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On similarity measures of interval-valued intuitionistic fuzzy sets and their application to pattern recognitions

Xu Zeshui

(School of Economics and Management, Southeast University, Nanjing 210096, China)

Abstract: The concept of the degree of similarity between interval-valued intuitionistic fuzzy sets (IVIFSs) is introduced, and some distance measures between IVIFSs are defined based on the Hamming distance, the normalized Hamming distance, the weighted Hamming distance, the Euclidean distance, the normalized Euclidean distance, and the weighted Euclidean distance, etc. Then, by combining the Hausdorff metric with the Hamming distance, the Euclidean distance and their weighted versions, two other similarity measures between IVIFSs, i. e., the weighted Hamming distance based on the Hausdorff metric and the weighted Euclidean distance based on the Hausdorff metric, are defined, and then some of their properties are studied. Finally, based on these distance measures, some similarity measures between IVIFSs are defined, and the similarity measures are applied to pattern recognitions with interval-valued intuitionistic fuzzy information.

Key words: interval-valued intuitionistic fuzzy set; similarity; pattern recognition

In Ref. [1], Atanassov generalized the notion of Zadeh's fuzzy set[2] to the notion of the intuitionistic fuzzy set, which is characterized by a membership function and a non-membership function. Atanassov and Gargov[3] introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the intuitionistic fuzzy set. The fundamental characteristics of IVIFS are that the values of its membership function and non-membership are intervals rather than exact numbers. Atanassov[4] defined some operations, relations and operators concerning IVIFSs. Bustince and Burillo[5] introduced the concepts of correlation and correlation coefficient of IVIFSs, and introduced two decomposition theorems of the correlation of IVIFSs. Hong[6] generalized the concepts of correlation and correlation coefficients of IVIFSs in a general probability space and also introduced some decomposition theorems of the correlation of IVIFSs in terms of the correlation of interval-valued fuzzy sets and the entropy of intuitionistic fuzzy sets. Hung and Wu[7] proposed a method for calculating the correlation coefficient of IVIFSs by means of "centroid". Xu[8] also developed a method for deriving the correlation coefficients of IVIFSs. The prominent characteristic of the method is that it can guarantee that the correlation coefficient of any two IVIFSs equals one if and only if these two IVIFSs are the same, and can relieve the influence of the unfair arguments on the final results. Mondal and Samanta[9] defined a topology of IVIFSs and studied some topological properties. Deschrijver and Kerre[10] established the relationships between intuitionistic fuzzy sets, L-fuzzy sets, interval-valued fuzzy sets and IVIFSs.

Similarity measures are very useful in many fields, such as pattern recognition, machine learning, decision making and market prediction[11]. Recently, many similarity measures have been proposed for measuring the degree of similarity between intuitionistic fuzzy sets[11-16]; however, all these measures cannot deal with the similarity measures between IVIFSs. Therefore, it is necessary to pay attention to this issue. In this paper, we define some similarity measures of IVIFSs on the bases of the Hausdorff distance, the Euclidean distance, and the Hamming distance, etc., and investigate some properties of these measures. Furthermore, we give the application of the similarity measures to pattern recognitions under interval-valued intuitionistic fuzzy information.

1 Similarity Measures between IVIFSs

Let X be a universe of discourse. An IVIFS A over X is an object having the form[3]:

$$A=\{\langle x,\tilde{\mu}_A(x),\tilde{v}_A(x)\rangle \mid x\in X\} \tag{1}$$

where $\tilde{\mu}_A(x)\subset[0,1]$ and $\tilde{v}_A(x)\subset[0,1]$ are intervals, and for every $x\in X$:

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Biography: Xu Zeshui (1968—), male, doctor, professor, xu_zeshui@263.net.

$$\sup \tilde{\mu}_A(x) + \sup \tilde{v}_A(x) \leqslant 1 \tag{2}$$

Especially, if $\mu_A(x) = \inf \tilde{\mu}_A(x) = \sup \tilde{\mu}_A(x)$ and $v_A(x) = \inf \tilde{v}_A(x) = \sup \tilde{v}_A(x)$, then, the given IVIFS A is reduced to an ordinary intuitionistic fuzzy set.

For every two IVIFSs $A = \{\langle x, \tilde{\mu}_A(x), \tilde{v}_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \tilde{\mu}_B(x), \tilde{v}_B(x)\rangle \mid x \in X\}$, the following two relations are defined[3]:

1) $A \subset B$ if and only if $(\forall x \in X)$ $\sup \tilde{\mu}_A(x) \leqslant \sup \tilde{\mu}_B(x)$ & $\inf \tilde{\mu}_A(x) \leqslant \inf \tilde{\mu}_B(x)$ & $\sup \tilde{v}_A(x) \geqslant \sup \tilde{v}_B(x)$ & $\inf \tilde{v}_A(x) \geqslant \inf \tilde{v}_B(x)$;

2) $A = B$ if and only if $A \subset B$ & $B \subset A$.

In the following, we introduce the concept of the degree of similarity between IVIFSs:

Definition 1 Let s be a mapping s: IVIFS$(X) \times$ IVIFS$(X) \to [0, 1]$, and let $A \in$ IVIFS(X) and $B \in$ IVIFS(X), then $s(A, B)$ is called the degree of similarity between A and B, if the following properties are satisfied: ① $0 \leqslant s(A, B) \leqslant 1$; ② $s(A, B) = 1$ if and only if $A = B$; ③ $s(A, B) = s(B, A)$; ④ If $A \subset B \subset C$, $C \in$ IVIFS(X), then $s(A, C) \leqslant s(A, B)$ and $s(A, C) \leqslant s(B, C)$.

For convenience, from here we will assume that X is finite, i. e., $X = \{x_1, x_2, \ldots, x_n\}$.

Let $A = \{\langle x_j, \tilde{\mu}_A(x_j), \tilde{v}_A(x_j)\rangle \mid x_j \in X\}$ and $B = \{\langle x_j, \tilde{\mu}_B(x_j), \tilde{v}_B(x_j)\rangle \mid x_j \in X\}$ be two IVIFSs, and let $\tilde{\mu}_A(x_j) = [\tilde{\mu}_A^{\mathrm{L}}(x_j), \tilde{\mu}_A^{\mathrm{U}}(x_j)]$, $\tilde{\mu}_B(x_j) = [\tilde{\mu}_B^{\mathrm{L}}(x_j), \tilde{\mu}_B^{\mathrm{U}}(x_j)]$, $\tilde{v}_A(x_j) = [\tilde{v}_A^{\mathrm{L}}(x_j), \tilde{v}_A^{\mathrm{U}}(x_j)]$, $\tilde{v}_B(x_j) = [\tilde{v}_B^{\mathrm{L}}(x_j), \tilde{v}_B^{\mathrm{U}}(x_j)]$, where $\tilde{\mu}_A^{\mathrm{L}}(x_j) = \inf \tilde{\mu}_A(x_j)$, $\tilde{\mu}_A^{\mathrm{U}}(x_j) = \sup \tilde{\mu}_A(x_j)$, $\tilde{v}_A^{\mathrm{L}}(x_j) = \inf \tilde{v}_A(x_j)$, $\tilde{v}_A^{\mathrm{U}}(x_j) = \sup \tilde{v}_A(x_j)$, $\tilde{\mu}_B^{\mathrm{L}}(x_j) = \inf \tilde{\mu}_B(x_j)$, $\tilde{\mu}_B^{\mathrm{U}}(x_j) = \sup \tilde{\mu}_B(x_j)$, $\tilde{v}_B^{\mathrm{L}}(x_j) = \inf \tilde{v}_B(x_j)$, $\tilde{v}_B^{\mathrm{U}}(x_j) = \sup \tilde{v}_B(x_j)$, $x_j \in X$, then based on the geometric interpretation of the IVIFS, we define the following distances for A and B:

1) The Hamming distance:

$$d_1(A, B) = \frac{1}{4}\sum_{j=1}^{n}\left[\,|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)| + |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)| + |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)| + |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j)|\,\right] \tag{3}$$

2) The normalized Hamming distance:

$$d_2(A, B) = \frac{1}{4n}\sum_{j=1}^{n}\left[\,|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)| + |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)| + |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)| + |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j)|\,\right] \tag{4}$$

3) The Euclidean distance:

$$d_3(A, B) = \sqrt{\frac{1}{4}\sum_{j=1}^{n}\left[(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2 + (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2 + (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2 + (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j))^2\right]} \tag{5}$$

4) The normalized Euclidean distance:

$$d_4(A, B) = \sqrt{\frac{1}{4n}\sum_{j=1}^{n}\left[(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2 + (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2 + (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2 + (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j))^2\right]} \tag{6}$$

The Hausdorff metric[17] is a measure of how many two non-empty compact (closed and bounded) sets. In a metric space resemble each other with respect to their positions, which can be defined as follows:

Definition 2[15] Let $\tilde{\alpha}_1 = [a_1, b_1]$ and $\tilde{\alpha}_2 = [a_2, b_2]$ be any two intervals, then the Hausdorff distance $H(\tilde{\alpha}_1, \tilde{\alpha}_2)$ is given by

$$H(\tilde{\alpha}_1, \tilde{\alpha}_2) = \max\{|a_1 - b_1|, |a_2 - b_2|\} \tag{7}$$

Grzegorzewski[14], and Hung and Yang[15] applied the Hausdorff metric to the similarity measures between intuitionistic fuzzy sets, in the following, we define some similarity measures between IVIFSs combining the Hausdorff metric with the Hamming distance, the Euclidean distance and their normalized versions.

1) The Hamming distance based on the Hausdorff metric:

$$d_5(A, B) = \sum_{j=1}^{n}\max\{|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)|, |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|, |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)|, |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|\} \tag{8}$$

2) The normalized Hamming distance based on the Hausdorff metric:

$$d_6(A, B) = \frac{1}{n}\sum_{j=1}^{n}\max\{|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)|, |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|, |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)|, |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|\} \tag{9}$$

3) The Euclidean distance based on the Hausdorff metric:

$$d_7(A, B) = \sqrt{\sum_{j=1}^{n}\max\{(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2, (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2, (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2, (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2\}} \tag{10}$$

4) The normalized Euclidean distance based on the Hausdorff metric:

$$d_8(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n} \max\{(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2, (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2, (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2, (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2\}} \quad (11)$$

However, the elements in the universe may have differences of importance in pattern recognition. Therefore, we need to take the weights of the elements $x_j (j = 1, 2, \ldots, n)$ into account. In the following, we develop some weighted distance measures between IVIFSs.

Let $\boldsymbol{w} = \{w_1, w_2, \ldots, w_n\}^{\mathrm{T}}$ is the weight vector of the elements $x_j (j = 1, 2, \ldots, n)$, then, we have

1) The weighted Hamming distance:

$$d_9(A,B) = \frac{1}{4}\sum_{j=1}^{n} w_j[\,|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)| + |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)| + |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)| + |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j)|\,] \quad (12)$$

If $\boldsymbol{w} = \{1/n, 1/n, \ldots, 1/n\}^{\mathrm{T}}$, then Eq. (12) is reduced to the normalized Hamming distance Eq. (4).

2) The weighted Euclidean distance:

$$d_{10}(A,B) = \sqrt{\frac{1}{4}\sum_{j=1}^{n} w_i[(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2 + (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2 + (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2 + (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j))^2]} \quad (13)$$

If $\boldsymbol{w} = \{1/n, 1/n, \ldots, 1/n\}^{\mathrm{T}}$, then Eq. (13) is reduced to the normalized Euclidean distance Eq. (6).

3) The weighted Hamming distance based on the Hausdorff metric:

$$d_{11}(A,B) = \sum_{j=1}^{n} w_j \max\{|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)|, |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|, |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)|, |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|\} \quad (14)$$

If $\boldsymbol{w} = \{1/n, 1/n, \ldots, 1/n\}^{\mathrm{T}}$, then Eq. (14) is reduced to the normalized Hamming distance based on the Hausdorff metric Eq. (9).

4) The weighted Euclidean distance based on the Hausdorff metric:

$$d_{12}(A,B) = \sqrt{\sum_{j=1}^{n} w_j \max\{(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2, (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2, (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2, (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2\}} \quad (15)$$

If $\boldsymbol{w} = \{1/n, 1/n, \ldots, 1/n\}^{\mathrm{T}}$, then Eq. (15) is reduced to the normalized Euclidean distance based on the Hausdorff metric Eq. (11).

Similar to the distance measures between intuitionistic fuzzy sets[11-16], it can be easily proven that all the above distance measures satisfy the following properties: ① $d(A, B) \geqslant 0$; ② $d(A, B) = 0$ if and only if $A = B$; ③ $d(A, B) = d(B, A)$; ④ If $A \subset B \subset C$, $C \in \mathrm{IVIFS}(X)$, then $d(A, C) \geqslant d(A, B)$ and $d(A, C) \geqslant d(B, C)$.

Based on Eqs. (12)-(15), we define some similarity measures of A and B as follows:

$$s_1(A,B) = 1 - \frac{1}{4}\sum_{j=1}^{n} w_j[\,|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)| + |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)| + |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)| + |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j)|\,] \quad (16)$$

$$s_2(A,B) = 1 - \sqrt{\frac{1}{4}\sum_{j=1}^{n} w_i[(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2 + (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2 + (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2 + (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{v}_B^{\mathrm{U}}(x_j))^2]} \quad (17)$$

$$s_3(A,B) = 1 - \sum_{j=1}^{n} w_j \max\{|\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j)|, |\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|, |\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j)|, |\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j)|\} \quad (18)$$

$$s_4(A,B) = 1 - \sqrt{\sum_{j=1}^{n} w_j \max\{(\tilde{\mu}_A^{\mathrm{L}}(x_j) - \tilde{\mu}_B^{\mathrm{L}}(x_j))^2, (\tilde{\mu}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2, (\tilde{v}_A^{\mathrm{L}}(x_j) - \tilde{v}_B^{\mathrm{L}}(x_j))^2, (\tilde{v}_A^{\mathrm{U}}(x_j) - \tilde{\mu}_B^{\mathrm{U}}(x_j))^2\}} \quad (19)$$

All these $s_i(A, B)$ $(i = 1, 2, 3, 4)$ have the properties given in definition 1. It is clear that the larger the values of $s_i(A, B)$ $(i = 1, 2, \ldots, 6)$, the more the similarity between IVIFSs A and B.

2 Application of Similarity Measures to Pattern Recognitions

In this section, we apply the similarity measures presented above to the pattern recognition problem with interval-valued intuitionistic fuzzy information, which involves the following steps:

Step 1 For a pattern recognition problem, suppose that there exist m patterns which are represented by IVIFSs $A_i = \{\langle x_j, \tilde{\mu}_{A_i}(x_j), \tilde{v}_{A_j}(x_j)\rangle \mid x_j \in X\}$ $(i = 1, 2, \ldots, m)$ in the feature space $X = \{x_1, x_2, \ldots, x_m\}$, and suppose that there is a sample to be recognized which is represented by an IVIFS $B = \{\langle x_j, \tilde{\mu}_B(x_j), \tilde{v}_B(x_j)\rangle \mid x_j \in X\}$.

Step 2 Calculate the degree $s(A_i, B)$ of similarity between A_i and B by one of Eqs. (16)-(19).

Step 3 Select the largest one $s(A_{i_0}, B)$ from $s(A_i, B)\,(i = 1, 2, \ldots, m)$, and then the sample B belongs to the pattern A_{i_0} according to the principle of the maximum degree of similarity between IVIFSs.

In the following, a pattern recognition problem about the classification of building materials (adopted from Ref. [16]) is used to illustrate the proposed similarity measures.

Assume that there are four classes of building material, which are represented by the IVIFSs $A_i = \{\langle x_j, \tilde{\mu}_{A_i}(x_j), \tilde{v}_{A_i}(x_j)\rangle \mid x_j \in X\}$ $(i = 1, 2, 3, 4)$ in the feature space $X = \{x_1, x_2, \ldots, x_{12}\}$ whose weight vector is $\boldsymbol{w} = \{0.1, 0.05, 0.08, 0.06, 0.03, 0.07, 0.09, 0.12, 0.15, 0.07, 0.13, 0.05\}^{\mathrm{T}}$, and there is an unknown building material B:

$$
\begin{aligned}
A_1 = \{&\langle x_1, [0.1, 0.2], [0.5, 0.6]\rangle, \langle x_2, [0.1, 0.2], [0.7, 0.8]\rangle, \langle x_3, [0.5, 0.6], [0.3, 0.4]\rangle,\\
&\langle x_4, [0.8, 0.9], [0, 0.1]\rangle, \langle x_5, [0.4, 0.5], [0.3, 0.4]\rangle, \langle x_6, [0, 0.1], [0.8, 0.9]\rangle,\\
&\langle x_7, [0.3, 0.4], [0.5, 0.6]\rangle, \langle x_8, [1.0, 1.0], [0, 0]\rangle, \langle x_9, [0.2, 0.3], [0.6, 0.7]\rangle,\\
&\langle x_{10}, [0.4, 0.5], [0.4, 0.5]\rangle, \langle x_{11}, [0.7, 0.8], [0.1, 0.2]\rangle, \langle x_{12}, [0.4, 0.5], [0.4, 0.5]\rangle\}\\
A_2 = \{&\langle x_1, [0.5, 0.6], [0.3, 0.4]\rangle, \langle x_2, [0.6, 0.7], [0.1, 0.2]\rangle, \langle x_3, [1.0, 1.0], [0, 0]\rangle,\\
&\langle x_4, [0.1, 0.2], [0.6, 0.7]\rangle, \langle x_5, [0, 0.1], [0.8, 0.9]\rangle, \langle x_6, [0.7, 0.8], [0.1, 0.2]\rangle,\\
&\langle x_7, [0.5, 0.6], [0.3, 0.4]\rangle, \langle x_8, [0.6, 0.7], [0.2, 0.3]\rangle, \langle x_9, [1.0, 1.0], [0, 0]\rangle,\\
&\langle x_{10}, [0.1, 0.2], [0.7, 0.8]\rangle, \langle x_{11}, [0, 0.1], [0.8, 0.9]\rangle, \langle x_{12}, [0.7, 0.8], [0.1, 0.2]\rangle\}\\
A_3 = \{&\langle x_1, [0.4, 0.5], [0.3, 0.4]\rangle, \langle x_2, [0.6, 0.7], [0.2, 0.3]\rangle, \langle x_3, [0.9, 1.0], [0, 0]\rangle,\\
&\langle x_4, [0, 0.1], [0.8, 0.9]\rangle, \langle x_5, [0, 0.1], [0.8, 0.9]\rangle, \langle x_6, [0.6, 0.7], [0.2, 0.3]\rangle,\\
&\langle x_7, [0.1, 0.2], [0.7, 0.8]\rangle, \langle x_8, [0.2, 0.3], [0.6, 0.7]\rangle, \langle x_9, [0.5, 0.6], [0.2, 0.4]\rangle,\\
&\langle x_{10}, [1.0, 1.0], [0, 0]\rangle, \langle x_{11}, [0.3, 0.4], [0.4, 0.5]\rangle, \langle x_{12}, [0, 0.1], [0.8, 0.9]\rangle\}\\
A_4 = \{&\langle x_1, [1.0, 1.0], [0, 0]\rangle, \langle x_2, [1.0, 1.0], [0, 0]\rangle, \langle x_3, [0.8, 0.9], [0, 0.1]\rangle,\\
&\langle x_4, [0.7, 0.8], [0.1, 0.2]\rangle, \langle x_5, [0, 0.1], [0.7, 0.9]\rangle, \langle x_6, [0, 0.1], [0.8, 0.9]\rangle,\\
&\langle x_7, [0.1, 0.2], [0.7, 0.8]\rangle, \langle x_8, [0.1, 0.2], [0.7, 0.8]\rangle, \langle x_9, [0.4, 0.5], [0.3, 0.4]\rangle,\\
&\langle x_{10}, [1.0, 1.0], [0, 0]\rangle, \langle x_{11}, [0.3, 0.4], [0.4, 0.5]\rangle, \langle x_{12}, [0, 0.1], [0.8, 0.9]\rangle\}\\
B = \{&\langle x_1, [0.9, 1.0], [0, 0]\rangle, \langle x_2, [0.9, 1.0], [0, 0]\rangle, \langle x_3, [0.7, 0.8], [0.1, 0.2]\rangle,\\
&\langle x_4, [0.6, 0.7], [0.1, 0.2]\rangle, \langle x_5, [0, 0.1], [0.8, 0.9]\rangle, \langle x_6, [0.1, 0.2], [0.7, 0.8]\rangle,\\
&\langle x_7, [0.1, 0.2], [0.7, 0.8]\rangle, \langle x_8, [0.1, 0.2], [0.7, 0.8]\rangle, \langle x_9, [0.4, 0.5], [0.3, 0.4]\rangle,\\
&\langle x_{10}, [1.0, 1.0], [0, 0]\rangle, \langle x_{11}, [0.3, 0.4], [0.4, 0.5]\rangle, \langle x_{12}, [0, 0.1], [0.7, 0.9]\rangle\}
\end{aligned}
$$

Our aim is to justify which class the unknown pattern B belongs to. We first calculate the degree of similarity between A_i and B by Eq. (16), and obtain

$$s_1(A_1, B) = 0.597, \quad s_1(A_2, B) = 0.561, \quad s_1(A_3, B) = 0.833, \quad s_1(A_4, B) = 0.976$$

From the result above, we know that the degree of similarity between A_4 and B is the largest one, and thus, the unknown pattern B should belong to the pattern A_4.

Similarly, if we calculate the degree of similarity between A_i and B by Eqs. (17)-(19), and then we obtain

$$
\begin{aligned}
&s_2(A_1, B) = 0.530, \quad s_2(A_2, B) = 0.529, \quad s_2(A_3, B) = 0.734, \quad s_2(A_4, B) = 0.951\\
&s_3(A_1, B) = 0.545, \quad s_3(A_2, B) = 0.503, \quad s_3(A_3, B) = 0.810, \quad s_3(A_4, B) = 0.956\\
&s_4(A_1, B) = 0.473, \quad s_4(A_2, B) = 0.473, \quad s_4(A_3, B) = 0.712, \quad s_4(A_4, B) = 0.934
\end{aligned}
$$

The results above show that the degrees of similarity between A_4 and B derived by Eqs. (17)-(19) are also greater than all the others, and thus it is clear that the unknown pattern B should belong to the pattern A_4.

3 Conclusion

Recently, many similarity measures have been developed for measuring the degree of similarity between intuitionistic fuzzy sets. However, it seems that there have been no investigations on similarity measures of IVIFSs. In this paper, based on the Hausdorff distance, the Euclidean distance, and the Hamming distance, etc., we have proposed some similarity measures between IVIFSs. We have also applied these similarity measures to pattern recognitions under interval-valued intuitionistic fuzzy information. The feasibility and effectiveness of the developed measures have been verified by a pattern recognition problem concerning the classification of building materials. In the future, the probability distribution (density) function[18] can be considered as a possible tool to measure the similarity between IVIFSs.

References

[1] Atanassov K. Intuitionistic fuzzy sets [J]. *Fuzzy Sets and Systems*, 1986, **20**(1): 87 – 96.

[2] Zadeh L A. Fuzzy sets [J]. *Information and Control*, 1965, **8**(3): 338 – 353.

[3] Atanassov K, Gargov G. Interval-valued intuitionistic fuzzy sets [J]. *Fuzzy Sets and Systems*, 1989, **31**(3): 343 – 349.

[4] Atanassov K. Operators over interval-valued intuitionistic fuzzy sets [J]. *Fuzzy Sets and Systems*, 1994, **64**(2): 159 – 174.

[5] Bustince H, Burillo P. Correlation of interval-valued intuitionistic fuzzy sets [J]. *Fuzzy Sets and Systems*, 1995, **74**(2): 237 – 244.

[6] Hong D H. A note on correlation of interval-valued intuitionistic fuzzy sets [J]. *Fuzzy Sets and Systems*, 1998, **95**(1): 113 – 117.

[7] Hung W L, Wu J W. Correlation of intuitionistic fuzzy sets by centroid method [J]. *Information Sciences*, 2002, **144**(1/2/3/4): 219 – 225.

[8] Xu Z S. On correlation measures of intuitionistic fuzzy sets [C]//*The 7th International Conference on Intelligent Data Engineering and Automated Learning, Lecture Notes in Computer Science*. Burgos, Spain, 2006, **4224**: 16 – 24.

[9] Mondal T K, Samanta S K. Topology of interval-valued intuitionistic fuzzy sets [J]. *Fuzzy Sets and Systems*, 2001, **119**(3): 483 – 494.

[10] Deschrijver G, Kerre E E. On the relationship between some extensions of fuzzy set theory [J]. *Fuzzy Sets and Systems*, 2003, **133**(3): 227 – 235.

[11] Liang Z Z, Shi P F. Similarity measures on intuitionistic fuzzy sets [J]. *Pattern Recognition Letters*, 2003, **24**(15): 2687 – 2693.

[12] Szmidt E, Kacprzyk J. Distances between intuitionistic fuzzy sets [J]. *Fuzzy Sets and Systems*, 2000, **114**(3): 505 – 518.

[13] Li D F, Cheng C T. New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions [J]. *Pattern Recognition Letters*, 2002, **23**(1/2/3): 221 – 225.

[14] Grzegorzewski P. Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric [J]. *Fuzzy Sets and Systems*, 2004, **148**(2): 319 – 328.

[15] Hung W L, Yang M S. Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance [J]. *Pattern Recognition Letters*, 2004, **25**(14): 1603 – 1611.

[16] Wang W Q, Xin X L. Distance measure between intuitionistic fuzzy sets [J]. *Pattern Recognition Letters*, 2005, **26**(13): 2063 – 2069.

[17] Nadler S B Jr. *Hyperspaces of sets* [M]. New York: Marcel Dekker, 1978.

[18] Zhang Z M. The application of generalized entropy in planning time distribution [J]. *Mathematics in Practice and Theory*, 1982 (1): 31 – 33. (in Chinese)

区间直觉模糊集相似性测度及其在模式识别中的应用

徐泽水

(东南大学经济管理学院,南京 210096)

摘要:定义了区间直觉模糊集相似度的概念,并且基于 Hamming 距离、标准化的 Hamming 距离、加权的 Hamming Euclidean 距离、Euclidean 距离、标准化的 Euclidean 距离、加权的 Euclidean 距离等,定义了一些区间直觉模糊集距离测度. 然后,通过把 Hamming 距离和 Euclidean 距离以及它们的加权形式与 Hausdorff 度量相结合,给出了 2 种组合的区间直觉模糊集距离测度,即基于 Hausdorff 度量的加权 Hamming 距离和基于 Hausdorff 度量的加权 Euclidean 距离,并且研究了它们的性质. 最后,基于上述距离测度,给出了区间直觉模糊集相似性测度,并且把它们应用于模式识别领域.

关键词:区间直觉模糊集;相似性;模式识别

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