

# Bi-normalized response spectra and scalar periods for developing uniform seismic design spectra

Xu Longjun<sup>1,2</sup> Xie Lili<sup>2</sup>

(<sup>1</sup>Engineering College, Ocean University of China, Qingdao 266100, China)

(<sup>2</sup>School of Civil Engineering, Harbin Institute of Technology, Harbin 150090, China)

**Abstract:** Aiming at the uniform features of acceleration response spectra, two scalar periods—the response spectral predominant period  $T_p$  and the smoothed spectral predominant period  $T_o$  are employed to normalize the abscissa of the normalized response spectra (NRS) of ground motions, respectively. Engineering characteristics of 5% -damped NRS, and the bi-normalized response spectra (BNRS) are investigated accounting for the effects of soil condition and fault distance. Nearly 600 horizontal ground motion components during the Chi-Chi earthquake are included in the analysis. It shows that the NRS strongly depends on soil condition and fault distance. However, soil condition and distance have only a slight influence on two kinds of BNRS. Dispersion analysis indicates that such normalization can reduce scatter in the derivation of response spectral shapes. Finally, a parametric analysis of the scalar periods ( $T_p$ ,  $T_o$ ) is performed and then compared with those of previous studies. These special and particular aspects of earthquake response spectra and scalar periods need to be considered in developing earthquake-resistant design criteria.

**Key words:** ground motion; normalized response spectrum; bi-normalized response spectra; scalar period

The earthquake response spectrum is an important ground motion parameter because it is the basis on which to construct design spectra, which can greatly affect seismic safety and economy of structures. A uniform, yet reasonable representation of ground motion response spectra is useful in earthquake engineering practice to formulate guidelines for structure seismic design.

Earthquake factors include source mechanisms, epicentral distances, focal depths, geological conditions, magnitudes, soil conditions, rupture directivities, damping ratios and period effect spectral shapes and amplifications<sup>[1]</sup>. To consider the influences of some of the factors, previous studies generally computed the response spectra of ground motions with similar characteristics<sup>[2-3]</sup>. The differences and similarities between the spectra from classified ground motions are used to formulate recommendations for earthquake resistance design. This conventional procedure mainly underscores the discrepancies of spectral features rather than exploring the uniform features of response spectra. So, much work is needed to identify and quantify new characteristics of response spectra to address issues concerning the common aspects of ground motions.

A scalar period that can fully describe the fre-

quency content of ground motions is equally important to construct design spectra for applications under different seismic environment cases. Recent studies have described several frequency content periods and also developed predictive relationships for the periods<sup>[4]</sup>. In this paper, to extend the concepts and to explore the uniform feature about earthquake response spectra, two frequency content periods (the spectral predominant period  $T_p$  and the smoothed spectral predominant period  $T_o$ ) are used to normalize the abscissa of the conventional NRS, respectively. The characteristics of the NRS and the bi-normalized response spectrum (BNRS), as well as the distributions of the scale periods regarding ground motion factors are investigated. For this reason, nearly 600 horizontal acceleration components recorded in recent the 1999 Chi-Chi earthquake are involved in the analysis.

## 1 Bi-Normalized Response Spectra Concept

Researches on response spectra have recently begun to underscore the uniform features among ground motions<sup>[5-8]</sup>. Xu and Xie attempted to identify similarities of the acceleration response spectrum of wide frequency-band strong motions<sup>[5]</sup> and found that the existing NRS of ground motions with different soil conditions or distances could exhibit better similarities when eliminating the influence of spectral predominant period  $T_p$ , however, the BNRS defined by  $T_p$  also emerged large dispersion in the longer period ranges. For this

Received 2006-10-25.

**Foundation item:** China Postdoctoral Science Foundation (No. 20060400826).

**Biographies:** Xu Longjun (1976—), male, doctor, lecturer, xulongjun80@163.com; Xie Lili (1939—), male, professor, academician of Chinese Academy of Engineering.

reason, another scalar period,  $T_o$ , is introduced to normalize the abscissa of the NRS in this study.

The ordinate of the BNRS at a relative period  $T/T_p$  (or  $T/T_o$ ) can be written as<sup>[4]</sup>

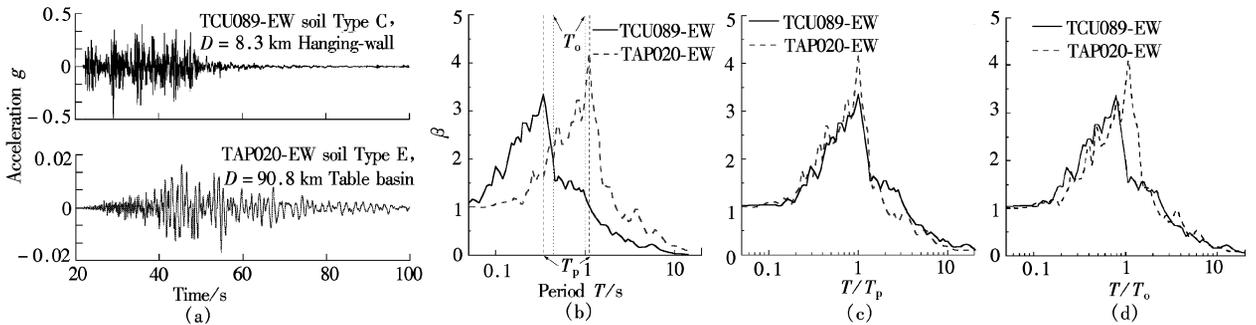
$$S_{bn}\left(\frac{T}{T_p}\right) = \frac{2\pi\omega'}{\text{PGA}\omega_p} \left| \int_0^t \ddot{x}(\tau) e^{-\frac{2\pi\omega}{\omega_p}\xi(t-\tau)} \cdot \left[ \left(1 - \frac{\xi^2}{1 - \xi^2}\right) \sin \frac{2\pi\omega'}{\omega_p}(t - \tau) + \frac{2\xi}{\sqrt{1 - \xi^2}} \cos \frac{2\pi\omega'}{\omega_p}(t - \tau) \right] d\tau \right|_{\max} \quad (1)$$

where  $\xi$  is the damping ratio,  $\omega' = \omega \sqrt{1 - \xi^2}$ ,  $T = 2\pi/\omega$ ,  $\omega_p = 2\pi/T_p$  (or  $\omega_o = 2\pi/T_o$ ),  $T_p$  is the period corresponding to the maximum response of the normalized response spectrum (NRS), and  $T_o$  is calculated as<sup>[4]</sup>

$$T_o = \frac{\sum_i T_i \ln\left(\frac{S_a(T_i)}{\text{PGA}}\right)}{\sum_i \ln\left(\frac{S_a(T_i)}{\text{PGA}}\right)} \quad (2)$$

for  $T_i$  with  $S_a/\text{PGA} \geq 1.2$ . Where  $T_i$  is the discrete period in the acceleration response spectra equaled spaced,  $S_a(T_i)$  is the spectral acceleration at  $T_i$ .

Fig. 1(a) shows two ground acceleration time histories, the conventional NRS for both the motions are drawn in Fig. 1(b). Figs. 1(c) and (d) show the BNRS normalized by  $T_p$  and  $T_o$  for both ground motions, respectively. It is evident that the BNRS are in good accordance with each other, and are more regular than those of the NRS.



**Fig. 1** Acceleration time-history of two ground motions and their response spectra. (a) Two ground motions; (b) NRS; (c)  $T_p$ -based BNRS; (d)  $T_o$ -based BNRS (5% damping)<sup>[5]</sup>

The strong motion parameters included in the analysis are 5%-damped NRS at natural periods ranging from 0.05 to 12 s. The BNRS at dimensionless periods, which are defined as the natural period divided by the scalar periods ( $T_p$  or  $T_o$ ) of the ground acceleration considered, also range from 0.05 to 12 s. These period ranges are interesting to engineers and can meet the design requirements for most structural systems.

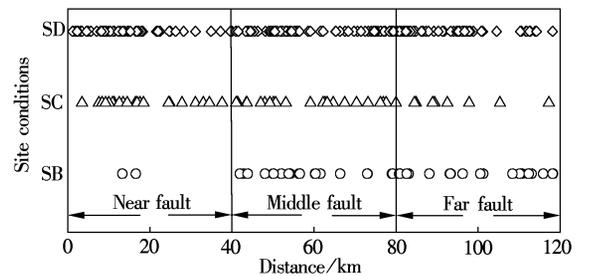
## 2 Ground Motion Data Set

The selected recordings are from the recent Chi-Chi earthquake and available from the CD-ROM compiled by Lee et al.<sup>[9]</sup>. This study used fault distance rather than an epicentral distance used in the previous study<sup>[5]</sup>. Fault distance “R” denotes the closest distance to the rupture surface. Motion recordings with fault distances greater than 120 km were excluded because motions with small peak ground acceleration (mostly  $\text{PGA} < 20$  gal) are basically irrelevant for the purpose of specifying design spectra. We also excluded the data obtained from deep alluvium plains and basin areas (arrays CHY, TCU, ILA and TAP) that show significant surface waves or basin responses. The data set

consists of totally 586 horizontal components and is divided into three classes according to station site conditions. To account for the influence of distance, each class recordings were subdivided into three groups. The classifications along with the number of recordings are provided in Tab. 1, and the distribution of the recording stations with respect to site condition and fault distance is shown in Fig. 2.

**Tab. 1** Numbers of stations for classified recordings

Site classes	Fault distance classification/km			Total
	Near ( $0 < \text{FD} \leq 40$ )	Middle ( $40 < \text{FD} \leq 80$ )	Far ( $80 < \text{FD} \leq 120$ )	
SB	2	21	23	46
SC	22	24	9	55
SD	57	82	53	192
Total	81	127	85	293

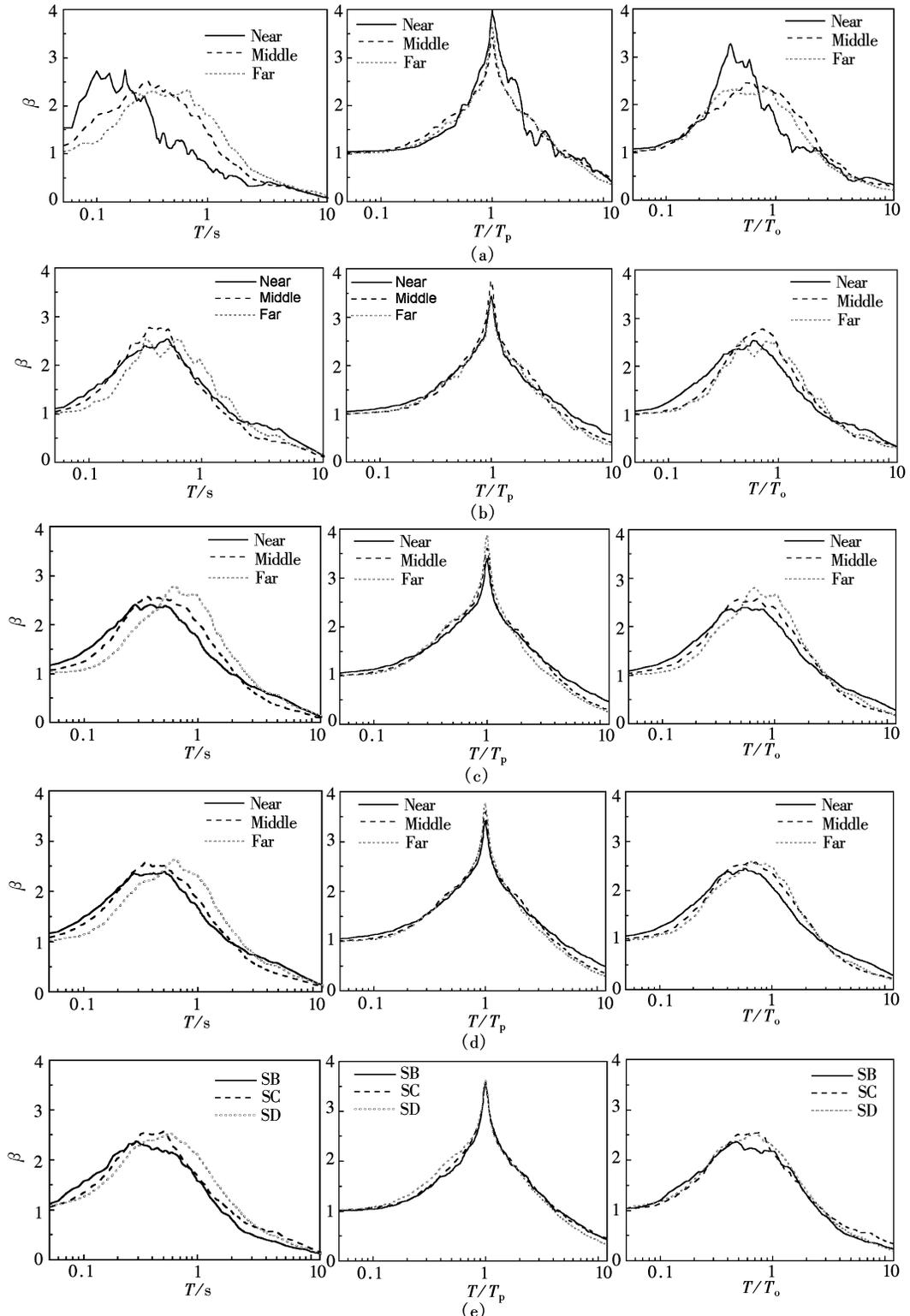


**Fig. 2** Distribution of station for classified recordings

### 3 Effects of Distance and Site Condition on Response Spectra

The mean NRS,  $T_p$ - and  $T_o$ -based BNRS within different fault distances at three types of site conditions

have been calculated and plotted in Figs. 3(a) to (c), respectively, Fig. 3(d) shows the influence of fault distance on the NRS and the BNRS disregarding the influence of soil condition, and Fig. 3(e) shows the effect of



**Fig. 3** The influences of distance and site condition on averaged NRS and BNRS. (a) Mean NRS and BNRS on site class SB; (b) Mean NRS and BNRS on site class SC; (c) Mean NRS and BNRS on site class SD; (d) Mean NRS and BNRS within different fault distance ranges; (e) Mean NRS and BNRS at different site conditions

soil condition on averaged NRS and BNRS. The influence of fault distance on the conventional NRS is evident. At short periods the normalized spectra of the near fault are higher than those of the far fault, but at middle and longer periods the normalized spectra of the near fault are lower than those of the far fault. However, when spectral periods are greater than 3 s, this trend changes; i. e., the spectra of the near fault are higher than those of the other distance classes at the long period range. In Fig. 3(e), soil condition affects the NRS to a significant degree. For soft soil sites, at small periods the normalized spectral values are less than those in stiff soil sites, while at middle to long periods the spectral values for stiff soil sites are lower than those in soft soil sites.

The influence of fault distance on  $T_p$ -based BNRS is evidently different from that of the NRS.  $T_p$ -based BNRS have consistent spectral shapes and all achieve their peaks at dimensionless periods,  $T/T_p$  of unit. Slight differences can be seen at relatively larger  $T/T_p$  over about 3, where the spectral ordinates of the near fault are higher than those of the other two distance classes. While at all the dimensionless periods, the difference between middle and far distances are negligible.

For  $T_o$ -based BNRS, they have similar characteristics compared to the NRS, but the discriminations of mean  $T_o$ -based BNRS between/at three site classes are relatively smaller than those of the conventional NRS, implying that distance has a less pronounced effect on  $T_o$ -based BNRS than that of the effect on the NRS. Comparing the two types of BNRS and NRS, it is obvious that the effect of soil conditions on the BNRS is much smaller. Averaged BNRS for different soil conditions take on uniform and consistent spectral shapes.

An important observation of  $T_p$ -based BNRS is

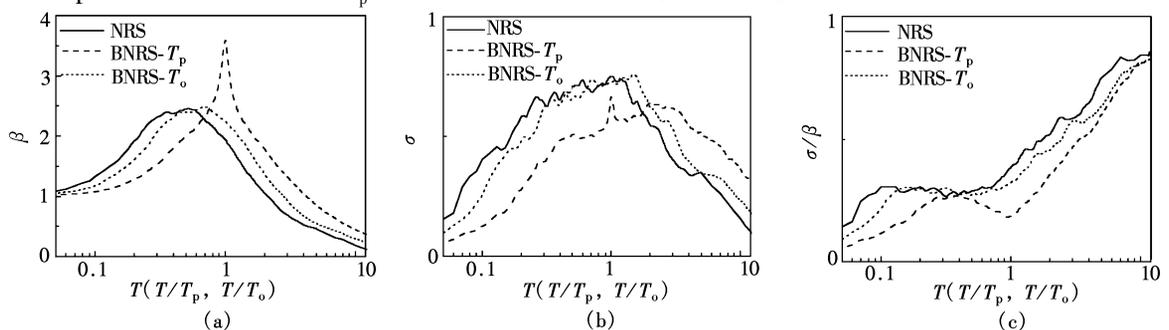


Fig. 4 Comparison among three kinds of spectra. (a) Mean curves; (b) Std curves; (c) CoV curves

## 5 Empirical Relationships for Scalar Periods

For period  $T_p$  or  $T_o$ , the corresponding period values of the two orthogonal horizontal components were

that their peak amplifications are significantly higher than those of the NRS and the  $T_o$ -based BNRS. It is because, in almost all the cases, normalized response spectral peak periods of different ground motions do not correspond to each other. When averaging the spectral curves, spectral peaks are chopped and valleys are filled up. As a consequence, the mean NRS and  $T_o$ -based BNRS exhibit flatter shapes around their peaks than the individual spectra<sup>[2]</sup>, while spectral peaks of  $T_p$ -based BNRS correspond to the same dimensionless periods,  $T/T_p$  of unit. As a result, peak amplifications of mean  $T_p$ -based BNRS will be absolutely greater than those of NRS and  $T_o$ -based BNRS.

## 4 Dispersion Analysis

To quantify the level of dispersion in the NRS and the BNRS, the standard deviation (Std) and coefficient of variation (CoV) are calculated. Neglecting the dimensions of the abscissa for these three kinds of response spectra, Fig. 4 illustrates the comparisons of the mean, the Std and CoV curves of the NRS and the BNRS for all of the recordings, respectively. Noting that the height of Std curve of the  $T_p$ -based BNRS is lower than that of NRS and the  $T_o$ -based BNRS, which indicate that the normalization to  $T_p$  can effectively reduce scatter in the spectral peak period ranges. CoV curves increase as the spectral period or dimensionless period increases, which indicate that dispersion is particularly higher for long period ranges of these spectra. The CoV of the NRS exhibits an overall larger dispersion in the period range; however, the CoV of  $T_p$ -based BNRS takes on the lowest curve values in the normalized period range. Thus, the normalization by  $T_p$  and  $T_o$  effectively reduces scatter in the NRS, particularly in the short to moderate spectral period range.

combined and simply calculated using the arithmetical mean  $0.5(T_1 + T_2)$ . To avoid the near fault effects influenced by rupture directivity, only recordings with fault distances of more than 20 km were used for

event-regression analysis. The functional form incorporated in this study is

$$\ln(T) = a + bR \quad R > 20 \quad (5)$$

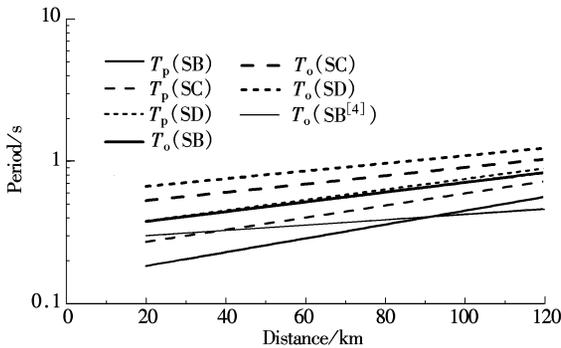
where  $T$  is the scalar periods  $T_p$  or  $T_o$ ;  $R$  is the fault distance; and  $a$  and  $b$  are regression coefficients.

The regression coefficients for the empirical models for  $T_p$  and  $T_o$  are listed in Tab. 2, along with the standard deviations of the fitting and standard errors for each coefficient. Parameter  $a$  is sufficiently significant for differentiating the effects of site conditions and parameter  $b$  reflects the effect of distance on ground motions. Standard deviations of the regression and the standard errors for each coefficient varies with site classes, with more scatter observed for rock sites (SB) than for soil sites (SC, SD). Standard deviations and standard errors for  $T_o$  are less than those for  $T_p$ , indicating that  $T_o$  can be more reliably predicted than  $T_p$ . Fig. 5 illustrates the predic-

tions of  $T_p$  and  $T_o$  versus distance for different site classes. These regression lines in log-normal coordinates are presumably parallel, and the predicted  $T_p$  are lower than those of the corresponding  $T_o$  of a site class. For the same scalar period, the predicted value increases when the site soil becomes soft. A comparison of the relationships predicted in this study with those developed by Rathje et al.<sup>[4]</sup> has also been provided in Fig. 5. It is evident that the slopes of the regression lines for the current study are larger than those of previous work. It indicates that distance affected  $T_o$  more significantly during the Chi-Chi earthquake than in other earthquakes.  $T_o$  values of site class SB are much higher than those of Ref. [4]. This trend is most likely attributable to the fact that large earthquake motions contain many moderate components and will be significantly amplified in the long distance range.

**Tab. 2** Regression coefficients and standard errors for the regression coefficients

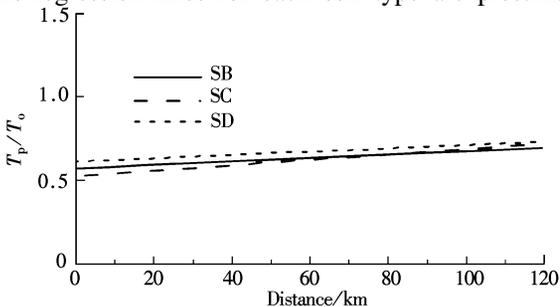
Coefficient	SB		SC		SD	
	$T_p$	$T_o$	$T_p$	$T_o$	$T_p$	$T_o$
$a$	-1.9182(0.2533)	-1.1299(0.1782)	-1.4981(0.1295)	-0.7684(0.1442)	-1.1340(0.0934)	-0.5279(0.0825)
$b$	0.0112(0.0030)	0.0079(0.0021)	0.0098(0.0018)	0.0067(0.0020)	0.0085(0.0012)	0.0062(0.0011)
Std	0.5217	0.3669	0.3464	0.3858	0.4004	0.3535



**Fig. 5** Prediction of  $T_p$  and  $T_o$ , and a comparison of this study with Rathje et al.

## 6 Relationship between Two Scalar Periods

To give the relationship between the scalar periods,  $T_p$  and  $T_o$ , a simple linear fit of a  $T_p/T_o$  ratio for each soil type is drawn in Fig. 6. It is interesting that the regression lines for each soil type are presumably



**Fig. 6** Relationship between  $T_p$  and  $T_o$

consistent with each other and that the  $T_p/T_o$  ratios vary gently from 0.6 to 0.7 when the fault distance increases. It indicates that the relationship between  $T_p$  and  $T_o$  is site-independent and only slightly related to fault distance.

## 7 Conclusion

This paper investigates the discrepant and uniform features of response spectra by analyzing nearly 600 free field ground motions during the recent Chi-Chi earthquake. The discrepant aspects of ground motions are examined by observing the characteristics of the conventional NRS of classified strong motions. Acceleration of the NRS strongly depends on soil condition and fault distance. The uniform aspects of motions are investigated by comparing the spectral shapes of the BNRS, whose abscissa have been normalized by scalar periods  $T_p$  or  $T_o$  based on the NRS. Both  $T_p$ - and  $T_o$ -based BNRS display better uniform characteristics than the NRS for different site conditions and distance ranges. The estimated scalar periods are more sensitive to distance than those in previous estimates.  $T_o$  are significantly higher than those of other earthquakes. Finally, the relationship between  $T_p$  and  $T_o$  are considered.  $T_p$  to  $T_o$  ratios are site-independent and only slightly related to fault distance.

The BNRS exhibit uniform spectral shapes for different site conditions and fault distances, implying that

the BNRS are more appropriate for being the basic criteria for construct seismic design spectra.  $T_p$ -based BNRS veritably display the spectral aspects of the predominant frequency of ground motions and may be best suited for engineering projects where this frequency range is of interest (e. g., short period structures). While  $T_0$ -based BNRS can be used for ordinary structures with short to moderate vibration periods. In spite of the limitations caused by analyzing data from only the Chi-Chi earthquake, we believe the findings of this work will be useful not only in revealing the characteristics of strong ground motions, but also in unifying current design spectra and developing seismic environment-specified design spectra when it comes time to evaluate code provisions.

## References

- [1] Chopra A K. *Dynamics of structures: theory and application to earthquake engineering* [M]. 2nd ed. Upper Saddle River, NJ: Prentice-Hall, 2001.
- [2] Seed H B, Ugas C, Lysmer J. Site-dependent spectra for earthquake-resistance design [J]. *Bull Seismol Soc Am*, 1976, **66**(1): 221 – 243.
- [3] Mohsen T, Farzaneh H. Influence of earthquake source parameters and damping on elastic response spectra for Iranian earthquakes [J]. *Engineering Structures*, 2002, **24**: 933 – 943.
- [4] Rathje E M, Faraj F, Russell S, et al. Empirical relationships for frequency content parameters of earthquake ground motions [J]. *Earthquake Spectra*, 2004, **20**(1): 119 – 144.
- [5] Xu Longjun, Xie Lili. Bi-normalized response spectral characteristics of the 1999 Chi-Chi earthquake [J]. *Earthquake Engineering and Engineering Vibration*, 2004, **3**(2): 147 – 155.
- [6] Chopra A K, Chintanapakdee C. Comparing response of SDF systems to near-fault and far-fault earthquake motions in the context of spectral regions [J]. *Earthquake Engng Struc Dyn*, 2001, **30**: 1769 – 1789.
- [7] Mavroeidis G P, Dong G, Papageorgiou A S. Near-fault ground motions, and the response of elastic and inelastic single-degree-of-freedom (SDOF) systems [J]. *Earthquake Engng Struc Dyn*, 2004, **33**: 1023 – 1049.
- [8] Xu Longjun, Xie Lili. Site-dependent bi-normalized earthquake acceleration response spectra [J]. *Journal of Harbin Institute of Technology*, 2004, **36**(8): 1061 – 1064. (in Chinese)
- [9] Lee W H K, Shin T C, Kuo K W, et al. CWB free-field strong-motion data from the 921 Chi-Chi earthquake: processed acceleration files on CD-ROM, strong-motion data series CD-001 [C/OL]. Seismological Observation Center, Central Weather Bureau, 2001.

# 统一抗震设计谱的双规准反应谱和标定周期

徐龙军<sup>1,2</sup> 谢礼立<sup>2</sup>

(<sup>1</sup> 中国海洋大学工程学院, 青岛 266100)

(<sup>2</sup> 哈尔滨工业大学土木工程学院, 哈尔滨 150090)

**摘要:** 为了考察地震动加速度反应谱的统一性, 用 2 种标定周期(反应谱卓越周期  $T_p$  和平滑化反应谱卓越周期  $T_0$ ) 分别对传统规准反应谱的横坐标进行规准化. 考虑场地条件和断层距的影响, 对集集地震近 600 水平向分量地震动 5% 阻尼比的规准反应谱和 2 种双规准反应谱进行了分析. 结果表明场地条件和断层距对规准谱的影响明显, 但对 2 种双规准谱的影响甚小. 离散性分析表明对规准反应谱横坐标的规准化得到的双规准谱具有离散性小的特点. 最后, 对标定周期( $T_p, T_0$ ) 的分布进行了参数分析并与以往结果进行了比较. 地震动反应谱和标定周期的新特性可以为规范设计谱的应用和发展提供参考依据.

**关键词:** 地震动; 规准反应谱; 双规准反应谱; 标定周期

**中图分类号:** P315.9