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OFDM blind channel estimation based on polynomial models

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Abstract: A two-dimensional (2-D) polynomial regression model is set up to approximate the time-frequency response of slowly time-varying orthogonal frequency-division multiplexing (OFDM) systems. With this model the estimation of the OFDM time-frequency response is turned into the optimization of some time-invariant model parameters. A new algorithm based on the expectation-maximization (EM) method is proposed to obtain the maximum-likelihood (ML) estimation of the polynomial model parameters over the 2-D observed data. At the same time, in order to reduce the complexity and avoid the computation instability, a novel recursive approach (RPEMTO) is given to calculate the values of the parameters. It is further shown that this 2-D polynomial EM-based algorithm for time-varying OFDM (PEMTO) can be simplified mathematically to handle the one-dimensional sequential estimation. Simulations illustrate that the proposed algorithms achieve a lower bit error rate (BER) than other blind algorithms.

Key words: orthogonal frequency-division multiplexing; expectation-maximization; polynomial model; recursive

Orthogonal frequency-division multiplexing (OFDM) is an effective technique for broadband wireless communication. It has been accepted by many wireless standards, such as wireless local area network (WLAN) standards (IEEE 802.11a)[1]. To detect the transmitted data sequence in the OFDM system, it is essential for the receiver to obtain reliable channel information[2]. However, blind channel estimation of time-varying OFDM systems is a challenging task as the OFDM time-frequency response of the i-th subchannel at the k-th symbol interval (denoted by $H_{i,k}$) changes with index i and k.

It is illustrated in Refs. [3 - 4] that $H_{i,k}$ is closely correlated with its neighbors, i. e. $H_{i,k+1}$ or $H_{i+1,k}$. In other words, although $H_{i,k}$ varies with i and k, there exits no abrupt change between adjacent $H_{i,k}$[3]. With this property some parametric models of $H_{i,k}$ and related blind algorithms are proposed[2,5-6]. These blind detectors are known as GLRT detectors[7], not ML detectors. Notice that these blind detectors only utilize the correlation information between sequential time-slots or subchannels, ignoring the 2-D correlation between adjacent subchannels or time-slots. Hence, the 2-D estimation is carried out based on the 1-D estimation.

In this paper we propose a new polynomial EM-based algorithm for time-varying OFDM systems (PEMTO). It exploits the 2-D correlation information between adjacent $H_{i,k}$ and it approximates $H_{i,k}$ of OFDM systems with the 2-D polynomial model introduced in Ref. [5]. In contrast to the pilot-based method given in Ref. [5], the new algorithm implements the blind estimation of $H_{i,k}$, and obtains the ML estimation of the polynomial model parameters iteratively. As shown in the simulation, it can provide better performance than other blind algorithms. It should be noted that this new algorithm is different from the algorithm given in Ref. [8] which focuses on the channel impulse response instead of $H_{i,k}$.

The PEMTO given above involves matrix inversion at every iterative step, which increases both complexity and instability. Exploiting the special structure of the matrix, we can derive a recursive variant of the PEMTO (RPEMTO) by applying the matrix inversion theorem. The RPEMTO requires no matrix inversion and handles the 2-D observed data sequentially. The simulations demonstrate that it works well for small Doppler shifts.

Mathematically, the 2-D PEMTO can be simplified to carry out the 1-D estimation of OFDM systems. Note that for static OFDM systems the change of $H_{i,k}$ in frequency direction can also be approximated by 1-D polynomial models. Therefore, the 1-D version of the PEMTO can also be employed to implement the channel estimation of static OFDM systems. Some blind estimation algorithms for static OFDM systems have been proposed[8-9]. However, this newly proposed 1-D method has some particular advantages for its performance

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improvement and complexity reduction.

1 System Model

Under the wide sense stationary uncorrelated scattering (WSSUS) mobile channel model[10], the time-varying characteristic of $H_{i,k}$ and the OFDM inter-subcarrier interference (ICI) are mainly caused by the Doppler frequency shift, especially the maximum Doppler frequency shift f_D[4,11]. For simplicity, we denote the normalized Doppler frequency shift as f_d with $f_d = f_D T_s$ and T_s representing the OFDM symbol interval. It has been proved in Ref. [11] that under the WSSUS channel model, the ratio of signal to ICI (SIC) is greater than 36 dB when f_d is less than 0.01. As pointed out in Ref. [1], f_D is usually on the order of 100 Hz in the practical communication system. Hence, the corresponding f_d is very small and the channels change slowly[6,8]. As a result, for a regular signal to noise ratio (SNR), the ICI is negligible compared to the additive Gaussian noise[2,6,8]. From Refs. [2, 6], we thus get the following expression about the OFDM received symbol:

$$y_{i,k} = H_{i,k} x_{i,k} + N_{i,k} \tag{1}$$

where $x_{i,k}$ is the transmitted data symbol, $y_{i,k}$ is the demodulated/received data symbol, and $N_{i,k}$ represents the white complex Gaussian process with a zero mean and a variance σ^2[5-6]. With the 2-D polynomials, $H_{i,k}$ can be projected over a $(2I+1)(2K+1)$ time-frequency window around a center point (i_0, k_0)[5]:

$$H_{i,k} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} C(n,m)(i-i_0)^m (k-k_0)^n + R_{N,M}$$
$$k_0 - K \leqslant k \leqslant k_0 + K;\ i_0 - I \leqslant i \leqslant i_0 + I \tag{2}$$

where $R_{N,M}$ is the model error and M and N are polynomial orders for frequency and time, respectively. It is pointed out in Ref. [5] that $H_{i,k}$ can be approximated by a few polynomial coefficients $C(n,m)$ with a small penalty on model error:

$$H_{i,k} \approx \boldsymbol{q}_{i,k}^{\mathrm{T}} \boldsymbol{C} \tag{3}$$

where

$$\boldsymbol{q}_{i,k} = \{(i-i_0)^0 (k-k_0)^0, \dots, (i-i_0)^{N-1}(k-k_0)^0, \dots, (i-i_0)^{N-1}(k-k_0)^{M-1}\}^{\mathrm{T}}$$

$$\boldsymbol{C} = \{C(0,0), \dots, C(N-1,0), C(0,1), \dots, C(N-1,M-1)\}^{\mathrm{T}}$$

We define the following vector to represent all the transmitted data in the $(2I+1)(2K+1)$ time-frequency window.

$$\boldsymbol{X} = \{x_{-I+i_0,-K+k_0}, \dots, x_{I+i_0,-K+k_0}, x_{-I+i_0,-K+1+k_0}, \dots, x_{I+i_0,K+k_0}\}^{\mathrm{T}} \tag{4}$$

If $\boldsymbol{X}$ is defined in this way, then $\boldsymbol{Y}$, $\boldsymbol{H}$ and $\boldsymbol{N}$ are used in the same way to denote the received data vector, the response vector and the Gaussian noise vector, respectively. From (4) and (3), we thus obtain

$$\boldsymbol{Y} = \mathrm{diag}(\boldsymbol{X})\boldsymbol{H} + \boldsymbol{N} \tag{5}$$

where $\boldsymbol{H} = \boldsymbol{RC}$, $\boldsymbol{R} = \{\boldsymbol{q}_{-I+i_0,-K+k_0}, \dots, \boldsymbol{q}_{I+i_0,-K+k_0}, \boldsymbol{q}_{-I+i_0,-K+1+k_0}, \dots, \boldsymbol{q}_{I+i_0,K+k_0}\}^{\mathrm{T}}$, and $\mathrm{diag}(\boldsymbol{X})$ denotes a zero matrix except that $\boldsymbol{X}_i$ is the matrix's (i, i) entry. With the above equation, the ML estimation of $\boldsymbol{H}$ is turned into the estimation of $\boldsymbol{C}$. Following the ML principle, we try to find an estimation of $\boldsymbol{C}$ that maximizes the likelihood function $f(\boldsymbol{Y} \mid \boldsymbol{C})$. However, as the observed data $\boldsymbol{Y}$ is not enough for deriving a close-form of $f(\boldsymbol{Y} \mid \boldsymbol{C})$, the maximization of $f(\boldsymbol{Y} \mid \boldsymbol{C})$ is not easy to manipulate. We hence resort to the EM algorithm.

2 EM-Based Algorithms and Recursive Variant

The key to the EM algorithm is to define "complete" data and "incomplete" data[8,12]. We take $\boldsymbol{Y}$ as "incomplete" data and $\boldsymbol{Z} = (\boldsymbol{X}, \boldsymbol{Y})$ as "complete" data. The algorithm can be broken down into two steps: the E-step and the M-step. The E-step calculates

$$Q(\boldsymbol{C} \mid \boldsymbol{C}^{(p)}) = E[\log f(\boldsymbol{Z} \mid \boldsymbol{C}) \mid \boldsymbol{Y}, \boldsymbol{C}^{(p)}] \tag{6}$$

The M-step then gets

$$\boldsymbol{C}^{(p+1)} = \arg\max_{\boldsymbol{C}} Q(\boldsymbol{C} \mid \boldsymbol{C}^{(p)}) \tag{7}$$

This procedure is repeated until $\boldsymbol{C}^{(p)}$ converges.

2.1 Polynomial EM-based algorithm for time-varying OFDM (PEMTO)

Without loss of generality, we assume that the transmitted data $x_{i,k}$ represents a PSK or QAM signal with constellation size D and we denote the signal by $\{x_d, 1 \leqslant d \leqslant D\}$. With the existence of the Gaussian noise, we have the following likelihood function

$$f(y_{i,k} \mid \boldsymbol{C}, x_{i,k}) = \frac{1}{\pi\sigma^2} \exp\left\{ -\frac{1}{\sigma^2} \mid y_{i,k} - x_{i,k} \boldsymbol{q}_{i,k}^{\mathrm{T}} \boldsymbol{C} \mid^2 \right\} \tag{8}$$

Since $N_{i,k}$ is independent from each other, the conditional probability density function (pdf) of $\boldsymbol{Y}$ is as follows:

$$f(\boldsymbol{Y} \mid \boldsymbol{C}, \boldsymbol{X}) = \prod_{k=-K+k_0}^{K+k_0} \prod_{i=-I+i_0}^{I+i_0} f(y_{i,k} \mid \boldsymbol{C}, x_{i,k}) \tag{9}$$

Following the E-step given in Ref. [8], we calculate

$$Q(\boldsymbol{C} \mid \boldsymbol{C}^{(p)}) = E[\log f(\boldsymbol{Z} \mid \boldsymbol{C}) \mid \boldsymbol{Y}, \boldsymbol{C}^{(p)}] =$$
$$E[\log f(\boldsymbol{Y} \mid \boldsymbol{C}, \boldsymbol{X}) f(\boldsymbol{X} \mid \boldsymbol{C}) \mid \boldsymbol{Y}, \boldsymbol{C}^{(p)}] =$$
$$\sum_{d=1}^{D} \sum_{k=-K+k_0}^{K+k_0} \sum_{i=-I+i_0}^{I+i_0} \left(\log \frac{f(Y_{i,k} \mid \boldsymbol{C}, X_d)}{D} \right) \frac{f(Y_{i,k} \mid X_d, \boldsymbol{C}^{(p)})}{D f(Y_{i,k} \mid \boldsymbol{C}^{(p)})} \tag{10}$$

We continue with the M-step. Differentiating

(10) with respect to $\boldsymbol{C}$ and setting it to zero, we get

$$\boldsymbol{C}^{(p+1)} = (\boldsymbol{P}^{(p)})^{-1}\boldsymbol{S}^{(p)} \tag{11}$$

where

$$\boldsymbol{P}^{(p)} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{U}^{(p)}\boldsymbol{R},\ \boldsymbol{S}^{(p)} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{V}^{(p)},\ \boldsymbol{U}^{(p)} = \mathrm{diag}(\boldsymbol{U}_u^{(p)})$$

$$\boldsymbol{U}_u^{(p)} = \{u^{(p)}_{-I+i_0,\,-K+k_0}, \ldots, u^{(p)}_{I+i_0,\,-K+k_0}, u^{(p)}_{-I+i_0,\,-K+1+k_0}, \ldots, u^{(p)}_{I+i_0,\,K+k_0}\}^{\mathrm{T}}$$

$$u^{(p)}_{i,k} = \sum_{d=1}^{D} |x_d|^2 \frac{1}{f(y_{i,k} \mid \boldsymbol{C}^{(p)})} f(y_{i,k} \mid x_{\mathrm{d}}, \boldsymbol{C}^{(p)})$$

$$\boldsymbol{V}^{(p)} = [v^{(p)}_{-I+i_0,\,-K+k_0}, \ldots, v^{(p)}_{I+i_0,\,-K+k_0}, v^{(p)}_{-I+i_0,\,-K+1+k_0}, \ldots, v^{(p)}_{I+i_0,\,K+k_0}]^{\mathrm{T}}$$

$$v^{(p)}_{i,k} = \sum_{d=1}^{D} y_{i,k}x_d^* \frac{1}{f(y_{i,k} \mid \boldsymbol{C}^{(p)})} f(y_{i,k} \mid x_d, \boldsymbol{C}^{(p)})$$

$$f(Y_{i,k} \mid \boldsymbol{C}) = \sum_{d=1}^{D} \frac{1}{\pi D\sigma^2}\exp\left\{-\frac{1}{\sigma^2}|y_{i,k} - x_d\boldsymbol{q}^{\mathrm{T}}_{i,k}\boldsymbol{C}|^2\right\}$$

The iterative procedure should be terminated as soon as the difference between $\boldsymbol{C}^{(p+1)}$ and $\boldsymbol{C}^{(p)}$ is sufficiently small. Once $\boldsymbol{C}$ is obtained, $\boldsymbol{H}$ can be recovered by (5).

2.2 Recursive PEMTO (RPEMTO)

Following (11), an $MN \times MN$ matrix must be inversed to obtain $\boldsymbol{C}^{(p+1)}$ for every p. However, matrix inversion often leads to computing instability and, in the worst case, is impossible. In order to make sure that the algorithm can be implemented successfully, we should avoid matrix inversion in the computing process. Considering the particular structure of $\boldsymbol{P}$, we give a recursive approach (RPEMTO) by exploiting the matrix inversion theorem as

$$(\boldsymbol{A} + \boldsymbol{BCB}^{\mathrm{T}})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{B} + \boldsymbol{C}^{-1})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}^{-1} \tag{12}$$

We define $T_0 = (2I+1)(2K+1)$ and rename the matrix entry in $\boldsymbol{P}^{(p)}$ as

$$\boldsymbol{R}^{\mathrm{T}} = \{\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{T_0}\},\ \boldsymbol{V}^{(p)} = \{v^{(p)}_{s_1}, \ldots, v^{(p)}_{s_{T_0}}\}^{\mathrm{T}}$$

$$\boldsymbol{U}^{(p)} = \mathrm{diag}(\boldsymbol{U}_{\mathrm{s}}^{(p)}),\quad \boldsymbol{U}_{\mathrm{s}}^{(p)} = \{u^{(p)}_{s_1}, \ldots, u^{(p)}_{s_{T_0}}\}^{\mathrm{T}}$$

In order to get a recursive method, for $1 \leqslant i \leqslant T_0$, we define the following vectors or matrices:

$$\delta_i^{(p)} = (\boldsymbol{P}_i^{(p)})^{-1},\ \boldsymbol{P}_i^{(p)} = \boldsymbol{R}_i^{\mathrm{T}}\ \boldsymbol{U}_i^{(p)}\ R_i$$

$$\boldsymbol{U}_i^{(p)} = \mathrm{diag}(\boldsymbol{U}_{u,i}^{(p)}),\ \boldsymbol{U}_{u,i}^{(p)} = \{u^{(p)}_{s_1}, \ldots, u^{(p)}_{s_i}\}^{\mathrm{T}}$$

$$\boldsymbol{R}_i^{\mathrm{T}} = \{\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_i\},\quad \boldsymbol{S}_i^p = \boldsymbol{R}_i^{\mathrm{T}}\boldsymbol{V}_i^{(p)}$$

$$\boldsymbol{V}_i^{(p)} = \{v^{(p)}_{s_1}, \ldots, v^{(p)}_{s_i}\}^{\mathrm{T}}$$

Thus, for $1 \leqslant i \leqslant T_0$, we have the following expressions:

$$\boldsymbol{R}^{\mathrm{T}}_{i+1} = [\boldsymbol{R}_i^{\mathrm{T}}, r_{i+1}],\ \boldsymbol{V}^{(p)}_{i+1} = \begin{bmatrix} \boldsymbol{V}_i^{(p)} \\ v^{(p)}_{s_{i+1}} \end{bmatrix},\ \boldsymbol{U}^{(p)}_{i+1} = \begin{bmatrix} \boldsymbol{U}_i^{(p)} & 0 \\ 0 & u^{(p)}_{s_{i+1}} \end{bmatrix}$$

$$\boldsymbol{S}^{(p)}_{i+1} = \boldsymbol{S}_i^{(p)} + \boldsymbol{r}_{i+1}v^{(p)}_{s_{i+1}}$$

$$\boldsymbol{P}^{(p)}_{i+1} = \boldsymbol{R}_i^{\mathrm{T}}\ \boldsymbol{U}_i^{(p)}\ \boldsymbol{R}_i + \boldsymbol{r}_{i+1}u^{(p)}_{s_{i+1}}\boldsymbol{r}^{\mathrm{T}}_{i+1}$$

Applying the matrix inversion theorem given in (12), we have

$$\boldsymbol{\delta}^{(p)}_{i+1} = (\boldsymbol{P}^{(p)}_{i+1})^{-1} = \boldsymbol{\delta}_i^{(p)} - \frac{\boldsymbol{\delta}_i^{(p)}\ \boldsymbol{r}_{i+1}\boldsymbol{r}^{\mathrm{T}}_{i+1}\boldsymbol{\delta}_i^{(p)}}{(u^{(p)}_{s_{i+1}})^{-1} + \boldsymbol{r}^{\mathrm{T}}_{i+1}\boldsymbol{\delta}_i^{(p)}\boldsymbol{r}_{i+1}} \tag{13}$$

Finally, we get

$$\boldsymbol{C}^{(p)}_{i+1} = \boldsymbol{\delta}^{(p)}_{i+1}\ \boldsymbol{S}^{(p)}_{i+1} = \boldsymbol{C}_i^{(p)} + \boldsymbol{K}^{(p)}_{i+1}((u^{(p)}_{s_{i+1}})^{-1}v^{(p)}_{s_{i+1}} - \boldsymbol{r}^{\mathrm{T}}_{i+1}\ \boldsymbol{C}_i^{(p)}) \tag{14}$$

where

$$\boldsymbol{K}^{(p)}_{i+1} = \frac{\boldsymbol{\delta}_i^{(p)}\boldsymbol{r}_{i+1}}{(u^{(p)}_{s_{i+1}})^{-1} + \boldsymbol{r}^{\mathrm{T}}_{i+1}\ \boldsymbol{\delta}_i^{(p)}\ \boldsymbol{r}_{i+1}}$$

$$\boldsymbol{\delta}^{(p)}_{i+1} = \boldsymbol{\delta}_i^{(p)} - \boldsymbol{K}^{(p)}_{i+1}\boldsymbol{r}^{\mathrm{T}}_{i+1}\ \boldsymbol{\delta}_i^{(p)}$$

A recursive approach for $\boldsymbol{C}^{(p+1)}$ is given in (14). But, all the recursive steps in (14) are repeated whenever p changes. This leads to great computing complexity since it consists of both iterative (with respect to p) and recursive steps (with respect to i).

Similar to the way the generalized EM algorithm takes[13], we apply some strategy to reduce computing complexity and derive a truly recursive variant of PEMTO. Actually, the steps in (14) mean that $u^{(i)}_{s_{i+1}}$ and $v^{(i)}_{s_{i+1}}$ are used to update $\boldsymbol{C}^{(p)}$ recursively. We do not need to carry out many recursive steps just for the computing of (7) with different p indices. We aim at just increasing the value of $Q(\boldsymbol{C} \mid \boldsymbol{C}^{(p)})$ instead of trying to obtain its maximum at every step. The recursive steps presented by (14) can be merged with the iterative steps presented by (6) and (7). Briefly speaking, we can replace the iterative index p with the recursive index i. For $1 \leqslant i \leqslant T_0$, the recursive computing steps are rewritten as follows:

$$\boldsymbol{C}^{(i+1)} = \boldsymbol{C}^{(i)} + \boldsymbol{K}^{(i+1)}((u^{(i)}_{s_{i+1}})^{-1}v^{(i)}_{s_{i+1}} - \boldsymbol{r}^{\mathrm{T}}_{i+1}\boldsymbol{C}^{(i)}) \tag{15}$$

where

$$\boldsymbol{K}^{(i+1)} = \frac{1}{(u^{(i)}_{s_{i+1}})^{-1} + \eta^{(i+1)}r_{i+1}}(\boldsymbol{r}^{\mathrm{T}}_{i+1}\boldsymbol{\delta}^{(i)})^{\mathrm{T}}$$

$$\boldsymbol{\delta}^{(i+1)} = \boldsymbol{\delta}^{(i)} - \boldsymbol{K}^{(i+1)}\boldsymbol{r}^{\mathrm{T}}_{i+1}\boldsymbol{\delta}^{(i)}$$

This idea has been presented and applied in many papers such as Refs. [13 – 14]. Lots of simulations show this recursive method achieves nearly the same performance as the direct application of (11). Although the performance of RPEMTO is not as good as the PEMTO, its complexity is far less than that of the PEMTO. At the same time the RPEMTO is a robust approach as it avoids matrix inversion.

2.3 1-D PEMTO or block PEMTO (BPEMTO)

Now we consider a particular case for PEMTO. If we select $K = 0$, the original 2-D estimation problem is turned into a 1-D estimation problem with some vectors being redefined as follows:

$$\boldsymbol{C} = \{C(0), \ldots, C(M-1)\}^{\mathrm{T}}$$

$$\boldsymbol{H} = \{H_{-I+i_0}, \ldots, H_{i_0}, \ldots, H_{I+i_0}\}^{\mathrm{T}}$$
$$\boldsymbol{q}_i = \{(i-i_0)^0, \ldots, (i-i_0)^{M-2}, \ldots, (i-i_0)^{M-1}\}^{\mathrm{T}}$$
$$\boldsymbol{R} = \{q_{-I+i_0}, \ldots, q_{I+i_0}\}^{\mathrm{T}}$$

Other vectors and matrices such as $\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{N}, \boldsymbol{V}$ and $\boldsymbol{U}$ are redefined in the same way. Although the vectors and matrices are modified, the framework of PEMTO remains the same and the EM iteration given in Eq. (12) is still valid. We call it block PEMTO (BPEMTO) since only one block of observed data $\boldsymbol{Y}$ is involved. Notice that the recursive approach given in Eq. (14) is also applicable.

The BPEMTO has the advantage of handling the observed data time-slot by time-slot (block by block). It requires no data storing in advance as the algorithm in Ref. [9] does. It is more appropriate for the sequential estimation. Compared to the blind GLRT detector given in Ref. [2], the BPEMTO is based on the EM method, resulting in better performance.

The initial value of $\boldsymbol{C}$ is obtained by the eigenvector-based method proposed in Ref. [6] which can be directly used for the BPEMTO. To make it fit for the PEMTO, we must run this method two times around the center point (i_0, k_0).

2.4 Cramer-Rao lower bound (CRLB)

The CRLB provides the MMSE bound of the estimator[8]. Defining $\boldsymbol{X}_M = \mathrm{diag}(\boldsymbol{X})$ and assuming $\boldsymbol{X}_M$ is known, we get the likelihood function from (5):

$$f(\boldsymbol{Y} \mid \boldsymbol{C}) = \frac{1}{(\pi\sigma^2)^{T_0}} \exp\left\{ -\frac{1}{\sigma^2} \|\boldsymbol{Y} - \boldsymbol{X}_M \boldsymbol{R}\boldsymbol{C}\|^2 \right\}$$

Define the t-th entry of $\boldsymbol{C}$ as C_t. The CRLB of C_t is

$$\mathrm{CRLB}(C_t) = I^{-1}(\boldsymbol{C})_{t,t} \tag{16}$$

where

$$I(\boldsymbol{C}) = -E\left[\frac{\delta}{\delta \boldsymbol{C}}\left(\frac{\partial \log f(\boldsymbol{Y} \mid \boldsymbol{C})}{\partial \boldsymbol{C}}\right)^{\mathrm{H}}\right] = \frac{\boldsymbol{R}^{\mathrm{H}} \boldsymbol{X}_M^{\mathrm{H}} \boldsymbol{X}_M \boldsymbol{R}}{\sigma^2}$$

is the Fisher information matrix and $\boldsymbol{I}^{-1}(\boldsymbol{C})_{t,t}$ is the (t, t) entry of the inversion matrix of $\boldsymbol{I}(\boldsymbol{C})$[9]. During the derivation process of (16), we assume $\boldsymbol{X}$ is deterministic. In fact, every entry in $\boldsymbol{X}$ is random. We can employ the modified CRLB (MCRB)[9] to provide a valid bound of $\boldsymbol{C}$.

$$\mathrm{MCRB}(C_t) = \frac{1}{E[I(\boldsymbol{C})_{t,t}]} = \frac{\sigma^2}{\sum_{k=-K+k_0}^{K+k_0} \sum_{i=-I+i_0}^{I+i_0} E[\,|x_{i,k}|\,]^2 q_{i,k,t}^2} \tag{17}$$

where $q_{i,k,t}$ is the t-th entry of the vector $\boldsymbol{q}_{i,k}$.

3 Simulation and Discussion

Assume an OFDM system with a total bandwidth of 1 MHz (the carrier frequency is 1 GHz) and M = 32 equally-spaced subchannels. The signal constellation is QPSK. The time-varying Rayleigh fading channel is simulated according to the COST 207 TU model[10], which has the delay profile {0.0, 0.2, 0.5, 1.6, 2.3, 5.0} μs and the power profile {0.189, 0.379, 0.239, 0.095, 0.061, 0.037}.

Fig. 1 gives the MSE of the PEMTO and the RPEMTO with $I = 15$, $K = 5$, $M = 4$, $N = 2$ and f_d = 0.012. Four pilot symbols with equal space are inserted to eliminate the phase ambiguity over each of $2K+1$ frames[6]. The overhead caused by the pilot symbols is only 1/88 of the total transmitted data. It can be found that PEMTO gets close to the CRLB when SNR is high. In other words, the PEMTO achieves excellent estimation with high spectral efficiency.

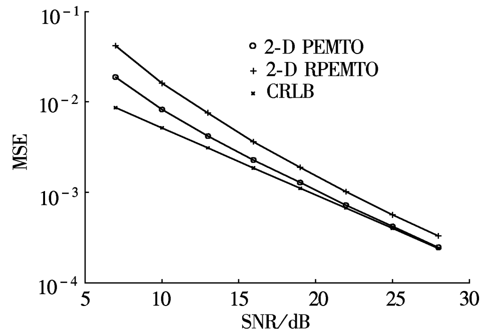

Fig. 1 SNR-MSE in time-varying OFDM systems (f_d = 0.012)

The average bit error rate (BER) versus SNR for different blind methods is shown in Fig. 2. These methods include the PEMTO, the RPEMTO and the method given in Ref. [6] which employs the 1-D polynomial-based branch and bound (BB) approach. For comparison, we also run simulations for the EM-based method offered in Ref. [9], which assumes that channels remain static. At the same time, the performance of one-tap equalization with perfect knowledge of the channel impulse response (CIR) is used as the benchmark. Notice that the SD method given in Ref. [2] and the BPEMTO are not simulated for 2-D estimation

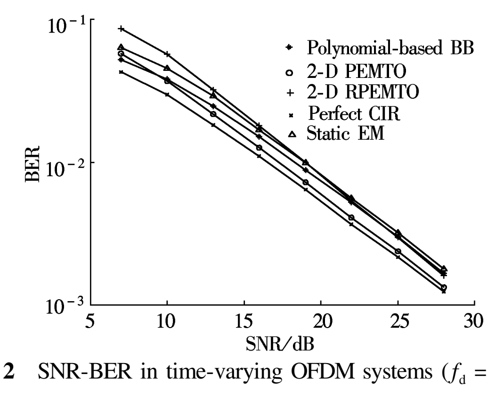

Fig. 2 SNR-BER in time-varying OFDM systems (f_d = 0.012)

as they are designed to carry out sequential estimation. If applied for 2-D observed data, they surely involve huge complexity, making them totally impractical.

Obviously, the PEMTO provides better performance than the EM-based method with static channel assumption. This may be explained by the fact that the PEMTO takes the channel variation into consideration. It can also be concluded that the PEMTO always offers better performance than the BB method since the PEMTO exploits the 2-D correlation information. When the SNR is high, the PEMTO achieves nearly a 1.5 dB gain over the BB-based method. The gap between the BER performance of the RPEMTO and the PEMTO is narrowed when the SNR increases.

Fig. 3 illustrates the BER versus the SNR for the BPEMTO, the recursive BPEMTO and the sphere decoder (SD) approach in Ref. [2] with $f_d = 0$, i. e, the static channel model. For the computing of the BPEMTO, we select $I = 15$, $K = 0$ and $M = 3$. The SD provides the best performance when the SNR is low. As the SNR rises, the BPEMTO outperforms the SD due to its ability to obtain the ML estimation. It can be found that the gap between the SD and the BPEMTO increases with the SNR. The increase of the SNR also makes the relative performance of the recursive BPEMTO become increasingly better. Notice that the performance gap between the recursive BPEMTO and the BPEMTO is less than 2 dB when the SNR is high.

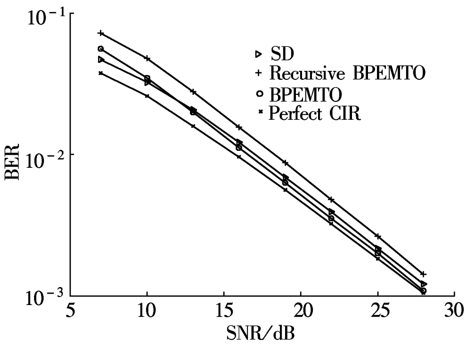

Fig. 3 SNR-BER for sequential estimation ($f_d = 0$)

The complexity versus the sequence size $(2I + 1)$ for the BPEMTO, the recursive BPEMTO and the SD is shown in Fig. 4. The number of float point operations (flops) is used as a measure of complexity[2]. The number of the flops of the BPEMTO is more than that of the SD when the sequence size is small. However, the BPEMTO involves less complexity than the SD with a large sequence size since the SD carries out an exhaustive search on the searching tree[2]. At the same time, the recursive PEMTO can be implemented with far less complexity.

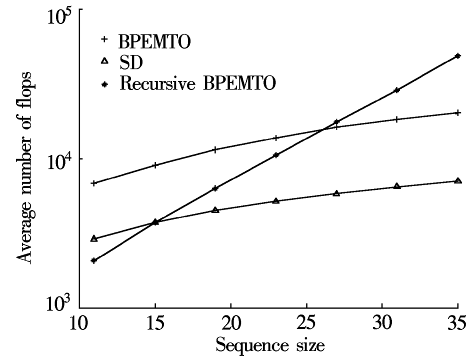

Fig. 4 Flops-sequence size $2I + 1$ ($f_d = 0$, SNR = 22 dB)

4 Conclusion

This paper investigates the application of the EM algorithm for OFDM channel estimation. Approximating the OFDM time-frequency response with a 2-D polynomial model, we turn the estimation of channel response into the estimation of some time-invariant coefficients which can be obtained effectively by the PEMTO/RPEMTO/BPEMTO algorithms presented in this paper. Simulations prove that these algorithms are appropriate for the blind estimation of OFDM systems.

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基于多项式模型的 OFDM 信道盲估计

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摘要:采用二元多项式模型对时变 OFDM 系统的时频响应进行建模. 在多项式模型的基础上,结合期望最大化(EM)方法的思想,提出了一种利用时频面上的二维数据来获取模型参数的最大似然(ML)估计值的算法(PEMTO). 为了降低计算复杂度,避免由于矩阵求逆而带来的风险,给出了 PEMTO 的一种迭代计算方法 (RPEMTO). PEMTO 算法在数学上进行简化后,可以用来进行一维序贯信道估计. 仿真结果显示,所提出算法的误码率低于其他类型的盲估计算法.

关键词:正交频分复用;期望最大化(EM);多项式模型;迭代

中图分类号:TN914. 3