

EM-based detection scheme for differential unitary space-time modulation

Du Zhengfeng Chen Jie Pan Wen Gao Xiqi

(National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China)

Abstract: The performance loss of an approximately 3 dB signal-to-noise ratio is always paid with conventional differential detection compared to the related coherent detection. A new detection scheme consisting of two steps is proposed for the differential unitary space-time modulation (DUSTM) system. In the first step, the data sequence is estimated by conventional unitary space-time demodulation (DUSTD) and differentially encoded again to produce an initial estimate of the transmitted symbol stream. In the second step, the initial estimate of the symbol stream is utilized to initialize an expectation maximization (EM)-based iterative detector. In each iteration, the most recent detected symbol stream is employed to estimate the channel, which is then used to implement coherent sequence detection to refine the symbol stream. Simulation results show that the proposed detection scheme performs much better than the conventional DUSTD after several iterations.

Key words: unitary space-time modulation; differential detection; expectation maximization (EM) algorithm

Recent information theory results show that the capacity of wireless channels can be substantially increased by employing multiple transmit and receive antennas, especially when the channel state information is known at the receiver^[1–2]. However, in many situations, the channel state information (CSI) may not be available to the receiver. Even if the receiver does not know the fading coefficients, a substantial increase in channel capacity is still possible^[3]. So it is of interest to develop techniques for modulation and coding that do not require CSI. Tarokh et al.^[4] first came up with a differential STBC scheme for a slow Rayleigh fading channel with two transmitter antennas. The scheme employs block-by-block detection, in which neither transmitter nor receiver knows the CSI. The same authors generalized the differential detection for STBC to more than two transmit antennas^[5]. In Ref. [6], a new class of signals called unitary space-time signals was proposed, which is well tailored for Rayleigh flat-fading channels where neither the transmitter nor the receiver knows the fading coefficients. Ref. [7] presented a systematic approach to designing unitary space-time signals. In particular, Ref. [8] introduced so-called diagonal codes that are simple to generate: every antenna

transmits a phase-shift keying (PSK) symbol in turn. Like other differential space-time codes, differential unitary space-time modulation (DUSTM) enables us to decode the received signals without the knowledge of the CSI^[6,8], but both suffer a loss of an approximately 3 dB signal to noise ratio when compared to their respective ideal coherent receivers when the CSI is perfectly known, so it is of great importance to devise new detection scheme for the DUSTM to improve system performance. In this paper, we present an expectation-maximization (EM)-based detection scheme for the DUSTM system.

The EM algorithm^[9] is a general approach for computing maximum likelihood (ML) estimates. For single-antenna channels, receivers based on the EM algorithm have been shown to perform well under fast fading^[10] and multipath fading^[11] conditions. In particular, EM-based detectors for space-time block codes have been derived in Refs. [12 – 13]. Ref. [14] proposed an iterative receiver for differential STBC using the EM algorithm; however, since the differential STBC employed was based on the Alamouti transmit diversity scheme^[15], only two transmit antennas were assumed.

In this paper, we employ the EM algorithm to derive an iterative detection scheme for DUSTM systems. We consider the detection of a differential unitary space-time constellation proposed in Ref. [8]. The scheme which we propose employs two steps. In the first step of our scheme, a data sequence is estimated by conventional differential unitary space-time demodulation (DUSTD) and is differentially encoded again

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Biographies: Du Zhengfeng (1976—), male, graduate; Gao Xiqi (corresponding author), male, doctor, professor, xqgao@seu.edu.cn.

to produce an initial estimate of the transmitted symbol stream. In the second step, the estimation of the symbol stream obtained from the first step is utilized to initialize an EM-based iterative detector. In each iteration, the most recent detected symbol stream is used to estimate the fading coefficients, which are then utilized to implement coherent detection to renew the detected symbol stream. The process of iterations improves the overall performance of the DUSTM system. Simulation results show that the detection scheme we propose performs better than the conventional DUSTD.

1 DUSTM and System Model

Consider a communication link comprising M transmitter antennas and N receiver antennas and operating in a Rayleigh flat-fading environment. Let s_m denote the transmitted signal by transmitter antenna $m = 1, 2, \dots, M$ at time slot $t = 1, 2, \dots, T$ and satisfy the power constraint $E \left[\sum_{m=1}^M |s_m|^2 \right] = 1$. The received signal x_m by the receiver antenna $n = 1, 2, \dots, N$ at time slot $t = 1, 2, \dots, T$ is given by

$$x_m = \sqrt{\rho} \sum_{n=1}^M h_{mn} s_m + w_m \quad (1)$$

where h_{mn} is the complex-valued fading coefficient between the m -th transmitter antenna and the n -th receiver antenna. The fading coefficients are assumed to be independent with respect to both m and n , and are CN $(0, 1)$ -distributed (complex normal zero-mean unit-variance distribution where the real and imaginary components of each random variable are independent and each has a variance $1/2$). In addition, h_{mn} is assumed to be constant over P symbol intervals, where P is the length of the received frame. In other words, a quasi-static fading case is considered in this paper. w_m is the additive noise at time t and at receiver antenna n , which is independently identically distributed CN $(0, 1)$, with respect to both t and n . ρ is the average signal-to-noise ratio (SNR) per receiver antenna. Alternatively we can write this in matrix form as

$$\mathbf{X}_i = \sqrt{\rho} \mathbf{S}_i \mathbf{H} + \mathbf{W}_i \quad (2)$$

where \mathbf{X}_i is the $T \times N$ symbol matrix of received signals x_m , \mathbf{S}_i is the $T \times M$ symbol matrix of transmitted signals s_m , and the subscript i denotes the i -th symbol within a frame; \mathbf{H} is the $M \times N$ matrix of the Rayleigh fading coefficients h_{mn} ; and \mathbf{W}_i is the $T \times N$ matrix of additive noise w_m . Now let matrices $\mathbf{S} = \{\mathbf{S}_1^\dagger, \dots, \mathbf{S}_P^\dagger\}^\dagger$ and $\mathbf{X} = \{\mathbf{X}_1^\dagger, \dots, \mathbf{X}_P^\dagger\}^\dagger$ denote the transmitted and received symbols of one entire frame, respectively, where “ \dagger ” denotes complex conjugate transpose, then the system be-

comes

$$\mathbf{X} = \sqrt{\rho} \mathbf{S} \mathbf{H} + \mathbf{W} \quad (3)$$

where $\mathbf{W} = \{\mathbf{W}_1^\dagger, \dots, \mathbf{W}_P^\dagger\}^\dagger$. For simplicity, we assume $T = M$ in this paper. Suppose that z_1, z_2, \dots with $z_t \in \{0, 1, \dots, L-1\}$ is a data sequence to be transmitted, each z_t corresponding to a constellation matrix from the constellation set $\{\Psi_l\}$, $l = 0, 1, \dots, L-1$, where $L = 2^{RM}$ for a data rate of R bit per channel use. The transmitter sends the symbol stream S_1, S_2, \dots , which is determined by the following fundamental differential transmission equation^[8]

$$\mathbf{S}_t = \Psi_{z_t} \mathbf{S}_{t-1} \quad t = 1, 2, \dots \quad (4)$$

where $\mathbf{S}_0 = \mathbf{I}_M$ and \mathbf{I}_M denotes the $M \times M$ identity matrix. At the receiver, the maximum-likelihood (ML) demodulator for DUSTM is given by^[8]

$$\hat{z}_t = \arg \min_{l=1,2,\dots,L-1} \|\mathbf{X}_t - \Psi_l \mathbf{X}_{t-1}\| \quad (5)$$

where $\|\cdot\|$ is the Frobenius norm which is defined as $\|\mathbf{A}\| = \sqrt{\text{tr}(\mathbf{A}^\dagger \mathbf{A})}$ and $\text{tr}(\cdot)$ denotes the trace operator.

2 New Detection Scheme for DUSTM

Due to the performance loss with the conventional DUSTD compared to the related coherent detection, we devise a new detection scheme for DUSTM in this section. In the first step of the proposed scheme, an initial estimate of the transmitted data sequence is derived by the conventional DUSTD, and then differentially encoded again to generate the estimate of the transmitted symbol stream, which is to be used to initialize an EM-based iterative detector in the next step. In the second step, we employ the EM algorithm for ML estimation to derive an iterative detector for the DUSTM system to improve overall performance. Fig. 1 provides an overview of the proposed detection scheme for DUSTM.

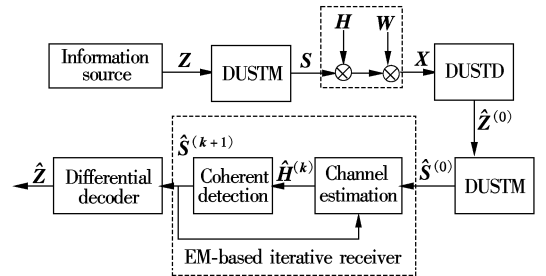


Fig. 1 Diagram of the proposed detection scheme

2.1 Initial estimate of the transmitted symbol stream

As shown in Fig. 1, $\hat{\mathbf{Z}}^{(0)} = \hat{z}_1^{(0)}, \hat{z}_2^{(0)}, \dots$ is an initial estimate of the data sequence, which can be obtained from the ML detector for DUSTM, i. e., Eq. (5), and is then differentially encoded to generate the initial estimate of the transmitted symbol stream $\hat{\mathbf{S}}_1^{(0)}$,

$\hat{\mathbf{S}}_2^{(0)}, \dots$. This estimate of the symbol stream is used to initialize the iterative receiver in the next step.

2.2 Iterative detection via EM algorithm

A detailed description of the EM algorithm can be found in Ref. [9]. Here we give a brief overview of the algorithm. Suppose that we wish to estimate a parameter $\mathbf{S} \in \mathcal{S}$ based on an observation \mathbf{X} . The EM algorithm, which is an iterative procedure, considers some larger sets of data \mathbf{Y} . Then we wish to find \mathbf{S} to maximize $\log f(\mathbf{Y} | \mathbf{S})$, but we do not have the data \mathbf{Y} to compute the log-likelihood. So instead, we maximize the expectation of $\log f(\mathbf{Y} | \mathbf{S})$ given the observation \mathbf{Y} and the current estimate of \mathbf{S} . The data \mathbf{X} and \mathbf{Y} are referred to incomplete data and complete data, respectively. Each iteration of the EM algorithm consists of two steps: an expectation step (E-step) and a maximization step (M-step). Let $\hat{\mathbf{S}}^{(k)}$ indicate the estimate of \mathbf{S} after the k -th iteration, $k = 1, 2, \dots$. The E-step computes

$$Q(\mathbf{S} | \hat{\mathbf{S}}^{(k)}) = E[\log p(\mathbf{Y} | \mathbf{S}) | \mathbf{X}, \hat{\mathbf{S}}^{(k)}] \quad (6)$$

where the expectation is with the conditional probability density function (pdf) $p(\mathbf{Y} | \mathbf{X}, \hat{\mathbf{S}}^{(k)})$. The M-step then computes the next estimate by

$$\hat{\mathbf{S}}^{(k+1)} = \arg \max_{\mathbf{S} \in \mathcal{S}} Q(\mathbf{S} | \hat{\mathbf{S}}^{(k)}) \quad (7)$$

Each iteration is guaranteed to increase the likelihood and the algorithm is guaranteed to converge to a local maximum of the likelihood function^[16].

For the problem at hand, let the complete data \mathbf{Y} be the incomplete data $\mathbf{X}_1, \mathbf{X}_2, \dots$ along with the fading coefficients \mathbf{H} , and let the unknown parameter to be estimated be the transmitted symbol stream $\mathbf{S}_1, \mathbf{S}_2, \dots$, where \mathbf{S}_i and \mathbf{X}_i are the i -th transmitted and received symbol matrices respectively within one frame. At the E-step of the iteration, the detector estimates the fading coefficients, conditioned on the received signals and the most recent estimate of the transmitted symbols. The E-step of the EM algorithm then yields

$$Q(\mathbf{S} | \hat{\mathbf{S}}^{(k)}) = E[\log p(\mathbf{X}, \mathbf{H} | \mathbf{S}) | \mathbf{X}, \hat{\mathbf{S}}^{(k)}] \quad (8)$$

Since \mathbf{H} and \mathbf{S} are independent matrices, the joint pdf $p(\mathbf{X}, \mathbf{H} | \mathbf{S})$ in Eq. (8) can be factored as

$$p(\mathbf{X}, \mathbf{H} | \mathbf{S}) = p(\mathbf{X} | \mathbf{H}, \mathbf{S}) p(\mathbf{H}) \quad (9)$$

The pdf $p(\mathbf{X} | \mathbf{H}, \mathbf{S})$ is given by

$$p(\mathbf{X} | \mathbf{H}, \mathbf{S}) = \frac{1}{\pi^{TN}} \exp\{-\text{tr}\{(\mathbf{X} - \sqrt{\rho}\mathbf{S}\mathbf{H})(\mathbf{X} - \sqrt{\rho}\mathbf{S}\mathbf{H})^\dagger\}\} \quad (10)$$

Then, substituting Eqs. (9) and (10) into Eq. (8) and dropping irrelevant factors finally yields

$$Q(\mathbf{S} | \hat{\mathbf{S}}^{(k)}) = \text{tr}\left\{\text{Re}(\mathbf{S}\hat{\mathbf{H}}^{(k)}\mathbf{X}^\dagger) - \frac{1}{2}\mathbf{S}\hat{\mathbf{\Omega}}^{(k)}\mathbf{S}^\dagger\right\} \quad (11)$$

where

$$\hat{\mathbf{H}}^{(k)} = E[\mathbf{H} | \mathbf{X}, \hat{\mathbf{S}}^{(k)}] = \frac{\sqrt{\rho}}{\rho P + 1} \sum_{i=1}^P [\hat{\mathbf{S}}_i^{(k)}]^\dagger \mathbf{X}_i \quad (12)$$

and

$$\hat{\mathbf{\Omega}}^{(k)} = E[\mathbf{H}\mathbf{H}^\dagger | \mathbf{X}, \hat{\mathbf{S}}^{(k)}] = \frac{1}{\rho P + 1} \mathbf{I} + \hat{\mathbf{H}}^{(k)} (\hat{\mathbf{H}}^{(k)})^\dagger \quad (13)$$

where $\hat{\mathbf{H}}^{(k)}$ and $\hat{\mathbf{\Omega}}^{(k)}$ are respectively the conditional mean and the second moment of the fading coefficients, given the received symbol $\mathbf{X}_1, \dots, \mathbf{X}_P$ and the most recent estimate of the transmitted symbols $\mathbf{S}_1^{(k)}, \dots, \mathbf{S}_P^{(k)}$ at the k -th iteration.

The M-step of the iteration then performs coherent data detection assuming that the channel estimation at the E-step is correct; i. e., finds $\hat{\mathbf{S}}^{(k+1)}$ that maximize $Q(\mathbf{S} | \hat{\mathbf{S}}^{(k)})$

$$\begin{aligned} \hat{\mathbf{S}}^{(k+1)} &= \arg \max_{\mathbf{S} \in \mathcal{S}} Q(\mathbf{S} | \hat{\mathbf{S}}^{(k)}) = \\ &\arg \max_{\mathbf{S} \in \mathcal{S}} \text{tr}\left\{\text{Re}(\mathbf{S}\hat{\mathbf{H}}^{(k)}\mathbf{X}^\dagger) - \frac{1}{2}\mathbf{S}\hat{\mathbf{\Omega}}^{(k)}\mathbf{S}^\dagger\right\} = \\ &\arg \max_{\mathbf{S} \in \mathcal{S}} \text{tr}\left\{\text{Re}\left(\sum_{i=1}^P \mathbf{S}_i \hat{\mathbf{H}}^{(k)} \mathbf{X}_i^\dagger\right)\right\} \end{aligned} \quad (14)$$

where \mathcal{S} is the collection of all possible transmitted symbol streams and $\text{Re}(\cdot)$ is the real operator. In other words, the M-step updates the decision on the symbol stream according to the most recent estimate of the fading coefficients. From the M-step of Eq. (14), it is known that the metric for optimization to find $\hat{\mathbf{S}}^{(k+1)}$ is independent of $\hat{\mathbf{\Omega}}^{(k)}$, so the E-step can be implemented only by computing Eq. (12). Since the received signal is known, the fading gain estimate is assumed to be correct, and the transmitted symbols are effectively memoryless^[14], the M-step can be solved on a symbol-by-symbol basis

$$\hat{\mathbf{S}}_i^{(k+1)} = \arg \max_{\mathbf{S}_i \in \mathcal{S}} \text{tr}\{\text{Re}(\mathbf{S}_i \hat{\mathbf{H}}^{(k)} \mathbf{X}_i^\dagger)\} \quad i = 1, 2, \dots, P \quad (15)$$

where \mathcal{S} is the collection of all possible transmitted symbols. The EM-based iterative receiver consists of choosing an initial value, then performing the E-step and the M-step successively. The iterations cease when the estimate of the symbol stream does not change during two subsequent iterations, or after a specified number of iterations. Finally, the estimate of the data sequence can be obtained by differentially decoding $\hat{\mathbf{S}}_1, \dots, \hat{\mathbf{S}}_P$.

At the same time, it should be pointed out that the proposed detection scheme introduces the latency and additional processing complexity at the receiver due to the EM-based iterative detection in the second step. Specifically, the complexity of the conventional

algorithm for computing Eq. (15) is $O(M^3)$ when $M = N$. However, compared with the conventional coherent detection scheme with training, the proposed scheme avoids the overhead used for channel sounding during transmitting. Since the bandwidth is a scarce resource and the training overhead may be excessive, especially for multiple-antenna communication, and with more and more powerful processing units emerging, obtaining higher bandwidth efficiency at the cost of some computing complexity is sometimes appealing.

3 Performance Simulations

We now apply the proposed detection scheme to the DUSTM system where the channel has unknown Rayleigh flat fading coefficients. For simplicity, in this paper we use a simple unitary space-time constellation proposed in Ref. [8].

$$\Psi_l = \text{diag} \left(e^{\frac{j2\pi k_1 l}{L}}, e^{\frac{j2\pi k_2 l}{L}}, \dots, e^{\frac{j2\pi k_M l}{L}} \right) \quad l = 0, 1, \dots, L-1 \quad (16)$$

where j denotes the imaginary unit and k_1, k_2, \dots, k_M are optimized integer parameters to achieve the maximum diversity product^[8]. The frame length $P = 1024$ is used in this paper. The symbol error rate (SER) performances as a function of the signal-to-noise ratio (SNR) when $M = 2$ or $M = 3$ are shown in Fig. 2. The integer q in the notation EM- q refers to the number of EM iterations. For simplicity, we assume that the data rate per channel use $R = 1$, which means $L = 2^M$. For comparison, we also plot the performance curves of the conventional DUSTD and give the results of the ML coherent detector when the channel \mathbf{H} is known perfectly at the receiver as a lower bound.

As shown in Fig. 2, the conventional DUSTD suffers approximately a 3 dB performance loss compared with its respective ideal coherent one when \mathbf{H} is known. The proposed detection scheme performs much better than the conventional DUSTD after a few iterations. Specifically, at a SER of 10^{-3} in Fig. 2 (e) when $M = 3$ and $N = 3$, the performance of the proposed detection scheme is roughly 1 dB better than the conventional DUSTD when the number of EM iterations is 4, and an approximately 1.9 dB gain is obtained when the number of EM iterations is 8. It is also obvious from these figures that the performance of the proposed detection scheme has moved a significant step toward the lower bound given by the ML coherent detector.

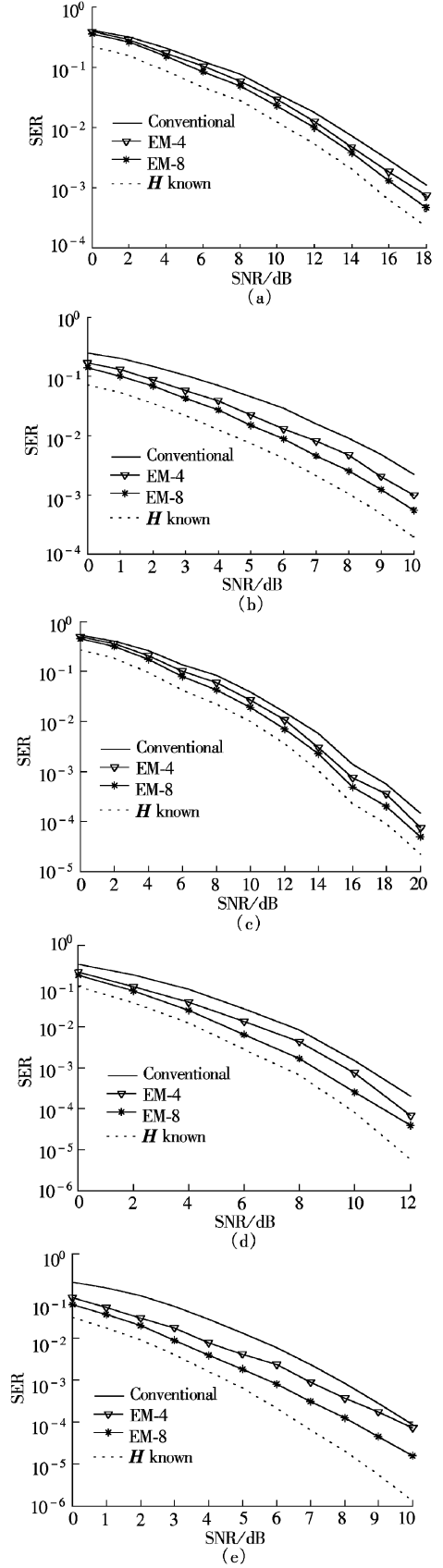


Fig. 2 The SER performances as a function of SNR. (a) $M = 2$ and $N = 1$; (b) $M = 2$ and $N = 2$; (c) $M = 3$ and $N = 1$; (d) $M = 3$ and $N = 2$; (e) $M = 3$ and $N = 3$

4 Conclusion

In this paper, a new detection scheme based on the EM algorithm is proposed for DUSTM, when neither the transmitter nor the receiver knows the channel fading coefficients. A data sequence produced by the conventional DUSTD is differentially recoded to obtain the initial estimate of the transmitted symbol stream in the first step, and then in the next step the EM algorithm is employed to derive an iterative detector, which implements channel estimation and coherent detection in each iteration, to improve the overall system performance, using the estimate of the transmitted symbol stream obtained from the first step as its initial value. We compare the performance of the proposed detection scheme with the conventional DUSTD and ML coherent detector with perfectly known fading coefficients through simulating. Simulation results demonstrate that the proposed detection scheme performs better than the conventional DUSTD. With the proposed detection scheme, the performance loss of the DUSTM system when compared to the related coherent detection is greatly reduced.

References

- [1] Telatar I E. Capacity of multi-antenna Gaussian channels [J]. *Eur Trans Telecommun*, 1999, **10**(6): 585 – 595.
- [2] Foschini G J, Gans M J. On limits of wireless communications in a fading environment when using multiple antennas [J]. *Wireless Pers Commun*, 1998, **6**(3): 311 – 335.
- [3] Marzetta T L, Hochwald B M. Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading [J]. *IEEE Trans Inform Theory*, 1999, **45**(1): 139 – 157.
- [4] Tarokh V, Jafarkhani H. A differential detection scheme for transmit diversity [J]. *IEEE J Select Areas Commun*, 2000, **18**(7): 1169 – 1174.
- [5] Jafarkhani H, Tarokh V. Multiple transmit antenna differential detection from generalized orthogonal designs [J]. *IEEE Trans Inform Theory*, 2001, **47**(9): 2626 – 2631.
- [6] Hochwald B M, Marzetta T L. Unitary space-time modulation for multiple-antenna communication in Rayleigh flat-fading [J]. *IEEE Trans Inform Theory*, 2000, **46**(2): 543 – 564.
- [7] Hochwald B M, Marzetta T L, Richardson T J, et al. Systematic design of unitary space-time constellations [J]. *IEEE Trans Inform Theory*, 2000, **46**(6): 1962 – 1973.
- [8] Hochwald B M, Sweldens W. Differential unitary space-time modulation [J]. *IEEE Trans Commun*, 2000, **48**(12): 2041 – 2052.
- [9] Dempster A P, Laird N M, Rubin D B. Maximum likelihood from incomplete data via the EM algorithm [J]. *J Royal Statist Soc: Series B*, 1977, **39**(1): 1 – 38.
- [10] Georgiades C N, Han J C. Sequence estimation in the presence of random parameters via the EM algorithm [J]. *IEEE Trans Commun*, 1997, **45**(3): 300 – 308.
- [11] Kaleh G. Joint parameter estimation and symbol detection for linear and nonlinear unknown channels [J]. *IEEE Trans Commun*, 1994, **42**(7): 2406 – 2413.
- [12] Li Y, Georgiades C N, Huang G. Iterative maximum-likelihood sequence estimation for space-time coded systems [J]. *IEEE Trans Commun*, 2001, **49**(6): 948 – 951.
- [13] Cozzo C, Hughes B L. Joint channel estimation and data detection in space-time communications [J]. *IEEE Trans Commun*, 2003, **51**(8): 1266 – 1270.
- [14] Riediger M L B, Ho P K M. A differential space-time code receiver using the expectation maximization algorithm [J]. *Canadian Journal of Electrical and Computer Engineering*, 2004, **29**(4): 227 – 230.
- [15] Alamouti S. A simple transmit diversity technique for wireless communications [J]. *IEEE J Select Areas Commun*, 1998, **16**(8): 1451 – 1458.
- [16] Wu C F. On the convergence properties of the EM algorithm [J]. *Ann Stat*, 1983, **11**(1): 95 – 103.

基于 EM 算法的差分酉空时调制检测方案

杜正锋 陈 杰 潘 文 高西奇

(东南大学移动通信国家重点实验室, 南京 210096)

摘要:与相干检测相比传统差分检测会带来约 3 dB 的性能损失. 提出一种新的差分酉空时调制检测方案. 该方案分为 2 步: 首先将传统差分检测获得的数据序列进行差分再编码, 作为对发送符号序列的初始估计; 然后由期望最大化(EM)算法进行迭代检测, 利用上一步得到的发送符号序列的初始估计值作为 EM 算法的初始值. 在每一次迭代时, 最新检测到的发送符号序列用来进行信道估计, 随后利用估计出的信道实施相干序列检测, 进一步提高对发送符号序列检测的准确性. 仿真结果表明, 经过几次迭代后, 提出的检测方案性能大大优于传统的差分酉空时调制检测方案.

关键词:酉空时调制; 差分检测; 期望最大化(EM)算法

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