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Multi-attribute decision-making based on subjective and objective integrated eigenvector method

Gong Yanbing Chen Senfa

(School of Economics and Management, Southeast University, Nanjing 210096, China)

Abstract: An integrated approach is proposed to investigate the fuzzy multi-attribute decision-making (MADM) problems, where subjective preferences are expressed by a pairwise comparison matrix on the relative weights of attributes and objective information is expressed by a decision matrix. An eigenvector method integrated the subjective fuzzy preference matrix and objective information is proposed. Two linear programming models based on subjective and objective information are introduced to assess the relative importance weights of attributes in an MADM problem. The simple additive weighting method is utilized to aggregate the decision information, and then all the alternatives are ranked. Finally, a numerical example is given to show the feasibility and effectiveness of the method. The result shows that it is easier than other methods of integrating subjective and objective information.

Key words: multi-attribute decision-making; eigenvector method; alternative ranking

In multi-attribute decision-making (MADM) problems, a decision maker (DM) is often faced with the problem of selecting or ranking alternatives associated with non-commensurate and conflicting attributes. MADM problems arise in many real-world situations. One of the hot research topics is how to solve MADM problems with fuzzy preference information[1–2]. Since fuzzy preference information on attributes involves a decision maker's subjective considerations, any decision-making tool chosen must incorporate them when ranking alternatives. However, such a ranking could not be considered final as it would lack objectivity. To determine the final ranking of alternatives, the weights of the attributes need to be assessed using subjective fuzzy preference information as well as information that is free from any bias the decision maker may introduce through his/her subjective input.

The main focus of this paper is on the method with information on attributes weight information. In a recent paper, Ma et al. [3] developed a subjective and objective integrated approach for MADM, where the multiplicative preference relation on the relative weights of attributes was integrated together with decision matrix information into an integrated decision model. Their approach utilized a quadratic programming technique to assess the attribute weights and was further discussed by Xu[4]. Fan et al. [5] proposed an approach to MADM based on the fuzzy preference relation on decision alternatives. In their approach the fuzzy preference relation on alternatives was incorporated together with decision matrix information through a quadratic programming model. Wang and Parkan[6] integrated the fuzzy preference relation and decision matrix information together in three different ways, which used the linear programming technique. Fan et al. [7] also incorporated them together using the linear programming technique, but with a different integrated model. Chiclana and Fan et al. [8–9] developed further a linear goal programming model and a two-objective optimization model to integrate fuzzy and multiplicative preference relations both on decision alternatives. It is quite clear that there has been no effort so far to develop a subjective and objective integrated approach with only an eigenvector method (EM) by the multiplicative preference relation on the relative weights of attributes and decision matrix information. The purpose of this paper is to develop a subjective and objective integrated EM (SOEM). It is a supplement or extension of the existing methods.

1 SAW Approach and Subjective EM

1.1 SAW approach for MADM

Suppose that an MADM problem has n decision alternatives $(A_1, A_2, \dots, A_n)$ and m decision attributes $(R_1, R_2, \dots, R_m)$. Each alternative is evaluated with respect to the m attributes, whose values constitute a decision matrix denoted by $\boldsymbol{X} = (x_{ij})_{n\times m}$. Due to the incommensurability among attributes, the decision matrix $\boldsymbol{X} = (x_{ij})_{n\times m}$ needs to be normalized. The most commonly used normalization method is as follows:

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Biographies: Gong Yanbing (1979—), male, graduate; Chen Senfa (corresponding author), male, professor, chensenfa@163.com.

$$z_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}} \qquad i = 1, 2, \ldots, n; j \in Q_1 \qquad (1)$$

$$z_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}} \qquad i = 1, 2, \ldots, n; j \in Q_2 \qquad (2)$$

where $x_j^{\min} = \min\limits_{1 \leqslant i \leqslant n} \{x_{ij}\}$, $x_j^{\max} = \max\limits_{1 \leqslant i \leqslant n} \{x_{ij}\}$; z_{ij} is the normalized attribute value; Q_1 and Q_2 are the sets of benefit attributes and cost attributes, respectively. The so-called benefit attributes are those for maximization, while the cost attributes are those for minimization.

Let $\boldsymbol{Z} = (z_{ij})_{n \times m}$ be the normalized decision matrix and $\boldsymbol{W} = \{w_1, w_2, \ldots, w_m\}$ be the normalized vector of attribute weights satisfying

$$\boldsymbol{e}^{\mathrm{T}} \boldsymbol{W} = 1 \qquad (3)$$

where $\boldsymbol{e}^{\mathrm{T}} = \{1, \ldots, 1\}$. According to the SAW approach, the overall weighted assessment value of alternative $A_i (i = 1, 2, \ldots, n)$ can be expressed as

$$d_i = \sum_{j=1}^{m} z_{ij} w_j \qquad i = 1, 2, \ldots, n \qquad (4)$$

where d_i is a linear function of weight variables $w_j (j = 1, 2, \ldots, m)$. The greater the d_i, the better the alternative A_i. The best alternative is the one with the greatest overall weighted assessment value. For brevity, Eq. (4) can be rewritten in matrix form as

$$\boldsymbol{D} = \boldsymbol{Z}\boldsymbol{W} \qquad (5)$$

where $\boldsymbol{D} = \{d_1, d_2, \ldots, d_n\}$ is a vector of the overall weighted assessment values for all the alternatives.

1.2 Subjective EM based on the multiplicative preference relation

Let the multiplicative preference relation on the relative weights of attributes be represented by

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & h_{12} & \ldots & h_{1m} \\ h_{21} & h_{22} & \ldots & h_{2m} \\ \vdots & \vdots & & \vdots \\ h_{m1} & h_{m2} & \ldots & h_{mm} \end{bmatrix}$$

where $h_{ji} = 1/h_{ij} > 0$ and $h_{ii} = 1 (i, j = 1, 2, \ldots, m)$. According to the Saaty's EM[10], the weight vector $\boldsymbol{W} = \{w_1, w_2, \ldots, w_m\}$ can be estimated by solving the following eigenvalue problem:

$$\boldsymbol{H}\boldsymbol{W} = \lambda_{\max} \boldsymbol{W} \qquad (6)$$

If the multiplicative preference relation $\boldsymbol{H}$ is a precise/consistent comparison matrix on the relative weights of attributes, then Eq. (6) can be simplified as

$$\boldsymbol{H}\boldsymbol{W} = m\boldsymbol{W} \qquad (7)$$

It is hoped that the multiplicative preference relation provided by DM should be as consistent as possible. However, an accurate estimate is nearly impossible. So Eq. (7) cannot hold exactly in most cases. An alternate method for obtaining the right-eigenvector $\boldsymbol{W}$ is to solve the following LP model[7],

$$\begin{aligned} \min \quad & \boldsymbol{J} = \boldsymbol{e}^{\mathrm{T}} \boldsymbol{E} \\ \text{s.t.} \quad & (\boldsymbol{H} - m\boldsymbol{I}) \widetilde{\boldsymbol{W}} - \boldsymbol{E} = \boldsymbol{0} \\ & \boldsymbol{e}^{\mathrm{T}} \widetilde{\boldsymbol{W}} = 1 \\ & \widetilde{\boldsymbol{W}}, \boldsymbol{E} \geqslant 0 \end{aligned} \qquad (8)$$

where $\widetilde{\boldsymbol{W}}$ is an estimate of $\boldsymbol{W}$, $\boldsymbol{E} = \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$ is a nonnegative vector of deviation variables.

2 SOEM for MADM

This approach combines the DM's subjective multiplicative preference relation matrix directly with the objective decision matrix in a way that is quite different from Wang et al.'s. It is known from Eq. (7) that for any multiplicative preference relation matrix $\boldsymbol{H}$ we can always find a positive normalized eigenvector $\widetilde{\boldsymbol{W}}$ such that $\boldsymbol{H}\widetilde{\boldsymbol{W}} = m\widetilde{\boldsymbol{W}}$. If the estimate is accurate then there must exist $\boldsymbol{H}\boldsymbol{W} = m\boldsymbol{W}$. In Eq. (7), left-product the normalized decision matrix $\boldsymbol{Z}$, we get

$$\boldsymbol{Z}\boldsymbol{H}\boldsymbol{W} = m\boldsymbol{Z}\boldsymbol{W} \qquad (9)$$

Due to the existence of fuzziness and subjectivity, however, an accurate estimate is nearly impossible. So Eq. (9) cannot hold exactly in most cases. In view of this, we define the deviation vector

$$\boldsymbol{E} = (\boldsymbol{Z}\boldsymbol{H} - m\boldsymbol{Z}) \boldsymbol{W} \qquad (10)$$

where $\boldsymbol{E} = \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$ and find the attribute weights that minimize the sum of the absolute values of the components of $\boldsymbol{E}$ by the following model

$$\begin{aligned} \min \quad & J = \sum_{i=1}^{n} |\varepsilon_i| \\ \text{s.t.} \quad & (\boldsymbol{Z}\boldsymbol{H} - m\boldsymbol{Z}) \boldsymbol{W} - \boldsymbol{E} = \boldsymbol{0} \\ & \boldsymbol{e}^{\mathrm{T}} \boldsymbol{W} = 1 \\ & \boldsymbol{W} \geqslant \boldsymbol{0} \end{aligned} \qquad (11)$$

Let

$$\varepsilon_i^+ = \frac{\varepsilon_i + |\varepsilon_i|}{2}, \quad \varepsilon_i^- = \frac{-\varepsilon_i + |\varepsilon_i|}{2} \qquad i = 1, 2, \ldots, n$$

ε_i can be written as

$$\varepsilon_i = \varepsilon_i^+ - \varepsilon_i^- \qquad i = 1, 2, \ldots, n$$

where the scalar product $\varepsilon_i^+ \varepsilon_i^- = 0$. Now, the optimization model (11) can be written as an LP model:

$$\begin{aligned} \min \quad & J = \boldsymbol{e}^{\mathrm{T}} (\boldsymbol{E}^+ + \boldsymbol{E}^-) \\ \text{s.t.} \quad & (\boldsymbol{Z}\boldsymbol{H} - m\boldsymbol{Z}) \boldsymbol{W} - \boldsymbol{E}^+ + \boldsymbol{E}^- = 0 \\ & \boldsymbol{e}^{\mathrm{T}} \boldsymbol{W} = 1 \\ & \boldsymbol{W}, \boldsymbol{E}^+, \boldsymbol{E}^- \geqslant \boldsymbol{0} \end{aligned} \qquad (12)$$

where $\boldsymbol{E}^+ = (\varepsilon_1^+, \varepsilon_2^+, \ldots, \varepsilon_n^+)^{\mathrm{T}}$ and $\boldsymbol{E}^- = (\varepsilon_1^-, \varepsilon_2^-, \ldots, \varepsilon_n^-)^{\mathrm{T}}$.

From Eq. (10), we can also find the attribute weights that minimize the maximum absolute values of the components of $\boldsymbol{E}$ by the following model:

$$\begin{aligned} \min \quad & \max_{1 \leqslant i \leqslant n} |\varepsilon_i| \\ \text{s.t.} \quad & (\boldsymbol{Z}\boldsymbol{H} - m\boldsymbol{Z}) \boldsymbol{W} - \boldsymbol{E} = \boldsymbol{0} \\ & \boldsymbol{e}^{\mathrm{T}} \boldsymbol{W} = 1 \end{aligned}$$

$$\boldsymbol{W} \geqslant \boldsymbol{0} \tag{13}$$

Let $\delta = \max_{1 \leqslant i \leqslant n} |\varepsilon_i|$, we have

$$-\delta \leqslant \varepsilon_i \leqslant \delta \qquad i = 1, 2, \ldots, n$$

That is

$$\varepsilon_i - \delta \leqslant 0 \qquad i = 1, 2, \ldots, n$$

$$-\varepsilon_i - \delta \leqslant 0 \qquad i = 1, 2, \ldots, n$$

Thus, the minimizing the maximum model (13) can be reconfigured as the following LP model:

$$\begin{aligned} \min \quad & \delta \\ \text{s.t.} \quad & (\boldsymbol{ZH} - m\boldsymbol{Z})\boldsymbol{W} - \delta \boldsymbol{e} \leqslant \boldsymbol{0} \\ & -(\boldsymbol{ZH} - m\boldsymbol{Z})\boldsymbol{W} - \delta \boldsymbol{e} \leqslant \boldsymbol{0} \\ & \boldsymbol{e}^{\mathrm{T}} \boldsymbol{W} = 1 \\ & \boldsymbol{W}, \delta \geqslant 0 \end{aligned} \tag{14}$$

where $\boldsymbol{e}^{\mathrm{T}} = \{1, \ldots, 1\}$. The optimum weight vector $\boldsymbol{W}^* = \{w_1^*, w_2^*, \ldots, w_m^*\}$ can be easily obtained by solving (12) and (14), which incorporates both subjective and objective information. With the attribute weights obtained by (12) and (14), the overall weighted assessment values for the alternatives can be computed by either (4) or (5), and the final decision can be obtained. We call the method described above SOEM.

3 Numerical Example

In this section, Ma et al.'s illustrative example[3] is re-examined to show how the DM's decision-making and ranking can be derived from the EM for integrating SOEM.

Example 1 A robot user intends to select a robot and there are four alternatives for him/her to choose. When making a decision, the attributes considered include: cost (\$ 10 000), velocity (m/s), repeatability (mm), load capacity (kg). Among four attributes, velocity and load capacity are of benefit type, cost and repeatability are of cost type. The decision matrix $\boldsymbol{X}$ for the MADM problem is

$$\boldsymbol{X} = \begin{bmatrix} 3.0 & 1.0 & 1.0 & 70 \\ 2.5 & 0.8 & 0.8 & 50 \\ 1.8 & 0.5 & 2.0 & 110 \\ 2.2 & 0.7 & 1.2 & 90 \end{bmatrix}$$

which can be normalized into matrix $\boldsymbol{Z}$ by using Eqs. (1) and (2),

$$\boldsymbol{Z} = \begin{bmatrix} 0 & 1 & \frac{5}{6} & \frac{1}{3} \\ \frac{5}{12} & \frac{3}{5} & 1 & 0 \\ 1 & 0 & 0 & 1 \\ \frac{2}{3} & \frac{2}{5} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

Suppose that the robot user gives his/her pairwise comparison matrix $\boldsymbol{H}$ about the weights

$$\boldsymbol{H} = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{2} & \frac{1}{5} \\ 3 & 1 & 2 & \frac{1}{2} \\ 2 & \frac{1}{2} & 1 & \frac{1}{2} \\ 5 & 2 & 2 & 1 \end{bmatrix}$$

When the relative importance of the subjective information and the objective information is equivalent, using Ma et al.'s approach, we obtain

$$\boldsymbol{W}^* = \{w_1^*, w_2^*, w_3^*, w_4^*\} = \{0.0943, 0.2705, 0.1988, 0.4365\}$$

The corresponding optimum rank value is

$$\boldsymbol{D}(\boldsymbol{W}^*) = \{d_1, d_2, d_3, d_4\} = \{0.5817, 0.4004, 0.5308, 0.5946\}$$

The application of SOEM, through the solution of model (12) and (14), produces

$$\boldsymbol{W}^* = \{w_1^*, w_2^*, w_3^*, w_4^*\} = \{0.0839, 0.2736, 0.1874, 0.4552\}$$

and

$$\boldsymbol{W}^* = \{w_1^*, w_2^*, w_3^*, w_4^*\} = \{0.0847, 0.2749, 0.1858, 0.4546\}$$

which incorporates both the subjective multiplicative preference relation and the objective decision matrix information. The corresponding overall weighted assessment value vector is

$$\boldsymbol{D}(\boldsymbol{W}^*) = \{d_1, d_2, d_3, d_4\} = \{0.5815, 0.3865, 0.5391, 0.5938\}$$

and

$$\boldsymbol{D}(\boldsymbol{W}^*) = \{d_1, d_2, d_3, d_4\} = \{0.5813, 0.3860, 0.5393, 0.5934\}$$

which are very close to the result of the Ma et al.'s approach and both lead to the same ranking order: $A_4 \succ A_1 \succ A_3 \succ A_2$.

In this example, our approach produces the same ranking as the Ma et al.'s approach, but it is more simple.

4 Conclusion

This paper proposes a subjective and objective integrated EM (SOEM) to determine attribute weights in MADM problems. The approach determines weights by solving two mathematical programming models and takes into consideration both the multiplicative preference relation on the relative weights of attributes and decision matrix information. It overcomes the shortages, which occur in a subjective EM. The SOEM is simpler than other models of integrating subjective and objective information. A numerical example provided by Ma et al. has been revisited to demonstrate the proposed method. It is shown that the method developed in this paper is rational, feasible and effective.

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多属性决策的主客观结合特征向量方法

龚艳冰　　陈森发

(东南大学经济管理学院,南京 210096)

摘要:研究了结合主观和客观信息的模糊多属性决策问题,其中主客观信息分别由属性权重的两两比较矩阵和决策矩阵组成.提出一种结合主观和客观信息的特征向量决策方法,给出了 2 种求解基于主客观特征向量法的模糊多属性决策方法.这种方法通过求解 2 个线性目标规划模型得到最优属性权重,然后,通过对决策信息进行简单的加权集结,得到所有方案的排序结果.最后,通过一个算例说明了该方法的实用性和有效性.结果表明,该方法要比其他主客观结合多属性决策方法简单.

关键词:多属性决策;特征向量方法;方案排序

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