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Synthetical condition assessment of long span suspension bridge based on closeness degree and FAHP

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Abstract: Based on the theory of pattern recognition, the concept of closeness degree between fuzzy sets is brought into the condition assessment of long span bridges. Using the fuzzy analytic hierarchy process (FAHP), a mathematical model of a multi-objective assessment of a long span suspension bridge is set up. An example is given to show the procedure in the synthetical condition assessment of the Runyang Suspension Bridge, which includes the hierarchical division, the definition of factor weights and fuzzy membership functions, and the calculation of closeness degrees, etc. The assessment combines both the data from the health monitoring system and the manual tests. The classification of evaluation items as well as the calculation of deterministic and nondeterministic items is presented. Compared with the traditional method of point rating, this method can better describe the discreteness of monitoring data and the fuzziness in the condition assessment.

Key words: pattern recognition; fuzzy analytic hierarchy process; closeness degree; synthetical condition assessment; suspension bridge

Much attention has been paid on the health monitoring of long span bridges, and many large bridges have been equipped with health monitoring systems. However, the work on data processing and assessment remains somewhat neglected. As a complex system, the long span bridge is influenced by many factors, and relationships among these factors and the behavior of the bridge are rather complicated. Most of these factors cannot be described quantitatively by a few functions or evaluation items. Furthermore, some items themselves have features of fuzziness and discreteness. In such cases, the traditional qualitative assessment methods are rather limited, and they need to be improved by the use of theories and methods of “soft science”[1-6]. Based on the theory of pattern recognition, the concept of closeness degree between fuzzy sets is brought into the synthetical assessment of bridges. Using the fuzzy analytic hierarchy process (FAHP), a mathematical model of multi-hierarchy assessment of a long span suspension bridge is set up and an example is also provided in this paper.

1 Closeness Degree

In the condition assessment of a long span bridge, sometimes the evaluation item is one or a series of deterministic numbers, while more often than not, the item turns out to be a continuous fuzzy set with a certain distribution character, or a series of deterministic and discrete numbers.

For a limited field $\boldsymbol{U}=\{x_1, x_2, x_3, \dots, x_n\}$, $\boldsymbol{A}$ or $\boldsymbol{B}$ is the fuzzy vector on $\boldsymbol{U}$, respectively, where $\boldsymbol{A}=\{a_1, a_2, a_3, \dots, a_n\}$, $\boldsymbol{B}=\{b_1, b_2, b_3, \dots, b_n\}$. Let $\boldsymbol{A}\cdot\boldsymbol{B}$ and $\boldsymbol{A}\odot\boldsymbol{B}$ be the inner product and outer product of fuzzy sets $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively, $\boldsymbol{A}\cdot\boldsymbol{B}$ and $\boldsymbol{A}\odot\boldsymbol{B}\in[0, 1]$, then there are formulas as follows:

$$\boldsymbol{A}\cdot\boldsymbol{B}=\bigvee_{k=1}^{n}(a_i\wedge b_i) \tag{1}$$

$$\boldsymbol{A}\odot\boldsymbol{B}=\bigwedge_{k=1}^{n}(a_i\vee b_i) \tag{2}$$

By Eqs. (1) and (2), the fuzzy closeness degree can be constructed to describe the proximity or distance of fuzzy sets $\boldsymbol{A}$ and $\boldsymbol{B}$ as follows:

$$(\boldsymbol{A},\boldsymbol{B})=\frac{1}{2}[\boldsymbol{A}\cdot\boldsymbol{B}+(1-\boldsymbol{A}\odot\boldsymbol{B})] \tag{3}$$

where $(\boldsymbol{A}, \boldsymbol{B})$ is called the Zadeh closeness degree. Provided that $\boldsymbol{A}$ is the data set of a stress gauge within a certain period, and $\boldsymbol{B}$ is the comment set “Excellent”, the closeness degree $(\boldsymbol{A},\boldsymbol{B})$ shows how close the evaluation item “stress” is to the condition grade “Excellent”.

To enhance the application of a closeness degree in a bridge assessment, sometimes the concept of a “Grill closeness degree”[7] can be adopted,

$$\zeta_{\mathrm{g}}(\boldsymbol{A},\boldsymbol{B})=(\boldsymbol{A}\cdot\boldsymbol{B})\wedge(1-\boldsymbol{A}\odot\boldsymbol{B}) \tag{4}$$

where $\zeta_{\mathrm{g}}(\boldsymbol{A},\boldsymbol{B})$ represents the Grill closeness degree.

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2 Mathematical Model of Multi-Objective Assessment

On the assumption that there are two limited fields $\boldsymbol{U} = \{x_1, x_2, x_3, \ldots, x_n\}$, $\boldsymbol{V} = \{y_1, y_2, y_3, \ldots, y_n\}$, where $\boldsymbol{U}$ is the factor set composed of a number of factors in a multi-objective assessment; $\boldsymbol{V}$ is the comment set composed of several assessment attributes. Moreover, different factors have various weights in the final evaluation, so the factor weight $\boldsymbol{W}$ is itself a fuzzy vector on $\boldsymbol{U}$, where $\boldsymbol{W} = \{w_1, w_2, w_3, \ldots, w_n\} \in F(\boldsymbol{U})$, $\sum_{i=1}^{n} w_i = 1$. Likewise, m numbers of comment are not absolute affirmation or denial in fuzzy cases, so the results of a synthetical assessment can also be regarded as a fuzzy set on $\boldsymbol{V}$, namely,

$$\boldsymbol{B} = \{b_1, b_2, b_3, \ldots, b_n\} \in F(\boldsymbol{U})$$

where b_j denotes the portion of the j-th remark in the whole assessment objective $\boldsymbol{V}$.

Let $\boldsymbol{R}$ be the fuzzy relationship between $\boldsymbol{U}$ and $\boldsymbol{V}$, $\boldsymbol{R} = (r_{ij})_{n \times m}$. Then by $\boldsymbol{R}$ the fuzzy transform T_{R} can be applied, and the corresponding mathematical model of a fuzzy multi-objective assessment can be established as follows:

① Factor set $\boldsymbol{U} = \{x_1, x_2, x_3, \ldots, x_n\}$. For example, the factor set "concrete quality" is made up of the concrete strength, the depth of carbonization, the potential of steel corrosion, the thickness of concrete cover, and the concrete crack.

② Comment set $\boldsymbol{V} = \{y_1, y_2, y_3, \ldots, y_n\}$. For example, the comments "Excellent (E), Good (G), Fair (F), Bad (B), Unacceptable (U)" make up the comment set "potential of steel corrosion". When the potential exceeds -200 mV, meaning no corrosion occurs in this area, the score is 100, and the corresponding comment is "Excellent", while when the potential is under -350 mV, the possibility of corrosion overruns 90%, the score is 60, and the corresponding comment is "Unacceptable". Medium situations can be treated by interpolation.

③ Fuzzy transform $T_{\mathrm{R}}: F(\boldsymbol{U}) \to F(\boldsymbol{V})$

$$\boldsymbol{A} \mapsto \boldsymbol{A} \circ \boldsymbol{R} \tag{5}$$

where $\boldsymbol{R}$ is the fuzzy relationship matrix between $\boldsymbol{U}$ and $\boldsymbol{V}$, $\boldsymbol{R} = (r_{ij})_{n \times m}$.

In this way, the ternary system $(\boldsymbol{U}, \boldsymbol{V}, \boldsymbol{R})$ forms a mathematical model of the fuzzy multi-objective assessment. Now, if a weight vector $\boldsymbol{W}$ is input, where $\boldsymbol{W} = \{w_1, w_2, w_3, \ldots, w_n\} \in F(\boldsymbol{U})$, through the fuzzy transform T_{R}, a synthetical assessment result $\boldsymbol{B}$ can be obtained, where $\boldsymbol{B} = \{b_1, b_2, b_3, \ldots, b_n\} \in F(\boldsymbol{U})$, namely,

$$\{b_1, b_2, b_3, \ldots, b_n\} = \{w_1, w_2, w_3, \ldots, w_n\}^{\mathrm{T}} \circ \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \tag{6}$$

Using the Zadeh operator or weighted average operator, the final assessment result b_k can be gained according to the principle of maximal membership, where $b_k = \max\{b_1, b_2, b_3, \ldots, b_n\}$.

For a long span bridge, there are many factors to be considered, which may belong to different hierarchies or categories, so the definition of factor weights must be carried out in different hierarchies. In this study, the hierarchies of a bridge assessment is partitioned using FAHP[8], and the triangular fuzzy number is substituted for the traditional "0 to 9" scale. Finally, based on the possibility formula, the factor weights are computed by the priority of a judgment matrix. The results may embody the fuzziness of expert judgment more effectively. Subsequently, the assessment will be made in different levels, using the evaluation model presented above. By summarizing these results level by level through T_{R}, the final statement on a bridge condition can be obtained.

3 Example

With a main span of 1 490 m, the Runyang Suspension Bridge is the longest bridge in China and the third longest in the world when it was opened to traffic in 2005. To get a comprehensive understanding of the performance of the bridge, a structural health monitoring system was established. The topological structure of the health monitoring system is shown in Fig. 1.

Provided that a thorough test is conducted after some years of service, the test includes the stress of suspenders and main cables, the line shape of steel box girder and tower, the corrosion of steel, and the concrete properties, etc. Now combined with the data from the monitoring system, a comprehensive assessment is made using FAHP.

First, the membership function of the comment set is set up. As for the traditional method of point rating, the comment set is deterministic. When the comment score is near the boundary of two grades, a little change may lead to absolutely different results, see Fig. 2 (a). Obviously, this is not acceptable. To solve this problem, a membership function of a comment set is defined in Fig. 2(b), which can provide a better description about the fuzziness of a comment.

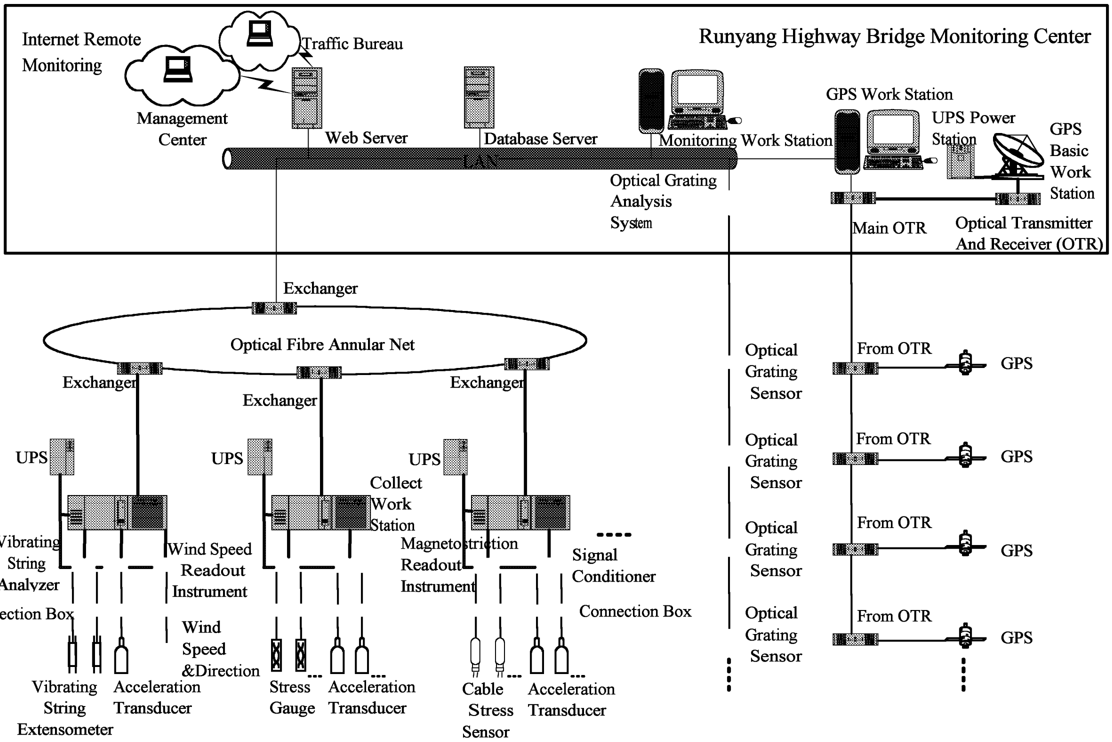

Fig. 1 Topological structure of the health monitoring system of Runyang Highway Bridge

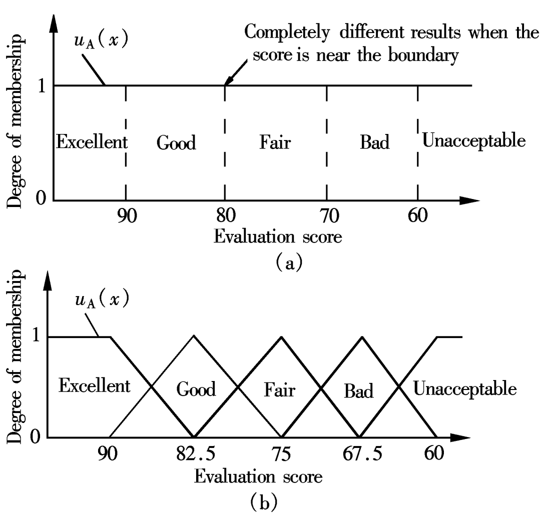

Fig. 2 Membership functions of deterministic comment set and fuzzy comment set. (a) Membership function of deterministic comment; (b) Membership function of fuzzy comment

According to the degree of certainty, the evaluation items can be divided into deterministic and nondeterministic ones. Deterministic items include the concrete strength and the stress of cables, which turn out to be a number or a series of numbers in a single test. Nondeterministic items include subjective nondeterministic items and objective nondeterministic items. The former are like the remarks on steel corrosion, where there is some subjective uncertainty on the part of the test personnel. The latter are like the data from a stress gauge, which turns out to be a statistical eigenvalue.

3.1 Assessment on deterministic items

If the evaluation item is only a single number, for example, the concrete strength, the score can be gained directly by test personnel according to Refs. [8 − 9]. If the test item is a series of numbers, for example, the suspender stress, the score will be the product of uniform and non-uniform change coefficients. Taking the suspender stresses of Runyang Suspension Bridge as an example, 40 of them are shown in Fig. 3 (360 in total).

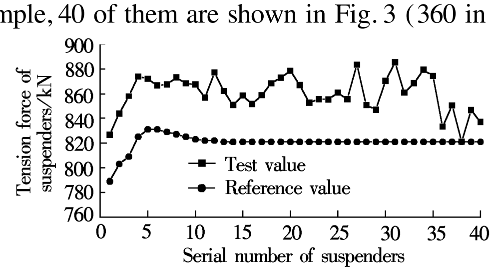

Fig. 3 Test values and reference values of suspender stresses

According to the characteristic of suspender stress assessment, it belongs to the moderate item, and the design value or the value from the completion test should be taken as the reference value. Here the former is taken. First, each suspender stress should be transferred into a non-dimensional number. Provided that the score is 100 if there is no change in suspender stress, and when the change reaches 40%, the score will decrease to 60, and the medium situation can be treated by a linear interpolation; hence, the evaluation score of each suspender can be computed. Meanwhile, considering the uniformity and non-uniformity in the change of suspender stress, the integral evaluation result can be computed by

$$S_{\mathrm{spd}}^{\mathrm{T}} = S_{\mathrm{spd}}^{\mathrm{M}} \eta(X_0, X_i) \tag{7}$$

where $S_{\mathrm{spd}}^{\mathrm{T}}$ is the integral evaluation score; $S_{\mathrm{spd}}^{\mathrm{M}}$ is the score by uniform change; $S_{\mathrm{spd}}^{\mathrm{M}} = \sum_{i=1}^{n} \frac{x_i}{n}$, n is the number of measuring points; $\eta(X_0, X_i)$ is the coefficient of non-uniform change, computed by the correlation degree of the slope ratio[10]; X_0 and X_i are the reference and test values, respectively.

By Eq. (7), the scores of these suspenders can be obtained, see Tab. 1.

Tab. 1 Evaluation results of suspender stresses

$S_{\mathrm{spd}}^{\mathrm{M}}$	$\eta(X_0, X_i)$	$S_{\mathrm{spd}}^{\mathrm{T}}$
95. 2	0. 987	94. 0

In the same way, the scores of other parts of suspenders can be obtained, provided that all the parts have the same weight, finally the total score is 94. 4 on average.

For a deterministic item, whose membership function turns out to be a vertical line (see Fig. 4), the closeness degree is the vertical coordinate at the intersecting point of the line and a comment set (oblique line).

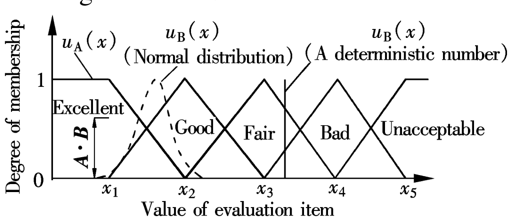

Fig. 4 Relationship of membership functions between fuzzy sets

3. 2 Assessment of nondeterministic item

3. 2. 1 Discrete item

For the discrete item, using the statistical results within a monitoring period, the statistical eigenvalue can be obtained, for example, the mean value μ and the standard deviation σ. Provided that the monitoring data follow normal distribution, the membership function of the monitoring data set and the comment set can be described by

$$u_{\mathrm{A}}(x) = ax + b \tag{8}$$

$$u_{\mathrm{B}}(x) = \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{9}$$

To get the value of the Grill closeness degree in Eq. (4), the inner product and outer product are to be calculated first. According to their meanings, $\boldsymbol{A} \odot \boldsymbol{B} = 0$, and the outer product $\boldsymbol{A} \cdot \boldsymbol{B}$ is the vertical intersection coordinate of the membership function, which can be calculated by Eq. (10). The solution can be obtained by iteration.

$$ax + b = \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{10}$$

Taking the assessment of maximal vertical displacement from No. 1 GPS sensor as an example, μ and σ are 2. 157 m and 0. 150 m, respectively, the condition grades are shown in Tab. 2. According to Eq. (10), the closeness degrees can be obtained by iteration, see Tab. 2. Calculating the closeness degree of each sensor and using the Zadeh operator, the final closeness degree of GPS can be achieved.

Tab. 2 Closeness degree of data from No. 1 GPS sensor

Item	E	G	F	B	U
Maximum vertical displacement/m	≤2. 031	2. 765	3. 499	4. 233	≥4. 967
Closeness degree of No. 1 GPS sensor	0. 914 6	0. 435	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮
Synthetical closeness degree of displacement (GPS)	0. 921 2	0. 356 4	0. 010 1	0	0

3. 2. 2 Subjective uncertain item

For the subjective uncertain item, the score can be gained according to Refs. [8 −9]. For the traditional method, the uncertainty of test personnel is ignored, while here the fuzzy indicator γ is used, which takes 0. 01, 0. 6, 1. 2, 1. 8 and 2. 5 according to the certainty categories of quite certain, moderately certain, certain, not quite certain and uncertain, respectively. Tab. 3 shows the remark of box girder (in the middle hierarchy).

Tab. 3 Comment result of girder in the middle hierarchy

Item	Crack of welding	Corrosion of steel	Pavement condition	Smooth of bridge
Mean value of score	85	85	90	90
Fuzziness index γ	1. 2	1. 8	0. 6	0. 6

By the calculation of the closeness degree between fuzzy sets, the results are obtained, as listed in Tab. 4. Notice that the evaluation items of line shape, stress and vibration can be calculated directly according to the data from the monitoring system.

Tab. 4 Closeness degree of girder in the middle hierarchy

Evaluation item	Weight	E	G	F	B	U
Crack of welding	0. 07	0. 515 1	0. 779 5	0. 005 5	0	0
Corrosion of steel	0. 05	0. 580 4	0. 818 6	0. 142 7	0	0
Pavement condition	0. 08	1	0. 155 3	0	0	0
Smooth of bridge	0. 07	1	0. 155 3	0	0	0
Line shape	0. 17	0. 921 2	0. 356 4	0. 010 1	0	0
Stress	0. 41	1	0	0	0	0
Vibration	0. 15	0. 935 6	0. 009 6	0	0	0
Synthetical closeness degree		0. 922 0	0. 180 8	0. 009 2	0	0

After all the closeness degrees in the bottom and middle hierarchies have been calculated, through a similar operation level by level, the results in the top hierarchy can be finally obtained (see Tab. 5).

Tab. 5 Evaluation result of suspension bridge in the top hierarchy

Evaluation item	Weight	E	G	F	B	U
Infrastructure	0.29	0.886 7	0.473 3	0.001 0	0	0
Main cable	0.19	0.932 2	0.110 4	0	0	0
Suspender	0.13	1	0	0	0	0
Box girder	0.12	0.922 0	0.180 8	0.009 2	0	0
Tower	0.24	0.947 1	0.089 9	0	0	0
Accessory	0.03	0	0.434 6	0.556 7	1.3×10^{-3}	0
Synthetical closeness degree		0.902 2	0.214 5	0.018 1	3.9×10^{-5}	0

Finally according to the principle of maximal membership in pattern recognition, the state of the suspension bridge is "Excellent".

4 Conclusions

1) The traditional method of point rating is deterministic on the choice of evaluation items and criteria, Taking the monitoring value of a stress gauge as an example, whether the mean value or the maximal value is taken, the discreteness and fuzziness can hardly be fully considered. However, using the FAHP and the closeness degree, the relationships between the evaluation target and their grades will be better described.

2) The synthetical assessment in this paper combines the information from both the monitoring system and the manual tests. The deterministic and nondeterministic evaluation items can be considered at the same time, so the content of the synthetical assessment has been enhanced.

3) Because the process of a fuzzy multilevel assessment of a suspension bridge is comparatively complicated and there are many evaluation items, usually certain software is necessary to solve these problems. Introduction to the software can be seen in Ref. [8].

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基于贴近度与模糊层次分析的大跨悬索桥综合状态评估

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摘要:基于模式识别理论,将模糊集贴近度的概念引入大跨桥梁状态评估中,利用模糊层次分析,建立起大跨悬索桥多目标评估的数学模型.以润扬大桥悬索桥的综合状态评估为例,介绍了评估的基本程序,包括评估指标的层次划分、指标权重的确定、模糊集隶属函数的定义以及贴近度的计算等.评估综合了健康监测系统的采集数据和人工监测结果2方面的信息,同时还给出了评估指标的分类方法和确定性指标、不确定性指标的计算方法.与传统的专家打分法相比,该方法更好地体现了监测信息的离散性、不确定性和评估过程的模糊性.

关键词:模式识别;模糊层次分析;贴近度;综合状态评估;悬索桥

中图分类号:U448.25