Journal of Southeast University (English Edition) Vol. 23, No. 1, pp. 92 – 97 Mar. 2007 ISSN 1003—7985

Probability evaluation method for cable safety of long-span cable-stayed bridges

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Abstract: A method of cable safety analysis is proposed for safety evaluation of long-span cable-stayed bridges. The Daniels' effect and the probability of broken wires in the cable are introduced to develop the cable strength model and the reliability assessment technique for long-span cable-stayed bridges based on the safety factors analysis of stay cables in service. As an application of the proposed model, the cable safety reliability of the cable No. 25 of Zhaobaoshan cable-stayed bridge in China is calculated. The effects of various parameters on the estimated cable safety reliability are investigated. The results indicate that the proposed method can be used to assess the safety level of stay cables in cable-stayed bridges effectively. The Daniels' effect should be taken into account for assessment, and the probability of broken wires can be used to simulate the deterioration of stay cables in service.

Key words: safety factor; probability; evaluation; cable-stayed bridge

Stay cables are the most important structural components of cable-stayed bridges, therefore, ensuring their safety under different load conditions is of great importance to engineers. Uncertainties which arise from variations in geometric properties (cross-sectional properties and dimensions), material mechanical properties (modulus and strength, etc.), load magnitude and distribution cannot be incorporated within the evaluation of stay cable safety levels. Probability analysis provides the tool for incorporating structural modeling uncertainties in the evaluation of the cable safety factor by describing the uncertainties as random variables.

Haight et al. [1] computed the cable safety factors for the main cables of four suspension bridges by the probabilistic method, and type Ⅰ extreme value distribution was used to estimate the true number of brittle wires in main cables. Cremona[2] presented a probabilistic approach for cable residual strength assessment. The approach is applied to the Tancarville suspension main cables by taking into account tensile test results, inspection data and weight-in-motion records. Faber et al. [3] gave an overview of reliability based assessment of parallel wire cables, both for design and assessment of cables in existing structures. Cheng et al. [4] applied the inverse reliability method to estimate cable safety factors of suspension bridges satisfying prescribed reliability.

Although the probabilistic approach has been found to have some applications in design and assessment for the main cables of suspension bridges, less work has so far been presented regarding the evaluation of stay cable safety factors of cable-stayed bridges. In the original study by the authors, a probabilistic cable strength model is proposed based on the work done by other researchers in estimating the cable safety factors of a long span pre-stressed concrete cable-stayed bridge. The variations of constitute law of wires, the reduction of ultimate stay tensile capacity due to the so-called length effect as well as the reduction due to the large number of individual wires working in parallel were taken into account in the approach. Whereas, the randomness of structural geometry, service loads conditions etc. were neglected. Therefore, the proposed paper attempts to present a framework for the assessment of cable safety factors of existing long span cable-stayed bridges in the framework of statistics. For this purpose, this paper first presents a deterministic model of cable safety factors and the analysis of its probabilistic properties. Then, the proposed approach is applied to evaluate the cable safety factors of an existing long span cable-stayed bridge in China.

Received 2006-08-07.

Foundation items: The Opening Fund of the Key Laboratory of Urban Security and Disaster Engineering of Ministry of Education (No. EESR200701), the Opening Fund of Beijing Laboratory of Earthquake Engineering and Structural Retrofit.

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1 Probability Properties of Cable Safety Factors

1.1 Deterministic model of cable safety factors

Traditionally, the cable safety factor γ is obtained as the ratio of the cable ultimate tension-carrying capacity T_u over the maximum cable tension due to dead and live loads T_s, namely,

$$\gamma = \frac{T_u}{T_s} \tag{1}$$

The cable strength T_u can be expressed as

$$T_u = A_c \sigma_c \tag{2}$$

where A_c is the cross-section area of the stay cable, and σ_c is the rupture strength of the stay cable.

The maximum cable tension due to dead and live loads is given as

$$T_s = T_D + T_L \tag{3}$$

where T_D is the cable tension due to dead loads. The stay cable carried the self-weight and the second dead loads of the structure, T_D should be designed according to the optimal method. The diversity of designed T_D and operational cable force is inevitable, therefore, T_D is gained from a field test such as the ambient vibration test. An optimal extrapolation method has been used to obtain cable force due to traffic load T_L in Ref. [5].

1.2 Cable strength model

The assessment of the required and sufficient safety of stay cables was based on many structural reliability analyses with a detailed modeling of the physical behavior of parallel wire cables.

Cables are represented in the present study by a model illustrated in Fig. 1. Each cable strand is considered to be composed of n individual parallel wires of the same diameter and length, and each wire is formed by m segments in series. The model includes a reduction of the ultimate stay tensile capacity due to the so-called length effect as well as a reduction due to the large number of individual wires working in parallel, the so-called Daniels' effect[6]. An arbitrary number of broken wires in the cable due to deterioration or construction errors is taken into account in the present study.

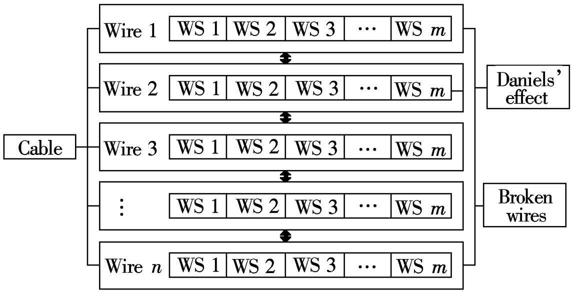

Fig. 1 Cable modeling (WS: wire segment)

1.2.1 Wire segment modeling

For obvious technical reasons, tensile tests are performed on wire segments. These segments represent the elementary length unit of a cable model. From a mechanical point of view, wire segments have intrinsic properties expressed in terms of idealized or simplified strain-stress constitutive laws. Different behaviors can be found in the literature, such as the perfect elastic, the elastic-brittle, the elastic-plastic, and the inelastic laws, etc[7-9].

In this paper, a simplified two-stage linear law is applied to simulate the behavior of wire segments according to the results of tests in the library. The first stage represents the pre-yield linear strain-stress relationship, and the second line represents the post-yield nonlinear behavior. This model is interesting because it introduces only four variables E, ε_e, ε_u, σ_u, as shown in Fig. 2. Tab. 1 provides some test results for two one-meter-length wire segments of two different stay cables from Zhaobaoshan cable-stayed bridge, respectively.

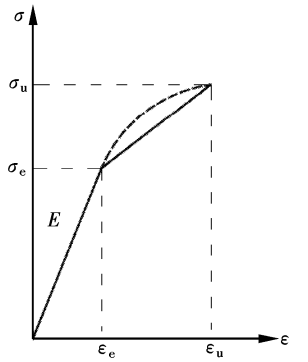

Fig. 2 Strain-stress relationship of wire segment

Tab. 1 Test results of wire segments

Number	Diameter/mm	Area/mm^2	Ultimate strength/GPa	Yield strength/MPa	Elongation percentage/%	Elastic modulus/GPa
C21-1	7	38.48	1710	1510	4.0	200
C22-1	7	38.48	1730	1550	1.5	200

A total of four random variables are used in the present model, namely E, ε_e, ε_u and σ_u. Due to the limited data used here, the correlation of these random variables cannot be estimated reliably. The Monte Carlo simulation is used to generate the samples. Therefore, the choice of variable distributions is also vital. The lognormal distribution has been chosen as Cremona[2] recommended for the Tancarville suspension bridge.

The proposed approach is to generate random samples. Each variable is described by a statistical distribution for which the parameters are fitted on experimental data from tensile tests in specific structures.

1.2.2 Wire modeling

A wire is simply modeled as a series of a system composed of wire segments. The wire strength is therefore the strength corresponding to the weakest segment. Intuitively, if the wire and segment lengths are respectively L and L_0, $m = L/L_0$ segments have to be simulated to constitute a wire.

An applied stress σ at the wire ends produces deformations (or displacements) different for all the segments. The problem is therefore reduced to a services problem. From a geometrical point of view, this problem consists of adding the m strains ε_j, for the specific stress σ. The latter should not exceed the minimum of the ultimate individual segment strengths σ_j^u:

$$\varepsilon_{\text{wire}}(\sigma_{\text{wire}}) = \frac{1}{m}\sum_{j=1}^{m}\varepsilon_j(\sigma_{\text{wire}})$$
$$0 \leqslant \sigma_{\text{wire}} \leqslant \sigma_{\max} = \min_j(\sigma_j^u) \tag{4}$$

From the above formula the wire constitutive law $\sigma_{\text{wire}} = f_{\text{wire}}(\varepsilon)$.

1.2.3 Cable modeling

1) The Daniels' model for the strength of a parallel wire cable

The strength of a parallel system with n components may, if n is large enough ($n > 150$), be shown to be normally distributed with mean value

$$\mu_n = nx_0[1 - F_z(x_0)] + c_n \tag{5}$$

and standard deviation

$$\sigma_n = x_0\{nF_z(x_0)[1 - F_z(x_0)]\}^{\frac{1}{2}} \tag{6}$$

where c_n may be assessed from

$$c_n = 0.966n^{\frac{1}{3}}a \tag{7}$$

and

$$a^3 = \frac{f_z^2(x_0)x_0^4}{2f_z(x_0) + x_0 f_z'(x_0)} \tag{8}$$

where $f_z(x_0)$ is the density function for the wire strength. The parameter x_0 is the solution of

$$x_0 = \max\{x[1 - F_z(x)]\} \tag{9}$$

provided that there is $f_z(x) = 0$ and $[1 - F_z(x)] \to 0$. A correction for the standard deviation can also be given but it is usually not necessary. In particular, if z is Weibull-distributed, the parameter x_0 may be determined from

$$x_0 = \left[l\frac{L_0}{Lk}\right]^{\frac{1}{k}} u \tag{10}$$

c_n may be considered as a correction term to the asymptotic solution (which is valid for large n).

For illustrating the Daniels' effect, an intact 100 meters long cable with 200 wires is analyzed by Faber et al. [3] The systematic reduction of the cable mean strength as a function of the number of wires in the cable is illustrated in Fig. 3(a) with the parameters $u = 1\,788.7$, $k = 72.62$ and $\lambda = 3$. The exponential attenuation of the mean strength to the number of wires for the same cable is gained by the Monte-Carlo simulation, as shown in Fig. 3(b).

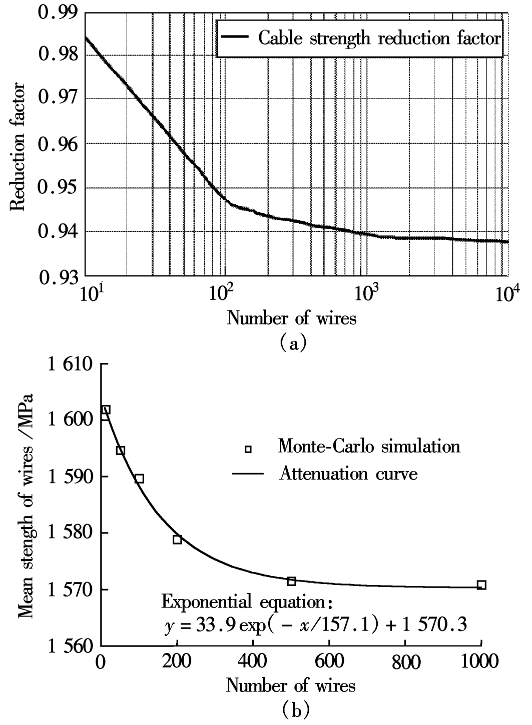

Fig. 3 Illustration of the Daniels' effect. (a) Ref. [3]; (b) Monte-Carlo simulation

The reduction factor decreases when the number of wires increases. The decreasing rate is more distinct when $n < 100$. However, as the mean value of the strength of the cable decreases, the standard deviation decreases even faster. Even for a moderate number of wires ($n > 100$) the strength of a parallel wire bundle can be considered to be deterministic. Together the two effects are often referred to as the Daniels' effect. A more straightforward attenuation curve is illustrated in Fig. 3(b) by the authors. Therefore, the Daniels' effect for the stay cables with the number of wires from 109 to 301 in Zhaobaoshan cable-stayed bridge cannot be neglected in the present study.

2) Broken wires in the cable

The wires breaking for a variety of reasons makes the topic more complex. In this paper the "broken wires" is described as the probability of broken wires in the cable P_f^w. P_f^w is defined as the ratio of the number of broken wires over the total number of the wires, and this is assumed to have a binomial distribution. In theory, the distribution function parameters of P_f^w can be gained from the results of non-damage identification in practical stay cables statistically. But up to now few detection techniques can fulfill it actually, so a simplified formula is applied to estimate P_f^w, namely,

$$P_f^w = \frac{\tilde{n}}{n} = P_0^w + T\zeta \tag{11}$$

where P_0^w is the connatural probability of broken wires due to molding in the factory or for many sorts of ambiguous reasons before the cables are fixed on the bridges, and is arbitrarily taken to be 1% in the present study. T is the period of service of stay cables, which is taken to be a maximum of 30 years. ζ is the rate of wires breaking, which is dependent on the environment and load conditions. According to the actual standard, $P_f^w = 5\%$ is defined as the failure criterion, namely, the cable is required to be replaced when 5% of the wires are broken.

3) The cable strength model

The constitutive law of the cable residual strength can be deduced by taking into account the Daniels' effect and the probability of broken wires as follows:

$$F_{\text{cable}} = \sum_{l=1}^{\tilde{n}} f_{\text{wire},l}(\varepsilon) A_l \qquad 0 \leqslant \varepsilon \leqslant \varepsilon_{\max} \tag{12a}$$

$$\varepsilon_{\max} = \max_{\text{wire}} [\varepsilon_{\max} = f_{\text{wire}}^{-1}(\sigma_{\text{wire, max}})] \tag{12b}$$

where $\tilde{n}$ is the number of unbroken wires in the cable section. Monte-Carlo simulations are sufficient to calculate the mean cable strength value, and the cumulative distribution function of the mean cable strength can also be defined.

1.3 Cable force due to service loads

The ambient vibration method is applied to measure the cable forces of existing cable-stayed bridges in the service stage. The cable force due to dead loads is gained by ambient vibration measurement with the exiguous traffic flow on the bridges, and the free hours are generally suitable for the measurement.

The experimental assessment of cable forces based on the vibrating chord theory, as shown in Eq. (13), has been successfully applied to cable-stayed bridges:

$$T_D = \frac{4w}{\varphi^2 g}(f_n L)^2 \tag{13}$$

where φ is the φ-th vibration mode of cable; f_φ is the frequency of the φ-th mode; L is the effective cable length; and w is the weight per unit length of the cable.

Bounding conditions, environment conditions, geometric properties etc. are the main factors which influence the frequencies of cables. A detailed analysis is conducted to gain the exact cable forces.

The cable force due to traffic load can be simplified as the given ratio to the cable force due to dead load, namely as $T_L/T_D = \eta$. The randomness of the load effect due to traffic conditions should be taken into account.

The mean values of cable force are gained from ambient vibration, and the coefficient of variation is taken to 0.02 according to the comparative studies by Chang et al. [10]

2 Application to Long Span Cable-Stayed Bridges

2.1 Reliability analysis

Zhaobaoshan bridge is located at the entrance of the Yong River to the Eastern Sea in Ningbo, crossing Zhaobao mountain in the district of Zhenhai and Jinji mountain in the district of Beilun. The bridge is a single-tower and double-cable plane pre-stressed concrete cable-stayed bridge. The main span is 258 m in length. The main tower is 148.4 m in height. There are 102 cables arranged upstream and downstream, respectively. Zhaobaoshan bridge opened to traffic on June 8, 2001. The general view of Zhaobaoshan cable-stayed bridge is illustrated in Fig. 4.

A four-year periodic inspection project has been conducted on the bridge, and the cable force records per six months has been applied to analyze the safety factors of the stay cables. In Chinese design code for cable-stayed bridges, 2.5 will be the design safety

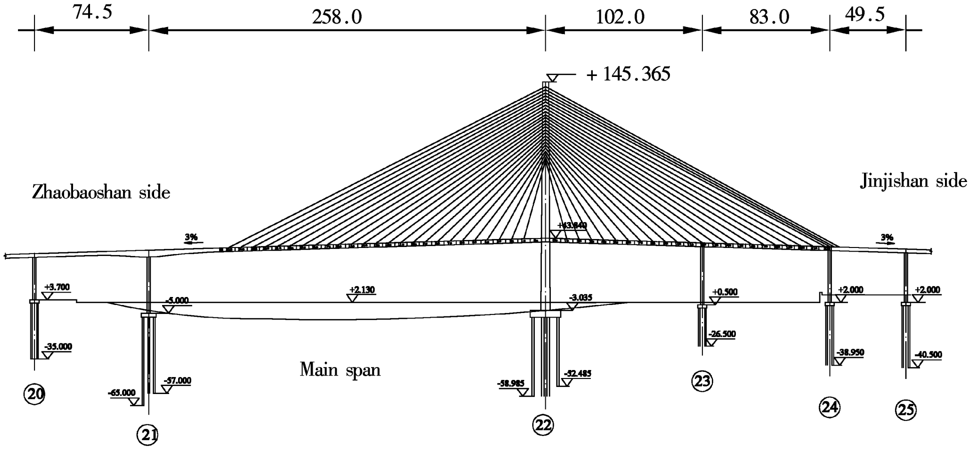

Fig. 4 General view of Zhaobaoshan cable-stayed bridge

factor of stay cables, so the limited state function is named as

$$f(A_w, \sigma_u, T_s, \eta) = \frac{\tilde{n} A_w \sigma_u}{(1+\eta) T_D} - 2.5 \qquad (14)$$

where A_w is the area of a wire in the cable, σ_u is the mean strength of the wires taking into account the Daniels' effect, T_D is the cable force due to dead load in service stage. The mean value of η is gained from the analysis in Ref. [11]. A_w, σ_u and T_s are assumed to be independent and have a standard normal distribution. The random variables and statistical properties for cable No. 25 is presented in Tab. 2.

Tab. 2 Statistical properties of random variables for cable No. 25

Random variable	Distribution type	Mean value	Variation coefficient
A_w/mm^2	Normal	38.5	0.05
σ_u/MPa	Normal	1620	0.05
T_D/kN	Normal	—*	0.02
η	Normal	0.025	0.05

Note: —* the mean values were gained from periodical detection, refer to Ref. [11].

The Monte-Carlo method is used to calculate the reliability indices. The results are illustrated in Fig. 5. From the figure, it can be seen that the reliability index of cable No. 25 has smooth change in different detection dates, other than the reliability index of upriver, cable No. 25 has a sharp decrease at the last detection, which causes the field detection engineers to pay more attention to the cable.

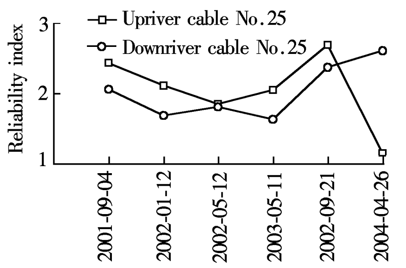

Fig. 5 Reliability indices of cable No. 25 in Zhaobaoshan Bridge

2.2 Sensitivity analysis

An important step in the structural reliability analyses is the sensitivity analysis of reliability indices. This helps to identify important parameters. The sensitivity analysis has been conducted regarding this problem. The results are listed in the following sections.

2.2.1 Effect of coefficient of variation of the cable force

The measured errors of cable force in the service stage is unavoidable. By changing the coefficient of variation of the cable force from 0.01 to 0.05, the reliability index of the safety is computed using the proposed algorithm. The results are plotted in Fig. 6(a). It can be seen that the reliability index of the safety of cable No. 25 decreases slightly as the measured error of cable force increases. The results indicate that the precision of cable force measurement has a major effect on the safety evaluation of cables.

2.2.2 Effect of probability of broken wires

Since the probability of broken wires P_f^w in the cable is the vital parameter influence on the ultimate strength of the cable, various values of P_f^w are considered for the considered parameter, i. e. six values are used: 0, 0.01, 0.02, 0.03, 0.04, 0.05. The variations of the reliability indices are shown in Fig. 6(b). It can be seen that the reliability index decreases as the probability of broken wires in the cable increases.

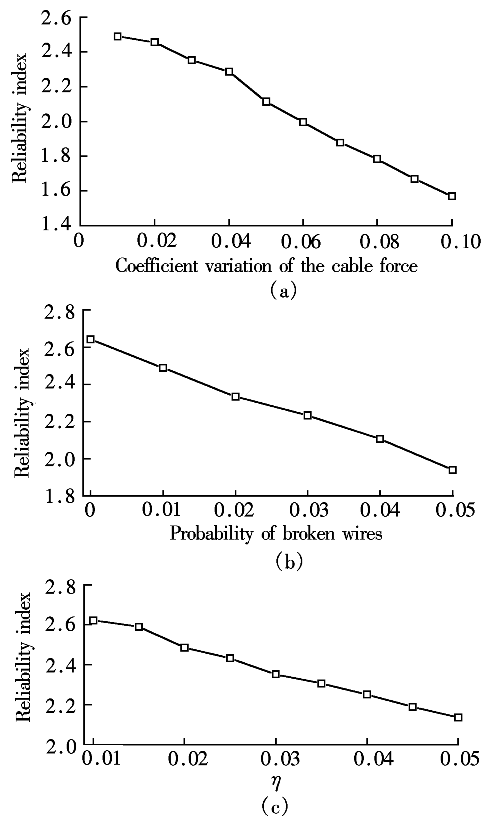

Fig. 6 Sensitivity of reliability indices with respect to (a) coefficient of variation of the cable force; (b) probability of broken wires; (c) ratio of the cable force due to live load to dead load

2.2.3 Effect of the ratio of the cable force due to live load to dead load

By changing the ratio of the cable force due to live load to dead load, the reliability indices are computed using the proposed algorithm. The reliability indices are compared in Fig. 6(c). As the ratio of the cable force due to live load to dead load increases, the estimated cable safety indices decrease. The results indicate that the traffic load is also a major influence factor on the safety assessment of stay cables.

3 Conclusions

1) Based on the presented models it is found that the strength of the stay cables may effectively be treated as random variables. A more reasonable reliability-based method is proposed for evaluating the safety level of cables for cable-stayed bridges. The Daniels' effect, i. e., the effect of having a large number of wires in the cable is further verified in the study.

2) The probability of broken wires in the cable is assumed linearly increasing during service, which should be an effective and simplified approach to take into account the effect of the deterioration mechanism of fatigue and/or corrosion.

3) The detection results of cable force of cable-stayed bridges during service are integrated with the proposed method to rate the performance of stay cables quantitatively.

This paper provides an improved understanding of the cable safety assessment of cable-stayed bridges. However, the target reliability index should be studied in the future for assessment of the practical safety level of the stay cables of bridges.

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大跨度斜拉桥拉索安全评估的概率方法

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摘要:为了对运营期斜拉桥安全性进行评估,提出了评估斜拉索安全性的方法. 考虑 Daniels 效应和断丝概率提出斜拉索强度模型与基于斜拉索运营期安全系数的大跨度斜拉桥可靠度评估方法. 作为模型的应用验证,分析了宁波招宝山斜拉桥最长边索,即第 25 号索在不同检测期的安全性,并调查了不同的参数对安全概率的影响. 分析结果表明:斜拉索 Daniels 效应不可忽略;运营期斜拉索的退化可以通过线性增长的索内断丝率来模拟;考虑斜拉桥拉索退化过程的随机性评估运营期斜拉桥拉索安全水平的方法是有效的.

关键词:安全系数;概率;评估;斜拉桥

中图分类号:U447