不同的性能. 根据此性质,构造了一种新的滤波算法,该算法可使滤波器在信号波形估计的非线性失真程度与它的抗噪能力之间进行折衷,这将大大推广量子随机滤波器的应用,例如,用于通信信号处理. 仿真结果表明了量子随机滤波器的优越性能.

关键词:通信信号处理;非线性滤波;量子随机滤波器

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