Journal of Southeast University (English Edition) Vol. 23, No. 2, pp. 313 – 316 June 2007 ISSN 1003—7985

Subspaces for weak mild solutions of the second order abstract differential equation

Wang Meiying

(Department of Applied Mathematics, Nanjing Audit University, Nanjing 210029, China)

Abstract: The topic on the subspaces for the polynomially or exponentially bounded weak mild solutions of the following abstract Cauchy problem $\frac{d^2}{dt^2}u(t,x)=Au(t,x)$; $u(0,x)=x$, $\frac{d}{dt}u(0,x)=0$, $x\in X$ is studied, where A is a closed operator on Banach space X. The case that the problem is ill-posed is treated, and two subspaces $Y(A,k)$ and $H(A,\omega)$ are introduced. $Y(A,k)$ is the set of all x in X for which the second order abstract differential equation has a weak mild solution $v(t,x)$ such that ess sup$\left\{(1+t)^{-k}\left|\frac{d}{dt}\langle v(t,x),x^*\rangle\right|: t\geqslant 0, x^*\in X^*, \|x^*\|\leqslant 1\right\}<+\infty$. $H(A,\omega)$ is the set of all x in X for which the second order abstract differential equation has a weak mild solution $v(t,x)$ such that ess sup$\left\{e^{-\omega t}\left|\frac{d}{dt}\langle v(t,x),x^*\rangle\right|: t\geqslant 0, x^*\in X^*, \|x^*\|\leqslant 1\right\}<+\infty$. The following conclusions are proved that $Y(A,k)$ and $H(A,\omega)$ are Banach spaces, and both are continuously embedded in X; the restriction operator $A|_{Y(A,k)}$ generates a once-integrated cosine operator family $\{C(t)\}_{t\geqslant 0}$ such that $\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h)-C(t)\|_{Y(A,k)}\leqslant M(1+t)^k$, $\forall t\geqslant 0$; the restriction operator $A|_{H(A,\omega)}$ generates a once-integrated cosine operator family $\{C(t)\}_{t\geqslant 0}$ such that $\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h)-C(t)\|_{H(A,\omega)}\leqslant Me^{\omega t}$, $\forall t\geqslant 0$.

Key words: second order abstract differential equation; polynomially bounded solution; cosine operator function

1 Basic Concepts

Let X be a Banach space and A a closed linear operator on X. Many physical problems may be modeled as a first order abstract Cauchy problem

$$\left.\begin{aligned}&\frac{d}{dt}u(t,x)=Au(t,x)\\&u(0,x)=x\end{aligned}\right\}\qquad(1)$$

The first order abstract Cauchy problem is well-posed if A is densely defined and generates a strongly continuous semigroup. When A is densely defined, but it does not generate a strongly continuous semigroup, then (1) is ill-posed[1-2].

Examples are the backwards heat equation, the Schrodinger equation on L^p, $p\neq 2$, etc[3-4].

To deal with the ill-posed abstract Cauchy problem, Davies and Pang[5] introduced the concept of exponentially bounded C-semigroups; Arendt[6] introduced the concept of integrated semigroups. de Laubenfels et al. [3] introduced two subspaces $Z(A,k)$ and $Y(A,k)$. $Z(A,k)$ is the set of all x in X for which (1) has a mild solution such that $(1+t)^{-k}u(t,x)$ is uniformly continuous and bounded on $[0,\infty)$; $Y(A,k)$ is the set of all x in X for which (1) has a weak mild solution such that

$$\text{ess sup}\left\{(1+t)^{-k}\left|\frac{d}{dt}\langle v(t,x),x^*\rangle\right|: t\geqslant 0, x^*\in X^*, \|x^*\|\leqslant 1\right\}<+\infty$$

Also in Ref. [3] the restriction operator $A|_{Z(A,k)}$ generates a strongly continuous semigroup, and $A|_{Y(A,k)}$ generates a once-integrated semigroup.

Motivated by de Laubenfels et al. [7], author introduced the subspace $W(A,k)$, that is the set of all x in X for which the second order abstract differential equation has a mild solution $u(t,x)$ such that $(1+t)^{-k}u(t,x)$ is uniformly continuous and bounded on $[0,\infty)$. We will consider the following second order abstract Cauchy problem

$$\left.\begin{aligned}&\frac{d^2}{dt^2}u(t,x)=Au(t,x)\\&u(0,x)=x,\ \frac{d}{dt}u(0,x)=0\qquad x\in X\end{aligned}\right\}\qquad(2)$$

where A is a closed linear operator on a Banach space X.

In this paper, we introduce the subspace $Y(A,k)$ that is the set of all x in X for which (2) has a weak

Received 2006-11-06.

Foundation item: The Natural Science Foundation of Department of Education of Jiangsu Province (No. 06KJD110087).

Biography: Wang Meiying (1950—), female, professor, nsjwangmy@sina. com.

mild solution $v(t,x)$ such that

$$\text{ess sup}\left\{(1+t)^{-k}\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x),x^*\rangle\right|: t\geqslant 0, x^*\in X^*, \|x^*\|\leqslant 1\right\} < +\infty$$

Also note that the restriction operator $A\mid_{Y(A,k)}$ generates a once-integrated cosine operator family $\{C(t)\}_{t\geqslant 0}$.

Throughout this paper, $B(X)$ stands for the Banach algebra of all bonded linear operators on X, A for a closed linear operator on X, $\rho(A)$ for the resolvent set of A, $D(A)$ and $\mathrm{Im}(A)$ for the domain and image of A, respectively. $\mathbf{R}^+ = [0,\infty)$.

Definition 1 A weak mild solution of (2) has a form

$$v(t,x) = A\int_0^t (t-s)v(s)x\mathrm{d}s + tx \tag{3}$$

so that $t\to v(t,x)$ is locally Lipschitz continuous, and $\int_0^t (t-s)v(s)\mathrm{d}s \in D(A)$, for all $t\geqslant 0$.

Definition 2 Let W be a subspace of X. We write $A\mid_W$ for the restriction operator of A in W; that is, $D(A\mid_W) = \{x\in W\cap D(A): Ax\in W\}$ and $A\mid_W x = Ax$ for $\forall x\in D(A\mid_W)$.

Definition 3 The strongly continuous family of operators $\{C(t)\}:\mathbf{R}^+\to B(X)$ is called a once-integrated cosine operator family, if

① $C(0)=0$;

② For every $x\in X$, we have

$$2C(s)C(t)x = \int_0^{s+t} C(r)x\mathrm{d}r - \int_0^{|s-t|} C(r)x\mathrm{d}r \qquad 0\leqslant s,t<+\infty$$

Definition 4 Let $k\in\mathbf{N}\cup\{0\}$, $Y(A,k)$ be the set of all $x\in X$ for which (2) has a weak mild solution $v(\cdot,x)$ so that

$$\text{ess sup}\left\{(1+t)^{-k}\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x),x^*\rangle\right|: t\geqslant 0, x^*\in X^*, \|x^*\|\leqslant 1\right\} < +\infty$$

Define

$$\|x\|_{Y(A,k)} = \text{ess sup}\left\{(1+t)^{-k}\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x),x^*\rangle\right|, |\langle x,x^*\rangle| : t\geqslant 0, x\in X, x^*\in X^*, \|x^*\|\leqslant 1\right\}$$

Definition 5 Let $\omega>0$, $H(A,\omega)$ be the set of all $x\in X$ for which (2) has a weak mild solution $v(\cdot,x)$ so that

$$\text{ess sup}\left\{\mathrm{e}^{-\omega t}\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x),x^*\rangle\right|: t\geqslant 0, x^*\in X^*, \|x^*\|\leqslant 1\right\} < +\infty$$

Define

$$\|x\|_{H(A,\omega)} = \text{ess sup}\left\{\mathrm{e}^{-\omega t}\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x),x^*\rangle\right|, |\langle x,x^*\rangle| : t\geqslant 0, x\in X, x^*\in X^*, \|x^*\|\leqslant 1\right\}$$

Definition 6 Suppose V is a Banach space. We say that $\{g(s)\}_{s>0}\subset V$ is the once-integrated Laplace transform of G, if $G:[0,\infty)\to V$ is continuous, $G(0)=0$ and

$$g(s) = s\int_0^\infty \mathrm{e}^{-st}G(t)\mathrm{d}t \qquad s>0$$

2 Main Results

Theorem 1 The following are true:

① $Y(A,k)$ is a Banach space, and is continuously embedded in X;

② $A\mid_{Y(A,k)}$ generates a once-integrated cosine operator family $\{C(t)\}_{t\in\mathbf{R}^+}$, such that

$$\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h)-C(t)\|_{Y(A,k)} \leqslant M(1+t)^k \qquad \forall t\geqslant 0$$

where $M>0$ is a constant.

Theorem 2 The following are true:

① $H(A,\omega)$ is a Banach space, and is continuously embedded in X;

② $A\mid_{H(A,\omega)}$ generates a once-integrated cosine operator family $\{C(t)\}_{t\in\mathbf{R}^+}$, such that

$$\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h)-C(t)\|_{H(A,\omega)} \leqslant M\mathrm{e}^{\omega t} \qquad \forall t\geqslant 0$$

where $M>0$ is a constant.

To prove theorem 1 we need the following lemmas:

Lemma 1 Suppose $t\mapsto W(t)\in C([0,\infty),X)$ and is $O(\mathrm{e}^{\alpha t})(t\to 0^+)$, $s^2I-A: X\to X$ is injective, then the following are equivalent:

① $(s^2I-A)^{-1}x = \int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t,\ s>\alpha$;

② For all $t\geqslant 0$, $\int_0^t (t-r)W(r)x\mathrm{d}r \in D(A)$, with $W(t)x = A\int_0^t (t-r)W(r)x\mathrm{d}r + tx$.

Proof ① $\Rightarrow$ ②. From $(s^2I-A)^{-1}x = \int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t$, we have

$$s^2\int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t = A\int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t + x = sA\int_0^\infty \mathrm{e}^{-st}\left[\int_0^t W(s)x\mathrm{d}s\right]\mathrm{d}t + x = s^2A\int_0^\infty \mathrm{e}^{-st}\left[\int_0^t (t-s)W(s)x\mathrm{d}s\right]\mathrm{d}t + x$$

Then

$$\int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t = \int_0^\infty \mathrm{e}^{-st}\left[A\int_0^t (t-s)W(s)x\mathrm{d}s\right]\mathrm{d}t + \frac{x}{s^2} = \int_0^\infty \mathrm{e}^{-st}\left[A\int_0^t (t-s)W(s)x\mathrm{d}s + tx\right]\mathrm{d}t$$

By the uniqueness of the Laplace transform, we have for $t\geqslant 0$, $\int_0^t (t-s)W(s)x\mathrm{d}s \in D(A)$, and

$$A\int_0^t (t-s)W(s)x\mathrm{d}s + tx = W(t)x \qquad \forall x \in X$$

②⇒①. From $W(t)x = A\int_0^t (t-s)W(s)x\mathrm{d}s + tx$, we have

$$\int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t = A\int_0^\infty \mathrm{e}^{-st}\Big[\int_0^t (t-s)W(s)x\mathrm{d}s\Big]\mathrm{d}t + \int_0^\infty t\mathrm{e}^{-st}x\mathrm{d}t = \frac{A}{s}\int_0^\infty \mathrm{e}^{-st}\Big[\int_0^t W(s)x\mathrm{d}s\Big]\mathrm{d}t + \frac{x}{s^2} = \frac{A}{s^2}\int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t + \frac{x}{s^2}$$

Thus $(s^2I - A)\int_0^\infty \mathrm{e}^{-st}W(t)\mathrm{d}t = x$. This implies that

$$(s^2I - A)^{-1}x = \int_0^\infty \mathrm{e}^{-st}W(t)x\mathrm{d}t$$

As in the proof of Ref. [8], we have the following lemma:

Lemma 2 The following are equivalent, if $k \in \mathbf{N}$,

① There exists a constant M_1 so that A generates a once-integrated cosine operator family $\{C(t)\}_{t\geqslant 0}$ such that $\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h) - C(t)\| \leqslant M(1+t)^k$, for all $t \geqslant 0$.

② There exists a constant M_2 so that $(0,\infty) \subset \rho(A)$, and for $n \in \mathbf{N}$

$$\left\|\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}[\lambda(\lambda^2I - A)^{-1}]\right\| \leqslant M_2\Big[\frac{n!}{\lambda^{n+1}} + \frac{(n+k)!}{\lambda^{n+k+1}}\Big]$$

Proof of Theorem 1

1) From the definition of $\|x\|_{Y(A,k)}$, we have

$$\|x\|_{Y(A,k)} = \operatorname{ess\,sup}\Big\{(1+t)^{-k}\,\Big|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x), x^*\rangle\Big|, |\langle x, x^*\rangle|;\, t\geqslant 0, x^* \in X^*, \|x^*\|\leqslant 1\Big\} \geqslant \|x\| \tag{4}$$

Thus $Y(A,k)$ continuously embedded in X. To see that $Y(A,k)$ is a Banach space, suppose $\{x_n\}$ is a Cauchy sequence in $Y(A,k)$. Then (4) implies that $\{x_n\}$ is a Cauchy sequence in X. There exists $x \in X$ such that $x_n \to x$ in X as $n\to\infty$. Since

$$v(t,x_n) = A\int_0^t (t-s)v(s,x_n)\mathrm{d}s + tx_n \tag{5}$$

then there exists Lipschitz continuous $W(t): [0,\infty) \to X$, such that $\{(1+t)^{-k}v(t,x_n)\}$ converge to $(1+t)^{-k}W(t)$ in the Lipschitz norm as $n\to\infty$. We set $n\to\infty$ in (5), then

$$W(t) = A\int_0^t (t-s)W(s)\mathrm{d}s + tx$$

Thus $x \in Y(A,k)$ with $W(t) = v(t,x)$. This implies that $Y(A,k)$ is a Banach space.

2) For all $t > 0$, $\forall x \in Y(A,k)$, we have

$$(\lambda^2I - A)\int_0^\infty \mathrm{e}^{-\lambda t}v(t)x\mathrm{d}t = \lambda^2\int_0^\infty \mathrm{e}^{-\lambda t}v(t)x\mathrm{d}t - A\int_0^\infty \mathrm{e}^{-\lambda t}v(t)x\mathrm{d}t = \lambda^2\int_0^\infty \mathrm{e}^{-\lambda t}v(t)x\mathrm{d}t - \lambda^2\int_0^\infty \mathrm{e}^{-\lambda t}\cdot\Big[A\int_0^t (t-s)v(s)x\mathrm{d}s\Big]\mathrm{d}t = \lambda^2\int_0^\infty \mathrm{e}^{-\lambda t}v(t)x\mathrm{d}t - \lambda^2\int_0^\infty \mathrm{e}^{-\lambda t}[v(t)x - tx]\mathrm{d}t = x$$

Thus $Y(A,k) \subseteq \mathrm{Im}(\lambda^2I - A) = D((\lambda^2I - A)^{-1})$.

To show that $(\lambda^2I - A)^{-1}$ maps $Y(A,k)$ into itself, for a fixed $x \in Y(A,k)$, we must construct a map

$$r \mapsto W(r, (\lambda^2I - A)^{-1}x)$$

such that $\frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t,x)$ is locally Lipschitz continuous and satisfies

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t,x) = A(W(t,x)) + tx \qquad t\geqslant 0$$

Let $\frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t,x) = v(t,x)$, then $v(t,x)$ satisfies

$$v(t,x) = A\int_0^t (t-r)v(r)x\mathrm{d}r + tx \qquad t\geqslant 0$$

Denote

$$u(r) = \int_0^\infty \mathrm{e}^{-\lambda t}\Big\{\frac{1}{2\lambda}[W(t+r,x) - W(t-r,x)] - \frac{r}{\lambda^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t,x)\Big\}\mathrm{d}t$$

then

$$u''(r) = \int_0^\infty \mathrm{e}^{-\lambda t}\frac{1}{2\lambda}\Big[\frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t+r,x) - \frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t-r,x)\Big]\mathrm{d}t$$

$$Au(r) = \int_0^\infty \mathrm{e}^{-\lambda t}\frac{1}{2\lambda}\Big[\frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t+r,x) - (t+r)x - \frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t-r,x) + (t-r)x\Big]\mathrm{d}t - \frac{r}{\lambda^2}A\int_0^\infty \mathrm{e}^{-\lambda t}\frac{\mathrm{d}^2}{\mathrm{d}t^2}W(t,x)\mathrm{d}t$$

$$u''(r) - Au(r) = \int_0^\infty \mathrm{e}^{-\lambda t}\frac{r}{\lambda}x\mathrm{d}t + \frac{r}{\lambda^2}(A - \lambda^2I + \lambda^2I)(\lambda^2I - A)^{-1}x = r(\lambda^2I - A)^{-1}x$$

This proves that $(\lambda^2I - A)^{-1}$ maps $Y(A,k)$ into itself, with $W(r, (\lambda^2I - A)^{-1}x) = u(r)$, for $x \in Y(A,k)$.

This implies that, for all $\lambda > 0$, $(\lambda^2I - A|_{Y(A,k)})$ is a bijection.

From lemma 1, we have

$$(\lambda^2I - A|_{Y(A,k)})^{-1}x = \int_0^\infty \mathrm{e}^{-\lambda t}v(t)x\mathrm{d}t \qquad \lambda > 0$$

Let $x \in Y(A,k)$, $x^* \in X^*$, then

$$\langle\lambda(\lambda^2I - A|_{Y(A,k)})^{-1}x, x^*\rangle = \lambda\int_0^\infty \mathrm{e}^{-\lambda t}\langle v(t)x, x^*\rangle\mathrm{d}t = \int_0^\infty \mathrm{e}^{-\lambda t}\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t)x, x^*\rangle\mathrm{d}t$$

Differentiating the above equality n times in λ, we obtain

$$\Big\langle\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}[\lambda(\lambda^2I - A|_{Y(A,k)})^{-1}]x, x^*\Big\rangle = (-1)^n\int_0^\infty \mathrm{e}^{-\lambda t}t^n\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t)x, x^*\rangle\mathrm{d}t$$

By the definition of $\|x\|_{Y(A,k)}$, we have

$$\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t)x, x^*\rangle\right| \leqslant M_1(1+t^k)\|x\|_{Y(A,k)}$$

Then

$$\left|\left\langle \frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}[\lambda(\lambda^2 I - A|_{Y(A,k)})^{-1}x], x^*\right\rangle\right| \leqslant M_2\left[\frac{n!}{\lambda^{n+1}} + \frac{(n+k)!}{\lambda^{n+k+1}}\right]\|x\|_{Y(A,k)}$$

Thus

$$\left\|\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}[\lambda(\lambda^2 I - A|_{Y(A,k)})^{-1}x]\right\|_{Y(A,k)} \leqslant M_2\left[\frac{n!}{\lambda^{n+1}} + \frac{(n+k)!}{\lambda^{n+k+1}}\right]\|x\|_{Y(A,k)}$$

For $x \in Y(A,k)$, let $C(t)x = v(t,x)$, then by lemma 2, $A|_{Y(A,k)}$ generates a once-integrated cosine operator family $\{C(t)\}_{t\geqslant 0}$ such that

$$\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h) - C(t)\|_{Y(A,k)} \leqslant M(1+t)^k$$

for all $t \geqslant 0$

The proof of theorem 2 is similar to that of theorem 1, the details are omitted.

References

[1] de Laubenfels R. Existence families, functional calculi and evolution equations[C]//*Lecture Notes in Mathematics*. Berlin: Springer-Verlag, 1994, **1570**: 27 − 54.

[2] Arendt W, Batty Ch J K, Hieber M, et al. *Vector-valued Laplace transforms and Cauchy problems* [M]. Birkhauser, 2001.

[3] de Laubenfels R, Huang Z, Wang S, et al. Laplace transforms of polynomially bounded vector-valued functions and semigroups of operators [J]. *Israel Journal of Mathematics*, 1997, **98**: 189 − 207.

[4] Wang Shenwang. Hille-Yosida type theorems for local regularized semigroups and local integrated semigroups[J]. *Studia Mathematica*, 2002, **152**(1): 45 − 67.

[5] Davies E B, Pang M M. The Cauchy problem and a generalization of the Hille-Yosida theorem[J]. *Proc London Math Soc*, 1987, **55**: 181 − 208.

[6] Arendt W. Vector-valued Laplace transforms and Cauchy problems [J]. *Israel Journal of Mathematics*, 1987, **59**: 327 − 352.

[7] Wang Meiying, Jiang Huikun. Maximal subspaces for polynomially bounded solutions of the second order abstract differential equation [J]. *Journal of Nanjing University*: *Natural Science Edition*, 2006, **42**(1): 11 − 16. (in Chinese)

[8] Wang Shenwang, Wang Meiying, Shen Yan. Perturbation theorems for local integrated semigroups and their applications [J]. *Studia Mathematica*, 2005, **170**(2): 121 − 146.

二阶抽象微分方程的次弱解空间

王梅英

(南京审计学院应用数学系,南京 210029)

摘要: 讨论 Banach 空间 X 上二阶抽象微分方程 $\frac{\mathrm{d}^2}{\mathrm{d}t^2}u(t,x) = Au(t,x)$; $u(0,x) = x, \frac{\mathrm{d}}{\mathrm{d}t}u(0,x) = 0, x \in X$ 的不适定情况,这里 A 是 X 上的闭算子;引进空间 $Y(A,k)$,即使得二阶抽象微分方程有次弱解 $v(t,x)$,且满足 $\mathrm{ess\ sup}\left\{(1+t)^{-k}\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x), x^*\rangle\right| : t\geqslant 0, x^* \in X^*, \|x^*\|\leqslant 1\right\} < +\infty$ 的 $x \in X$ 的全体,及空间 $H(A,\omega)$,即使得二阶抽象微分方程有次弱解 $v(t,x)$,且满足 $\mathrm{ess\ sup}\left\{\mathrm{e}^{-\omega t}\left|\frac{\mathrm{d}}{\mathrm{d}t}\langle v(t,x), x^*\rangle\right| : t\geqslant 0, x^* \in X^*, \|x^*\|\leqslant 1\right\} < +\infty$ 的 $x \in X$ 的全体. 证明了如下结论: $Y(A,k)$ 和 $H(A,\omega)$ 均为 Banach 空间,且 $Y(A,k)$ 和 $H(A,\omega)$ 均连续嵌入 X; A 在 $Y(A,k)$ 上的限制算子 $A|_{Y(A,k)}$ 生成一个一次积分 Cosine 算子函数 $\{C(t)\}_{t\geqslant 0}$,满足 $\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h) - C(t)\|_{Y(A,k)} \leqslant M(1+t)^k$, $\forall t \geqslant 0$; A 在 $H(A,\omega)$ 上的限制算子 $A|_{H(A,\omega)}$ 生成一个一次积分 Cosine 算子函数 $\{C(t)\}_{t\geqslant 0}$,满足 $\overline{\lim_{h\to 0^+}}\frac{1}{h}\|C(t+h) - C(t)\|_{H(A,\omega)} \leqslant M\mathrm{e}^{\omega t}$, $\forall t \geqslant 0$.

关键词: 二阶抽象微分方程; 多项式有界解; 余弦算子函数

中图分类号: O177.5