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Reasoning complexity for extended fuzzy description logic with qualifying number restriction

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Abstract: To solve the extended fuzzy description logic with qualifying number restriction (EFALCQ) reasoning problems, EFALCQ is discretely simulated by description logic with qualifying number restriction (ALCQ), and ALCQ reasoning results are reused to prove the complexity of EFALCQ reasoning problems. The ALCQ simulation method for the consistency of EFALCQ is proposed. This method reduces EFALCQ satisfiability into EFALCQ consistency, and uses EFALCQ satisfiability to discretely simulate EFALCQ sat-domain. It is proved that the reasoning complexity for EFALCQ satisfiability, consistency and sat-domain is PSPACE-complete.

Key words: extended fuzzy description logic; qualifying number restriction; reasoning complexity

Description logics (DLs)[1] are a class of knowledge representation languages with well-defined semantics and determinable reasoning methods. Many applications often need to represent fuzzy knowledge, especially when dealing with text, multimedia or uncertain data. However, classical DLs are insufficient in dealing with such fuzzy knowledge. Therefore, it is necessary to add fuzzy features to description logics.

Straccia presented a fuzzy extension of typical ALC (FALC)[2], and gave a constraint propagation calculus for reasoning with empty TBoxes. However, FALC offers limited expressive power for fuzzy knowledge. To overcome its insufficiency, we presented a family of extended fuzzy description logics (EFDLs), in which cut sets of fuzzy concepts and fuzzy roles are considered as the atomic concepts and atomic roles[3-4]. Some reasoning techniques for EFDLs were discussed in Refs. [5 – 6]. This paper discusses the reasoning complexity for extended fuzzy description logic with qualifying number restriction and proves that the reasoning complexity for EFALCQ satisfiability, consistency and sat-domain is PSPACE-complete.

1 EFALCQ

ALCQ is an extension of ALC with qualifying number restriction[1]. Fuzzy extension of ALCQ is called EFALCQ[3], which introduces the cut sets of atomic fuzzy concepts and atomic fuzzy roles as atomic concepts and atomic roles. An EFALCQ knowledge base $\sum_{\mathrm{E}}(T_{\mathrm{E}}, H_{\mathrm{E}}, A_{\mathrm{E}})$ consists of TBox T_{E}, RBox H_{E} and ABox A_{E}. I satisfies a TBox T_{E}, iff I satisfies all cut concept axioms in T_{E}. I satisfies a RBox H_{E}, iff I satisfies all cut role axioms in H_{E}. I satisfies an ABox A_{E} iff I satisfies any cut assertion in A_{E}. I is a model of knowledge base $\sum_{\mathrm{E}}(T_{\mathrm{E}}, H_{\mathrm{E}}, A_{\mathrm{E}})$ iff I satisfies T_{E}, H_{E} and A_{E}. A knowledge base $\sum_{\mathrm{E}}(T_{\mathrm{E}}, H_{\mathrm{E}}, A_{\mathrm{E}})$ can support several reasoning problems. In this paper, we only consider reasoning problems with EFALCQ ABoxes. Since EFALCQ supports alterable cut concepts with suffix vectors that may contain variables and functions, we extend satisfiability into sat-domain[3].

Sat-domain For an alterable cut concept $C_{[f_1(n), \dots, f_k(n)]}$ in a given domain $n \in X_0 = [x_0, x_1]$, where $X_0 \subseteq (0, 1]$, and $f_i(n)$ is a linear function from domain X_0 to $(0, 1]$, the reasoning problem computes satisfiable and unsatisfiable sub-domains of X_0. For any $n_0 \in X_0$, if $C_{[f_1(n_0), \dots, f_k(n_0)]}$ is satisfiable, then n_0 is in the satisfiable sub-domain; otherwise, n_0 is in the unsatisfiable sub-domain.

2 Reasoning Properties for EFALCQ

Any cut concept or cut role of EFALCQ is composed of a prototype and a suffix vector. The suffix vector contains real numbers in $(0, 1]$ and brings new reasoning properties. We first give an obvious reasoning property dealing with suffix vectors of cut roles.

Theorem 1 For any two cut roles $R_{[n_1]}$ and $R_{[m_1]}$

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with the same prototype, if $n_1 \leqslant m_1$, then for any interpretation I, $(R_{[n_1]})^I \supseteq (R_{[m_1]})^I$ holds.

The similar property holds in the case of cut concepts. We introduce the definition of constraint signal sig( ) of prefixes: For any atomic fuzzy concept A that occurs in $C_{[n_1,\ldots,n_k]}$, $\mathrm{sig}(A) = (-1)^D \times (-1)^Q$, where D is the "$\leqslant N$" restriction depth of A in $C_{[n_1,\ldots,n_k]}$ and if $\neg$ occurs in front of A, $Q = 1$, otherwise $Q = 0$; for any atomic fuzzy role R that occurs in $C_{[n_1,\ldots,n_k]}$, $\mathrm{sig}(R) = (-1)^D \times (-1)^Q$, where D is the "$\leqslant N$" restriction depth of R in $C_{[n_1,\ldots,n_k]}$ and if R is restricted by $\forall$, $Q = 1$, otherwise $Q = 0$.

Theorem 2 Let $C_{[n_1,\ldots,n_k]}$ and $C_{[m_1,\ldots,m_k]}$ be two cut concepts with the same prototype C, and satisfy the following condition: For any the i-th prefix ($1 \leqslant i \leqslant k$), if its constraint signal is -1, then $n_i \geqslant m_i$, otherwise $n_i \leqslant m_i$, then for any interpretation I, the inclusion $(C_{[n_1,\ldots,n_k]})^I \supseteq (C_{[m_1,\ldots,m_k]})^I$ holds.

Proof We use length $(C_{[n_1,\ldots,n_k]})$ to denote the number of suffixes in the suffix vector of the cut concept $C_{[n_1,\ldots,n_k]}$, and $\mathrm{length}(C_{[n_1,\ldots,n_k]}) = k$ obviously.

① Induction base: When $k = 1$, the cut concepts have one of the following forms: $B_{[n_1]}$ and $\neg B_{[n_1]}$.

Case 1 For two cut concepts $B_{[n_1]}$ and $B_{[m_1]}$, $\mathrm{sig}(B) = 1$, then $n_1 \leqslant m_1$. For any interpretation I, $d \in (B_{[m_1]})^I \Rightarrow B^I(d) \geqslant m_1$. And for $B^I(d) \geqslant m_1 \geqslant n_1$, then $d \in (B_{[n_1]})^I$. Therefore, $(B_{[n_1]})^I \supseteq (B_{[m_1]})^I$ holds.

Case 2 For two cut concepts $\neg B_{[n_1]}$ and $\neg B_{[m_1]}$, $\mathrm{sig}(B) = -1$, then $n_1 \geqslant m_1$. For any interpretation I, for any $d \in (\neg B_{[m_1]})^I$, $B^I(d) < m_1 \Rightarrow B^I(d) < n_1$. Then $d \in (\neg B_{[n_1]})^I$. Therefore, $(\neg B_{[n_1]})^I \supseteq (\neg B_{[m_1]})^I$ holds.

② Induction hypothesis: Assume that when $k \leqslant L$ ($L \geqslant 1$), the theorem holds.

③ Induction step: When $k = L + 1$, the cut concepts have one of the following forms: $C_{[n_1,\ldots,n_h]} \cup (\cap) D_{[n_{h+1},\ldots,n_{L+1}]}$, $\exists(\forall) R_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]}$ and $\geqslant(\leqslant) NR_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]}$.

Case 1 For two cut concepts $C_{[n_1,\ldots,n_h]} \cup D_{[n_{h+1},\ldots,n_{L+1}]}$ and $C_{[m_1,\ldots,m_h]} \cup D_{[m_{h+1},\ldots,m_{L+1}]}$, obviously it is true that length $(C_{[n_1,\ldots,n_h]}) = h$, length $(C_{[n_{h+1},\ldots,n_{L+1}]}) = L - h + 1$ and $h, L - h + 1 \leqslant L$. For the constraint signals of all prefixes in $C_{[n_1,\ldots,n_h]}$ or $D_{[n_{h+1},\ldots,n_{L+1}]}$ and the ones in $C_{[n_1,\ldots,n_h]} \cup D_{[n_{h+1},\ldots,n_{L+1}]}$ are the same, two cut concepts satisfy the condition, which implies that $C_{[n_1,\ldots,n_h]}$ and $C_{[m_1,\ldots,m_h]}$ (respectively $D_{[n_{h+1},\ldots,n_{L+1}]}$ and $D_{[m_{h+1},\ldots,m_{L+1}]}$) satisfy the condition. From the induction hypothesis, for any I, $(C_{[n_1,\ldots,n_h]})^I \supseteq (D_{[m_1,\ldots,m_h]})^I$ and $(D_{[n_{h+1},\ldots,n_{L+1}]})^I \supseteq (D_{[m_{h+1},\ldots,m_{L+1}]})^I$. So $(C_{[n_1,\ldots,n_h]} \cup D_{[n_{h+1},\ldots,n_{L+1}]})^I \supseteq (C_{[m_1,\ldots,m_h]} \cup D_{[m_{h+1},\ldots,m_{L+1}]})^I$ holds from the definition of interpretation.

Case 2 For cut concepts $C_{[n_1,\ldots,n_h]} \cap D_{[n_{h+1},\ldots,n_{L+1}]}$ and $C_{[m_1,\ldots,m_h]} \cap D_{[m_{h+1},\ldots,m_{L+1}]}$, the proof is similar to that of case 1.

Case 3 For two cut concepts $\exists R_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]}$ and $\exists R_{[m_1]}.C_{[m_2,\ldots,m_{L+1}]}$, since the constraint signals of all prefixes in $C_{[n_2,\ldots,n_{L+1}]}$ and the ones in $\exists R_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]}$ are the same, if these two cut concepts satisfy the condition, $n_1 \leqslant m_1$ holds, and $C_{[n_2,\ldots,n_{L+1}]}$, $C_{[m_2,\ldots,m_{L+1}]}$ satisfy the condition. From theorem 1, for any interpretation I, we have $(R_{[n_1]})^I \supseteq (R_{[m_1]})^I$. From the induction hypothesis, $(C_{[n_2,\ldots,n_{L+1}]})^I \supseteq (C_{[m_2,\ldots,m_{L+1}]})^I$. If $d \in (\exists R_{[m_1]}.C_{[m_2,\ldots,m_{L+1}]})^I$, there exists $d' \in \Delta^I$ such that $(d, d') \in (R_{[m_1]})^I$ and $d' \in (C_{[m_2,\ldots,m_{L+1}]})^I$. It can be deduced that $(d, d') \in (R_{[n_1]})^I$ and $d' \in (C_{[n_2,\ldots,n_{L+1}]})^I$. This means that the assertion $d \in (\exists R_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]})^I$ holds. Then $(\exists R_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]})^I \supseteq (\exists R_{[m_1]}.C_{[m_2,\ldots,m_{L+1}]})^I$.

Case 4 For two cut concepts $\forall R_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]}$ and $\forall R_{[m_1]}.C_{[m_2,\ldots,m_{L+1}]}$, the proof is similar to that of case 3.

Case 5 For cut concepts $\geqslant NR_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]}$ and $\geqslant NR_{[m_1]}.C_{[m_2,\ldots,m_{L+1}]}$, since the constraint signals of all prefixes in $C_{[n_2,\ldots,n_{L+1}]}$ and the ones in $\geqslant NR_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]}$ are the same, if these two cut concepts satisfy the condition, $n_1 \leqslant m_1$, and $C_{[n_2,\ldots,n_{L+1}]}$ and $C_{[m_2,\ldots,m_{L+1}]}$ satisfy the condition. Similar to case 3, for any I, $(R_{[n_1]})^I \supseteq (R_{[m_1]})^I$ and $(C_{[n_2,\ldots,n_{L+1}]})^I \supseteq (C_{[m_2,\ldots,m_{L+1}]})^I$ hold. If $d \in (\geqslant NR_{[m_1]}.C_{[m_2,\ldots,m_{L+1}]})^I$, there are at least N individuals $d'_1, d'_2, \ldots, d'_N$ such that $(d, d'_i) \in (R_{[m_1]})^I$ and $d'_i \in (C_{[m_2,\ldots,m_{L+1}]})^I$, $1 \leqslant i \leqslant N$. It can deduce that $(d, d'_i) \in (R_{[n_1]})^I$ and $d'_i \in (C_{[n_2,\ldots,n_{L+1}]})^I$. Then $d \in (\geqslant NR_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]})^I$ holds, namely, $(\geqslant NR_{[n_1]}.C_{[n_2,\ldots,n_{L+1}]})^I \supseteq (\geqslant NR_{[m_1]}.C_{[m_2,\ldots,m_{L+1}]})^I$.

Case 6 Let $\leqslant NR_{[n_1]}.C_{[n_2,\ldots,n_k]}$ and $\leqslant NR_{[m_1]}.C_{[m_2,\ldots,m_k]}$ be two cut concepts, for any prefix in $C_{[n_2,\ldots,n_k]}$, its "$\leqslant N$" restriction depth increases by 1 and its constraint signal is multiplied by -1 in $\leqslant NR_{[n_1]}.C_{[n_2,\ldots,n_k]}$. Therefore, these two cut concepts satisfy the condition meaning $n_1 \geqslant m_1$ holds, and $C_{[m_2,\ldots,m_k]}$ and

$C_{[n_2,\ldots,n_k]}$, the pair in the reversed order, satisfies the condition. The following proof is similar to that of case 5.

From ① to ③, for any two cut concepts, the theorem holds.

3 Reasoning Complexity for EFALCQ

For any cut concept $C_{[n_1,\ldots,n_k]}$, size ($C_{[n_1,\ldots,n_k]}$) is defined as the number of symbols within its prototype C. For any cut assertion α, size (α) is defined as: If $\alpha = a: C_{[n_1,\ldots,n_k]}$, size($\alpha$) = size ($C_{[n_1,\ldots,n_k]}$); otherwise, size ($\alpha$) = 1. For any ABox A_E, size (A_E) is defined as the sum of the sizes of all cut assertions in A_E. We introduce the ALCQ simulation method for the consistency of EFALCQ ABox A_E. For any atomic fuzzy concept B and role R in A_E, we define $B^* = \{B_{[n]} \mid B_{[n]}$ appears in $A_E\}$ and $R^* = \{R_{[n]} \mid R_{[n]}$ appears in $A_E\}$, we order elements of B^* and R^* in an ascending order of their suffixes n. The i-th element in B^* (R^*) is denoted as $B_{[n_i]}$ ($R_{[n_i]}$). And we define $S_C = \{B^* \mid B$ appears in $A_E\}$ and $S_R = \{R^* \mid R$ appears in $A_E\}$. By the definition of S_C and S_R, we construct an ALCQ knowledge base $\sum(T, H, A) = \text{sim}(A_E)$ to simulate A_E, where sim(A_E) is defined as

$$T \overset{\text{def}}{=} \{B_{[n_{i+1}]} \subseteq B_{[n_i]} \mid B^* \in S_C \text{ and } 1 \leqslant i < |B^*|\}$$
$$H \overset{\text{def}}{=} \{R_{[n_{i+1}]} \subseteq R_{[n_i]} \mid R^* \in S_R \text{ and } 1 \leqslant i < |R^*|\}$$
$$A \overset{\text{def}}{=} A_E$$

In this case, $B_{[n_i]}$ and $R_{[n_i]}$ are considered as ALCQ atomic concept and atomic role. And obviously sim(A_E) can be constructed in linear time of size(A_E).

Theorem 3 For any EFALCQ ABox A_E, it is consistent iff the ALCQ knowledge base $\sum(T, H, A) = \text{sim}(A_E)$ has a model.

Proof $\Rightarrow$) Let $I_E(\Delta^{I_E}, \cdot^{I_E})$ be an EFALCQ model of A_E. We create an ALCQ interpretation $I(\Delta^I, \cdot^I)$ satisfying $\sum(T, H, A)$ from I_E in the following steps:

① $\Delta^I = \Delta^{I_E}$;

② For any individual a that occurs in A, $a^I = a^{I_E}$;

③ For any atomic concept $B_{[n_i]}$ and atomic role $R_{[n_i]}$ in A_E, $(B_{[n_i]})^I = (B_{[n_i]})^{I_E}$, $(R_{[n_i]})^I = (R_{[n_i]})^{I_E}$.

Now, we prove that I satisfies $\sum(T, H, A)$.

① For any $B_{[n_{i+1}]} \subseteq B_{[n_i]}$ in T: for $n_{i+1} > n_i$, $(B_{[n_{i+1}]})^{I_E} \subseteq (B_{[n_i]})^{I_E}$. For $(B_{[n_i]})^I = (B_{[n_i]})^{I_E}$, we can induce that $(B_{[n_{i+1}]})^I \subseteq (B_{[n_i]})^I$. Therefore I satisfies T.

② For any $R_{[n_{i+1}]} \subseteq R_{[n_i]}$ in H: the proof is similar to that of case ①.

③ For any assertion in A: the semantics of ALCQ concept constructors is unchanging in EFALCQ. Since $(B_{[n_i]})^I = (B_{[n_i]})^{I_E}$, $(R_{[n_i]})^I = (R_{[n_i]})^{I_E}$, for any $C_{[n_1,\ldots,n_k]}$ in A, ${C_{[n_1,\ldots,n_k]}}^I = {C_{[n_1,\ldots,n_k]}}^{I_E}$. And since the semantics of individuals are also equivalent ($a^I = a^{I_E}$) and I_E satisfies A_E and I satisfies A.

$\Leftarrow$) Let $I(\Delta^I, \cdot^I)$ be a model of $\sum(T, H, A)$. We create an EFALCQ interpretation g$I_E(\Delta^{I_E}, \cdot^{I_E})$ satisfying A_E from I in the following steps:

① $\Delta^{I_E} = \Delta^I$;

② For any individual a that occurs in A_E, $a^{I_E} = a^I$;

③ For any atomic fuzzy concept B and role R in A_E, for any $d, d' \in \Delta^{I_E}$, $B^{I_E}(d) = \max\{\{n_i \mid d \in (B_{[n_i]})^I\} \cup \{0\}\}$, $R^{I_E}(d, d') = \max\{\{n_i \mid (d, d') \in (R_{[n_i]})^I\} \cup \{0\}\}$.

Now, we prove I_E satisfies A_E.

Obviously, $(B_{[n_i]})^{I_E} = \{d \mid B^{I_E}(d) \geqslant n_i\} = \{d \mid \exists j, d \in (B_{[n_j]})^I \text{ and } j \geqslant i\} = \cup_{j \geqslant i}(B_{[n_j]})^I$. And for any i, $(B_{[n_{i+1}]})^I \subseteq (B_{[n_i]})^I$ holds, obviously for any $j \geqslant i$, $(B_{[n_j]})^I \subseteq (B_{[n_i]})^I$ holds. As a consequence, $(B_{[n_i]})^{I_E} = (B_{[n_i]})^I$. And $(R_{[n_i]})^{I_E} = (R_{[n_i]})^I$ is similarly proved. Since the semantics of ALCQ concept constructors are unchanging in EFALCQ, for any concept $C_{[n_1,\ldots,n_k]}$ in A_E, $(C_{[n_1,\ldots,n_k]})^{I_E} = (C_{[n_1,\ldots,n_k]})^I$. And since the semantics of individuals are also equivalent and I satisfies A, I satisfies A_E.

This theorem guarantees that consistency of EFALCQ ABox A_E can be converted into consistency of a semantically equivalent ALCQ knowledge base sim(A_E) in linear time of size(A_E).

Then, we can reduce satisfiability into consistency easily.

Theorem 4 For any EFALCQ cut concept $C_{n_1,\ldots,n_k}$, it is satisfiable iff the EFALCQ ABox $A_E = \{x: C_{[n_1,\ldots,n_k]}\}$ is consistent.

Guaranteed by the above theorem, satisfiability of cut concept $C_{[n_1,\ldots,n_k]}$ can be equivalently rewritten into consistency of $A_E = \{x: C_{[n_1,\ldots,n_k]}\}$. And such rewriting can be finished in linear time of size($C_{[n_1,\ldots,n_k]}$).

Finally, we use satisfiability to discretely simulate sat-domain. For any alterable cut concept $C_{[f_1(n),\ldots,f_k(n)]}$ $n \in X_0 = [x_0, x_1]$, the discrete simulation is based on the idea of equivalence partitioning of the given domain X_0. There are at most k functions in $C_{[f_1(n),\ldots,f_k(n)]}$'s suffix vector. We define the equivalence relation "$\equiv$": For any two points $n_x, n_y \in X_0$, $n_x \equiv n_y$ iff for any two functions $f_i(n)$ and $f_j(n)$, $1 \leqslant i < j \leqslant k$, $f_i(n_x)\#f_j(n_x) \Leftrightarrow f_i(n_y)\#f_j(n_y)$, where $\# \in \{<, >, =\}$. By using the equivalence relation "$\equiv$", we partition X_0 into some equivalence sub-domains.

Now we will prove the upper bound of the number of the equivalence sub-domains. For any two different linear functions $f_i(n)$ and $f_j(n)$, they have at most one point of intersection in X_0. Therefore, there are no more than $k(k-1)/2$ points of intersection for all k functions. The points of intersection of two equivalent functions $f_i()$ and $f_j()$ are ignored here, because they have no effect on the equivalence partitioning. For any $x \in X_0, f_i(x) = f_j(x)$ holds. We denote the set of points of intersection as P^*. Let P^* be in the increasing order of the value of the points and $N = |P^*|$. Now we prove that any $p_i \in P^*$, $\{p_i\}$ is an equivalence sub-domain, and the intervals (p_i, p_{i+1}) $1 \leqslant i < N$, $[x_0, p_1)$ and $(p_N, x_1]$ are equivalence sub-domains.

① For any $p_i \in P^*$, p_i must be the point of intersection of two different functions $f_s(n)$ and $f_l(n)$. Since $f_s(n)$ and $f_l(n)$ have at most one point of intersection, no another point y satisfies $f_s(y) = f_l(y)$. That means no another point y satisfies $x \equiv y$. Therefore, $\{p_i\}$ is an equivalence sub-domain.

② For any $n_x < p_i < n_y$, let p_i be the point of intersection of two different functions $f_s(n)$ and $f_l(n)$. Since $f(n) = f_s(n) - f_l(n)$ is also a linear function and $f(p_i) = 0$, $f(n_x)f(n_y) < 0$ holds. That means $f_s(n_x) > (<) f_l(n_x) \Rightarrow f_s(n_y) < (>) f_l(n_y)$. n_x and n_y are in two equivalence sub-domains.

③ For any n_x, n_y in (p_i, p_{i+1}), $f_s(n)$ and $f_l(n)$ are two different functions, let $f(n) = f_s(n) - f_l(n)$. Since $f(n)$ is also a linear function and there is no point z in (p_i, p_{i+1}) satisfying $f(z) = 0$, then $f(n_x)f(n_y) > 0$. That means $f_s(n_x)\#f_l(n_x)$ iff $f_s(n_y)\#f_l(n_y)$. Therefore, $n_x \equiv n_y$ holds. And from ②, (p_i, p_{i+1}) is an equivalence sub-domain.

④ For any x, y in $[x_0, p_1)$ or $(p_N, x_1]$, $n_x \equiv n_y$ obtains a similar result from the proof in ③. And from ②, $[x_0, p_0)$ or $(p_N, x_1]$ is an equivalence sub-domain.

Since $|P^*| \leqslant k(k-1)/2$ and the number of equivalence sub-domains is no more than $2|P^*| + 1$, then the upper bound of the number of equivalence sub-domains is $k(k-1) + 1$. After equivalence partitioning, we will prove that "$\equiv$" can cause the equivalent results of satisfiability.

Theorem 5 For any $n_x, n_y \in X_0$ and $n_x \equiv n_y$, the cut concept $C_{[f_1(n_x), \ldots, f_k(n_x)]}$ is satisfiable iff $C_{[f_1(n_y), \ldots, f_k(n_y)]}$ is satisfiable.

Proof $\Rightarrow$) Let $I(\Delta^I, \cdot^I)$ satisfy $C_{[f_1(n_x), \ldots, f_k(n_x)]}$. We create an interpretation $I_E(\Delta^{I_E}, \cdot^{I_E})$ satisfying $C_{[f_1(n_y), \ldots, f_k(n_y)]}$ from I in the following steps:

① $\Delta^{I_E} = \Delta^I$;

② For any atomic fuzzy concept B and atomic fuzzy role R that appear in C, for any $d, d' \in \Delta^{I_E}$,

$$B^{I_E}(d) = \max\{\{f_i(n_y) \mid d \in (B_{[f_i(n_x)]})^I\} \cup \{0\}\}$$

$$R^{I_E}(d, d') = \max\{\{f_i(n_y) \mid (d, d') \in (R_{[f_i(n_x)]})^I\} \cup \{0\}\}$$

Now, we prove I_E satisfies $C_{[f_1(n_y), \ldots, f_k(n_y)]}$.

From the definition of B^{I_E} and R^{I_E}, we can obtain

$$(B_{[f_i(n_y)]})^{I_E} = \{d \mid B^{I_E}(d) \geqslant f_i(n_y)\} =$$
$$\{d \mid \exists f_j, d \in (B_{[f_j(n_x)]})^I \text{ and } f_j(n_y) \geqslant f_i(n_y)\}$$

For $n_x \equiv n_y, f_j(n_y) \geqslant f_i(n_y) \Leftrightarrow f_j(n_x) \geqslant f_i(n_x)$,

$$(B_{[f_i(n_y)]})^{I_E} = \{d \mid \exists f_j, d \in (B_{[f_j(n_x)]})^I \text{ and } f_j(n_x) \geqslant f_i(n_x)\} =$$
$$\bigcup_{f_j(n_x) \geqslant f_i(n_x)} (B_{[f_j(n_x)]})^I = (B_{[f_i(n_x)]})^I$$

Similarly for any $R_{[f_i(n_y)]}$ in $C_{[f_1(n_y), \ldots, f_k(n_y)]}$, $(R_{[f_i(n_y)]})^{I_E} = (R_{[f_i(n_x)]})^I$. From the definition, $(C_{[f_1(n_x), \ldots, f_k(n_x)]})^{I_E} = (C_{[f_1(n_y), \ldots, f_k(n_y)]})^I \neq \varnothing$. Therefore, I_E satisfies $C_{[f_1(n_y), \ldots, f_k(n_y)]}$.

$\Leftarrow$) The proof is the same as $\Rightarrow$).

From theorem 5, we can obtain that any equivalence sub-domain is completely belonging to an either satisfiable or unsatisfiable sub-domain. Therefore, we can check satis-fiability of the alterable cut concept at a single point instead of in the whole equivalence sub-domain. Since the number of equivalence sub-domains is no more than $k(k-1)+1$, the sat-domain of $C_{[f_1(n), \ldots, f_k(n)]}$ in X_0 can be converted into at most $k(k-1)+1$ satisfiability of $C_{[f_1(n_i), \ldots, f_k(n_i)]}$, where n_i is in X_0.

From the above three theorems, we have proved that the three EFALCQ reasoning problems can be polynomially converted into consistency of ALCQ. Since consistency of ALCQ is PSPACE-complete[7], we have the following theorem.

Theorem 6 The reasoning complexity for EFALCQ satisfiability, consistency and sat-domain is PSPACE-complete.

4 Conclusion

The reasoning complexity for EFALCQ is discussed. To solve the EFALCQ reasoning problems, EFALCQ is discretely simulated by ALCQ, and ALCQ reasoning results are reused to prove the complexity of EFALCQ reasoning problems. It is proved that the reasoning complexity for EFALCQ satisfiability, consistency and sat-domain is PSPACE-complete. This work can be considered as a new idea of extending DLs with fuzzy features. Further work will focus on reasoning techniques for the fuzzy extension of more expressive DLs with more concept and role constructors.

References

[1] Baader F, Calvanese D, McGuinness D L, et al. *The description logic handbook: theory, implementation, and applications* [M]. Cambridge: Cambridge University Press, 2003.

[2] Straccia U. Reasoning within fuzzy description logics [J]. *Journal of Artificial Intelligence Research*, 2001, **14**: 137 − 166.

[3] Lu Jianjiang, Xu Baowen, Li Yanhui, et al. A family of extended fuzzy description logics [J]. *International Journal of Business Intelligence and Data Mining*, 2006, **1**(4): 384 − 400.

[4] Li Yanhui, Xu Baowen, Lu Jianjiang, et al. An extended fuzzy description logic [J]. *Journal of Southeast University: Natural Science Edition*, 2005, **35**(5): 683 − 687. (in Chinese)

[5] Li Yanhui, Xu Baowen, Lu Jianjiang, et al. On the computational complexity of the extended fuzzy description logic with numerical constraints [J]. *Journal of Software*, 2006, **17**(5): 968 − 975. (in Chinese)

[6] Li Yanhui, Lu Jianjiang, Xu Baowen, et al. A fuzzy extension of description logic ALCH [C]//*Lecture Notes in Artificial Intelligence*. Springer-Verlag, 2005, **3789**: 152 − 161.

[7] Baader F, Sattler U. An overview of tableau algorithms for description logics [J]. *Studia Logica*, 2001, **69**(1): 5 − 40.

限制数量约束的扩展模糊描述逻辑的推理复杂性

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摘要:为了解决限制数量约束的扩展模糊描述逻辑(EFALCQ)的推理复杂性问题,采用限制数量约束的描述逻辑(ALCQ)离散模拟 EFALCQ,并重用 ALCQ 的推理结论来证明 EFALCQ 推理问题的复杂性.提出了 EFALCQ 一致性推理问题的 ALCQ 模拟方法,将 EFALCQ 的可满足性推理问题转换为 EFALCQ 的一致性推理问题,并用 EFALCQ 的一致性推理问题离散模拟 EFALCQ 的可满足区间推理问题.最后证明了 EFALCQ 的可满足性、一致性、以及可满足区间推理问题的推理复杂性是 PSPACE-complete 问题.

关键词:扩展模糊描述逻辑;限制数量约束;推理复杂性

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